1.9 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. If the composite functions $f(g(x))$ and $g(f(x))$ both equal x, then the function g is the ________ function of f.
2. The inverse function of f is denoted by ________.
3. The domain of f is the ________ of f^{-1}, and the ________ of f^{-1} is the range of f.
4. The graphs of f and f^{-1} are reflections of each other in the line ________.
5. A function f is ________ when each value of the dependent variable corresponds to exactly one value of the independent variable.
6. A graphical test for the existence of an inverse function of f is called the _______ Line Test.

Skills and Applications

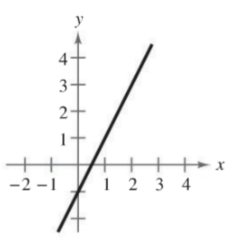
Finding an Inverse Function Informally In Exercises 7–12, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

7. $f(x) = 6x$
8. $f(x) = \frac{1}{3}x$
9. $f(x) = 3x + 1$
10. $f(x) = \dfrac{x-1}{5}$
11. $f(x) = \sqrt[3]{x}$
12. $f(x) = x^5$

Verifying Inverse Functions In Exercises 13–16, verify that f and g are inverse functions.

13. $f(x) = -\dfrac{7}{2}x - 3, \quad g(x) = -\dfrac{2x+6}{7}$
14. $f(x) = \dfrac{x-9}{4}, \quad g(x) = 4x + 9$
15. $f(x) = x^3 + 5, \quad g(x) = \sqrt[3]{x-5}$
16. $f(x) = \dfrac{x^3}{2}, \quad g(x) = \sqrt[3]{2x}$

Sketching the Graph of an Inverse Function In Exercises 17–20, use the graph of the function to sketch the graph of its inverse function $y = f^{-1}(x)$.

17.


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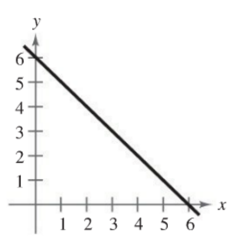

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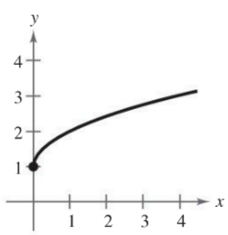

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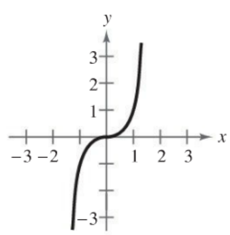


Verifying Inverse Functions In Exercises 21–32, verify that f and g are inverse functions (a) algebraically and (b) graphically.

21. $f(x) = 2x, \quad g(x) = \dfrac{x}{2}$
22. $f(x) = x - 5, \quad g(x) = x + 5$
23. $f(x) = 7x + 1, \quad g(x) = \dfrac{x-1}{7}$
24. $f(x) = 3 - 4x, \quad g(x) = \dfrac{3-x}{4}$
25. $f(x) = \dfrac{x^3}{8}, \quad g(x) = \sqrt[3]{8x}$
26. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x}$
27. $f(x) = \sqrt{x-4}, \quad g(x) = x^2 + 4, \quad x \ge 0$
28. $f(x) = 1 - x^3, \quad g(x) = \sqrt[3]{1-x}$
29. $f(x) = 9 - x^2, \quad x \ge 0, \quad g(x) = \sqrt{9-x}, \quad x \le 9$
30. $f(x) = \dfrac{1}{1+x}, \quad x \ge 0, \quad g(x) = \dfrac{1-x}{x}, \quad 0 < x \le 1$
31. $f(x) = \dfrac{x-1}{x+5}, \quad g(x) = -\dfrac{5x+1}{x-1}$
32. $f(x) = \dfrac{x+3}{x-2}, \quad g(x) = \dfrac{2x+3}{x-1}$

Using a Table to Determine an Inverse Function In Exercises 33 and 34, does the function have an inverse function?

33.

x	−1	0	1	2	3	4
$f(x)$	−2	1	2	1	−2	−6

34.

x	−3	−2	−1	0	2	3
$f(x)$	10	6	4	1	−3	−10

Using a Table to Find an Inverse Function In Exercises 35 and 36, use the table of values for $y = f(x)$ to complete a table for $y = f^{-1}(x)$.

35.

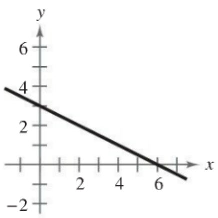
x	-2	-1	0	1	2	3
$f(x)$	-2	0	2	4	6	8

36.

x	-3	-2	-1	0	1	2
$f(x)$	-10	-7	-4	-1	2	5

Applying the Horizontal Line Test In Exercises 37–40, does the function have an inverse function?

37.



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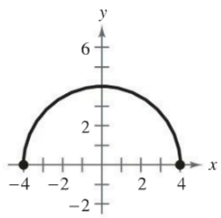

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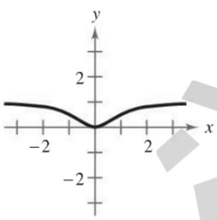
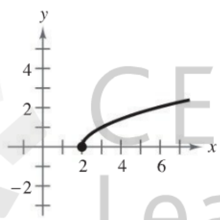


40.



Applying the Horizontal Line Test In Exercises 41–44, use a graphing utility to graph the function, and use the Horizontal Line Test to determine whether the function has an inverse function.

41. $g(x) = (x + 5)^3$

42. $f(x) = \frac{1}{8}(x + 2)^2 - 1$

43. $f(x) = -2x\sqrt{16 - x^2}$

44. $h(x) = |x + 4| - |x - 4|$

Finding and Analyzing Inverse Functions In Exercises 45–56, (a) find the inverse function of f, (b) graph both f and f^{-1} on the same set of coordinate axes, (c) describe the relationship between the graphs of f and f^{-1}, and (d) state the domains and ranges of f and f^{-1}.

45. $f(x) = 2x - 3$

46. $f(x) = 3x + 1$

47. $f(x) = x^5 - 2$

48. $f(x) = x^3 + 1$

49. $f(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2$

50. $f(x) = x^2 - 2, \quad x \le 0$

51. $f(x) = \dfrac{4}{x}$

52. $f(x) = -\dfrac{2}{x}$

53. $f(x) = \dfrac{x + 1}{x - 2}$

54. $f(x) = \dfrac{x - 3}{x + 2}$

55. $f(x) = \sqrt[3]{x - 1}$

56. $f(x) = x^{3/5}$

Finding an Inverse Function In Exercises 57–72, determine whether the function has an inverse function. If it does, then find the inverse function.

57. $f(x) = x^4$

58. $f(x) = \dfrac{1}{x^2}$

59. $g(x) = \dfrac{x}{8}$

60. $f(x) = 3x + 5$

61. $p(x) = -4$

62. $f(x) = \dfrac{3x + 4}{5}$

63. $f(x) = (x + 3)^2, \quad x \ge -3$

64. $q(x) = (x - 5)^2$

65. $f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \ge 0 \end{cases}$

66. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$

67. $h(x) = -\dfrac{4}{x^2}$

68. $f(x) = |x - 2|, \quad x \le 2$

69. $f(x) = \sqrt{2x + 3}$

70. $f(x) = \sqrt{x - 2}$

71. $f(x) = \dfrac{6x + 4}{4x + 5}$

72. $f(x) = \dfrac{5x - 3}{2x + 5}$

Restricting the Domain In Exercises 73–82, restrict the domain of the function f so that the function is one-to-one and has an inverse function. Then find the inverse function f^{-1}. State the domains and ranges of f and f^{-1}. Explain your results. (There are many correct answers.)

73. $f(x) = (x - 2)^2$

74. $f(x) = 1 - x^4$

75. $f(x) = |x + 2|$

76. $f(x) = |x - 5|$

77. $f(x) = (x + 6)^2$

78. $f(x) = (x - 4)^2$

79. $f(x) = -2x^2 + 5$

80. $f(x) = \frac{1}{2}x^2 - 1$

81. $f(x) = |x - 4| + 1$

82. $f(x) = -|x - 1| - 2$

Composition with Inverses In Exercises 83–88, use the functions $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$ to find the indicated value or function.

83. $(f^{-1} \circ g^{-1})(1)$

84. $(g^{-1} \circ f^{-1})(-3)$

85. $(f^{-1} \circ f^{-1})(6)$

86. $(g^{-1} \circ g^{-1})(-4)$

87. $(f \circ g)^{-1}$

88. $g^{-1} \circ f^{-1}$

Composition with Inverses In Exercises 89–92, use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the specified function.

89. $g^{-1} \circ f^{-1}$

90. $f^{-1} \circ g^{-1}$

91. $(f \circ g)^{-1}$

92. $(g \circ f)^{-1}$