AP Calculus BC Syllabus

In AP Calculus BC, students study Functions, Graphs, and Limits; Derivatives and their Applications; Integrals and their Applications; and Polynomial Approximations and Series. Calculus pulls together many of the concepts the students have studied in previous courses, and it also helps them to see the relevance of the material they were taught prior to Calculus. I believe that AP Calculus BC gives the students a strong foundation for the math and science courses they will take in college.

Course Planner

First Semester

Chapter 1: Limits and Their Properties (15 days – 1 Exam)

- Review of Pre-Calculus
- An introduction to limits, including an intuitive understanding of the limit process
- Using graphs and tables of data to determine limits
- Properties of limits
- Algebraic techniques for evaluating limits
- Comparing relative magnitudes of functions and their rates of change
- Continuity and one-sided limits
- Geometric understanding of the graphs of continuous functions
- Intermediate Value Theorem
- Infinite limits
- Using limits to find the asymptotes of a function

Chapter 2: Differentiation (30 days – 2 Exams)

- Zooming-in activity and local linearity
- Understanding of the derivative – graphically, numerically, and analytically
- Approximating rates of change from graphs and tables of data
- The derivative as: the limit of the average rate of change, an instantaneous rate of change, limit of the difference quotient, and the slope of a curve at a point
- The meaning of the derivative -- translating verbal descriptions into equations and vice versa
- The relationship between differentiability and continuity
- Functions which have a vertical tangent at a point
- Functions which have a point on which there is no tangent
- Differentiation rules for basic functions, including power functions and trigonometric functions
- Rules of differentiation for sums, differences, products, and quotients
- The chain rule
- Implicit differentiation
- Related rates

Chapter 3: Applications of Differentiation (32 days – 1 Exam)

- Extrema on an interval and the Extreme Value Theorem

- Rolle's Theorem and the Mean Value Theorem and their geometric consequences
- Lab on the First Derivative Test
- Increasing and decreasing functions and the First Derivative Test
- Lab on concavity and points of inflection
- Concavity and its relationship to the first and second derivatives
- Second Derivative Test
- Limits at infinity
- A summary of curve sketching---using geometric and analytic information as well as calculus to predict the behavior of a function
- Relating the graphs of f, f', and f''
- Optimization including both relative and absolute extrema
- Tangent line to a curve and linear approximations
- Newton's Method
- Differentials
- Application problems including position, velocity, acceleration, and rectilinear motion

Chapter 4: Integration (15 days – 2 Exams)

- Antiderivatives and indefinite integration, including antiderivatives following directly from derivatives of basic functions
- Basic properties of the definite integral
- Area under a curve
- Meaning of the definite integral
- Definite integral as a limit of Riemann sums
- Riemann sums, including left, right, and midpoint sums
- Trapezoidal sums
- Use of Riemann sums and trapezoidal sums to approximate definite integrals of functions that are represented analytically, graphically, and by tables of data
- Discovery lesson on the First Fundamental Theorem of Calculus
- Use of the First Fundamental Theorem to evaluate definite integrals
- Use of substitution of variables to evaluate definite integrals
- Integration by substitution
- Discovery lesson on the Second Fundamental Theorem of Calculus
- The Second Fundamental Theorem of Calculus and functions defined by integrals
- The Mean Value Theorem for Integrals and the average value of a function

Chapter 5: Logarithmic, Exponential, and Other Transcendental Functions (21 days – 1 Exam)

- The natural logarithmic function and differentiation
- The natural logarithmic function and integration
- Inverse functions
- Exponential functions: differentiation and integration
- Bases other than *e* and applications

- Inverse trig functions and differentiation
- Inverse trig functions and integration

Chapter 6: Differential Equations (12 days – 1 Exam)

- Solving separable differential equations
- Applications of differential equations in modeling, including exponential growth
- Use of slope fields to interpret a differential equation geometrically
- Drawing slope fields and solution curves for differential equations

Chapter 7: Applications of Integration (20 days – 1 Exam)

- Review of basic integration rules
- The integral as an accumulator of rates of change
- Area of a region between two curves
- Volume of a solid with known cross sections
- Volume of solids of revolution
- Arc length
- Applications of integration in physical, biological, and economic contexts
- Applications of integration in problems involving a particle moving along a line, including the use of the definite integral with an initial condition and using the definite integral to find the distance traveled by a particle along a line

First Semester Exam (two review days)
Second Semester

Chapter 8: Integration Techniques, L'Hopital's Rule, and Improper Integrals (17 days—2 exams)

- Review of basic integration rules
- Integration by parts
- Trigonometric integrals
- Integration by partial fractions
- Solving logistic differential equations and using them in modeling
- Discovery lab on L'Hopital's Rule
- L'Hopital's Rule and its use in determining limits
- Discovery activity on improper integrals
- Improper integrals and their convergence and divergence, including the use of
    - L'Hopital's Rule

Chapter 9: Infinite Series (17 days— 2 exams)

- Lab on Sequences
- Convergence and divergence of sequences
- Definition of a series as a sequence of partial sums

- Convergence of a series defined in terms of the limit of the sequence of partial sums of a series
- Introduction to convergence and divergence of a series by using technology on two examples to gain an intuitive understanding of the meaning of convergence
- Geometric series and applications
- The nth-Term Test for Divergence
- The Integral Test and its relationship to improper integrals and areas of rectangles
- Use of the Integral Test to introduce the test for p-series
- Comparisons of series
- Alternating series and the Alternating Series Remainder
- The Ratio and Root Tests
- Taylor polynomials and approximations---introduction using the graphing calculator
- Power series and radius and interval of convergence
- Taylor and Maclaurin series for a given function
- Maclaurin series for sin x, cos x,
- Manipulation of series, including substitution, addition of series, multiplication of series by a constant and/or a variable, differentiation of series, integration of series, and forming a new series from a known series
- Taylor's Theorem with the Lagrange Form of the Remainder (Lagrange Error Bound)

Chapter 10: Plane Curves, Parametric Equations, and Polar Curves (12 days—2 exams)

- Plane curves and parametric equations
- Parametric equations and Calculus
- Parametric equations and vectors—motion along a curve, position, velocity, acceleration,
    - speed, distance traveled
- Analysis of curves given in parametric and vector form
- Polar coordinates and polar graphs
- Analysis of curves given in polar form
- Area of a region bounded by polar curves

AP Exam Review (minimum of 15 days – 1 exam)
AP Exam

After the AP Exam:

- Appendix C: Differential Equations (6 days—1 exam)
    - Definitions and basic concepts of differential equations
    - First order linear differential equations
    - Second order homogeneous linear equations
- Chapter 5: Hyperbolic Functions (4 days—1 exam)
    - Hyperbolic functions and applications

Second Semester Exam (two review days)

Teaching Strategies

Graphing Calculator. Many of the exercises described above rely heavily on the use of the graphing calculator. The calculator helps students develop a visual understanding of the material that they would not otherwise have.

Examples:
1) When I introduce limits, I have the students use their graphing calculator to make a table of values to help them determine the limit of $f(x) = \frac{\sin(3x)}{x}$ as x gets closer and closer to zero.

2) When the students are introduced to the Squeeze Theorem, I have them graph $y = x,\ y = -x,\ \text{and}\ y = x\cos\left(\frac{50\pi}{x}\right)$ in radian mode on the same screen over the x-interval from -1 to 1, and then they zoom in on the graph at $x = 0$ to see how the Squeeze Theorem can be used to find $\lim_{x\to 0} x\cos\left(\frac{50\pi}{x}\right)$.

3) We discover the meaning of local linearity and differentiability by using the zooming in feature of the calculator to zoom in on the graphs of several functions and to determine which ones will look linear if we zoom enough times and which ones will not.

4) When I introduce derivatives and local linearity, I have the students zoom in on a particular graph at a given point until their graph looks linear. Then they estimate the slope at the given point by using a second point very close to the original point. One group of students chooses a second point with a smaller *x*-coordinate then the original point, and another group chooses a second point with a larger *x*-coordinate. Then we discuss how we could estimate the actual value of the derivative at the given point by using our results and how the number of times the students zoomed in effects the accuracy of their estimate.

5) The students discover the relationship between the graph of a function and the graph of its derivative by graphing several functions and their derivatives on their calculators and then discussing what happens on the graph of the derivative for the points where the function is increasing and decreasing and where the function is concave up and concave down.

6) When we study Riemann sums and how they can be used to estimate the area bounded by a curve and the x-axis on a given interval, we use a calculator program to see how increasing the number of subintervals affects the accuracy of our estimate.

My students use the TI-89 graphing calculator almost every day in class and also on homework. However, many homework problems and about half of the problems on quizzes and tests are done without the use of the graphing calculator. Since the AP Exam is half calculator and half non-calculator, I feel that it is very important for students to have practice working problems both ways. We spend time in class discussions talking about the types of questions that they must know how to work *with* their calculators and the types of questions that they must know how to work *without* their calculators. We also discuss the techniques needed to use the calculator most efficiently (storing functions in the y = screen, storing values that will be used later in the problem, etc.).

Technology. I like to incorporate technology within the curriculum. I hope to allow students to explore different ways of learning by utilizing what is given to them. For example, students can use iPads to use for Geogebra to explore the limit of a process. Students can visit my website to get necessary homework and assignments.

Teacher In-service. Teachers attend a summer AP Calculus Conference which is invaluable as preparation in teaching AP Calculus. Sometimes, the district provides funds for the conference and sometimes the teacher pays their own registration. Teachers are consistently involved and improving our skills in covering the AP curriculum. Humble ISD also offers vertical alignment for all district teachers who teach the same class and collaborate at least once per semester.

Rule of Four. I give my students many opportunities to work problems presented in a variety of ways: graphical, numerical, analytical, and verbal. Many of the problems in my primary textbook are written with an analytical representation so I frequently supplement these problems with problems giving a graph or tabular data. Students are frequently given word problems in which they have the opportunity to apply the concepts they have learned to a real-world situation. Students are often asked for verbal explanations, both oral and written, to give them the opportunity to communicate their work and their reasoning in words. They are asked to discuss the meaning of their answers, particularly when working related rates problems, optimization problems, and applications of integration problems. I also ask them to discuss the steps they have used to arrive at their answers. They have additional opportunities for discussion when they work in small groups on the discovery exercises and when they present homework problems to the entire class.

Justification of Answers. I ask my students to justify their answers on homework, quizzes, and tests, and I prefer that they write the justifications in sentences. We talk a lot about the amount of work they need to show and about the correct way to justify their work on various types of problems.

Homework. My students have homework each night, usually requiring about 30 to 45 minutes of their time. In order to cover all of the Calculus AB topics, we move fairly quickly through each chapter; it is very important that students do their homework each night so that they gain the maximum benefit from the homework discussion that occurs the next day in class.

Assessment. Our school district requires a minimum of four major grades for every nine-week period. The formative score weighs 25% of their total and summative score weighs 75% of a student's nine week grade. All exams are under the summative score weight. Quizzes, projects, and homework are all under the formative category.

AP Review. During the third nine-week period, I start giving the students an AP Review Sheet each week, which consists of free-response questions and multiple choice questions from previous AP Exams. The students work a few problems each night along with their homework on the topic being studied. I try to pick questions from topics that the students learned several weeks (or months) earlier so that they are reviewing as they work the problems. For example, as they are learning applications of integration, I might give them a related rates question or a problem in which they are given a graph of the derivative and asked questions about it. On each major test, I also include a problem that is similar to the AP Review problems that they have worked on recently. My students feel that the AP Review Sheets provide them with constant review for the free-response section of the AP Exam, so they do not forget the topics they learned earlier in the year.

I try to allot a minimum of three weeks before the AP Exam to devote to review. During this three-week period, students work on the sample questions in the *AP Calculus Course Description* and on multiple-choice and free-response questions from Released Exams. Some of these are assigned for homework, while others are given as a quiz or test. The calculus teachers in my district also administer a practice AP Exam on a Saturday morning about a week and a half before the actual AP Exam is given. For the practice exam, we use the multiple-choice section from the most recent Released Exam and the previous year's free-response questions. I grade the practice exams, and we spend the next few days discussing the problems. The practice exam lets students see which topics they need to review. It also gives them an idea of how exhausting the "real" exam will be, so that they realize they need to get a good night's sleep before exam day and eat a good breakfast before taking the exam.

Teacher Resources

Primary Textbook
Calculus, 8th edition, by Houghton Mifflin, 2006, ISBN 0-618-50304-8

Supplementary Texts
I supplement quite a bit with materials from other calculus textbooks, AP Released Exams, and free-response questions from AP Central. My resources include the following textbooks:

- Anton, H. *Calculus—A New Horizon.* 7th edition, New York: John Wiley & Sons.
- Dick, T.P., and Charles M. Patton. *Calculus of a Single Variable.* Boston: PWS Publishing Company.
- Finney, R., et al. *Calculus—Graphical, Numerical, Algebraic.* 3rd edition, Menlo Park, Calif.: Scott-Foresman/Addison-Wesley.
- Foerster, P. A. *Calculus—Concepts and Applications.* Emeryville, Calif.: Key Curriculum Press.
- Hughes-Hallett, D. et al. *Calculus—Single Variable.* 4th edition. New York: John Wiley & Sons.
- Ostebee, A. and Zorn, P. *Calculus from Graphical, Numerical, and Symbolic Points of View*. 2nd ed. Boston: Houghton Mifflin.
- Rokowski, J. and Cannon, R. *Rowgawski's Calculus for AP.* 2nd edition, New York: W.H. Freeman and Company.
- Stewart, J. *Calculus: Concepts and Contexts*. 6th edition. Pacific Grove, Calif.: Brooks/Cole Publishing Co.

More Resources

- Edwards, B. H., Ron Larson, and Robert P. Hostetler. *Themes for Advanced Placement Calculus.* 8th ed. Boston: Houghton Mifflin.
- Foerster, P.A. *Instructor's Resource Book*. Emeryville, Calif.: Key Curriculum Press.

- Hockett, S. O. and Bock, D. *Barron's AP Calculus.* Hauppauge, NY : Barron's Group.

- Howell, M. and Montgomery, M. *Be Prepared for the AP Calculus Exam*, Skylight Publications.

- Kelley, M. *The Humongous Book of Calculus Problems*. New York : Penguin Group.

- Ruby, T., Sellers, J., Korf, L., Van Horn, J., and Munn, M. *Kaplan's AP Calculus AB & BC : 2008 Edition*. New York : Kaplan Publishing.

- Venture Publications
  - *AP Calculus with the TI-83*
  - *AP Calculus with the TI-89 Graphing Calculator*
  - *Preparing for the Calculus AB Exam*
  - Preparing for the Calculus BC Exam

- Wheater, C. *My Max Score AP Calculus AP/BC*. Naperville, IL : SourceBooks, Inc.