Calculus BC Review Sheet KEY – Fall Semester Rev 2018

Chapter 1: Limits

1a) $\frac{1}{6}$ 1b) 4, 1c) –1, 1d) $-\infty$, 1e) $\frac{1}{4}$, 1f) 125, 1g) DNE, 1h) 2, 1i) 3

2a) 3, 2b) 1, 2c) DNE, 2d) 3, 2e) DNE

3) f is continuous, $f(-3) \neq f(0)$, and $f(-3) < 0$ and $f(0) > 0$ f guarantees a zero in $I[-3,0]$

4) f is continuous, $f(a) \neq f(b)$, and $f(c) = k$ where c is in between a and b and $f(c)$ is in between $f(a)$ and $f(b)$

5) $f(c)$ is defined, limit of $f(x)$ where x approaches to c exists, and limit of $f(x)$ where it approaches to c equals $f(c)$

6a) –21, 6b) –125, 6c) 2, 6d) 0

7) $k = -2$

Chapter 2: Differentiation

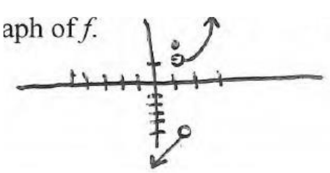
1) $y - 16 = 5(x - 2)$

2)
$$\lim_{h\to 0} \frac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h}$$
$$\lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 5h}{h}$$
$$\lim_{h\to 0} 4x + 2h - 5 = \boxed{4x - 5}$$

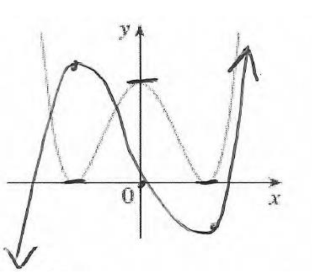
3a) No. $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

3b) No, differentiation implies continuity and the piecewise function is not continuous

3c)



4)



5a) $3x^2 - 6x$, 5b) $\frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$, 5c) $\frac{2x^3+2}{x^3}$,

5d) $-\frac{2}{(x-1)^2}$, 5e) e^{x+6}, 5f) $e^x x^5 + 5x^4 e^x$,

5g) $4\left(x^4 + 7x^2 - 3\right)^3\left(4x^3 + 14x\right)$,

5h) $\frac{-72x\left(x^2+3\right)^5}{(x^2-3)^7}$, 5i) $20x^3\cos(5x^4)$, 5j) $40x^3\sin\left(5x^4\right)\cos\left(5x^4\right)$

6a) –6, 6b) –9/8, 6c) 18

7a) 30, 7b) 90

8) 42

9a) 20, 9b) $\sec x\left(\sec^2 x + \tan^2 x\right)$, 9c) $4\left(\sin^2 x - \cos^2 x\right)$

10a) $\frac{-3x^2y - y^3}{x^3 + 3xy^2}$, 10b) $\frac{y-x}{3y-x}$, 10c) $\frac{e^y \sin x + y\cos(xy)}{e^y \cos x - x\cos(xy)}$

11) $\frac{dA}{dt} = 20\pi\ cm^2/\sec$

12) $\frac{dy}{dt} = -\frac{9}{4}\ ft/\sec$; The ladder is changing at a rate of $-\frac{9}{4} ft/sec$

13) Water is decreasing at a rate of $\frac{1}{2\pi} in/\min$

14a) –162 gal/min, 14b) 0 gal/min, 14c) $t = 0$ min, 14d) $t = 50$ min

Chapter 3: Applications

1) f has Absolute Maximum at $x = 4$ and Absolute Minimum at $x = \frac{2}{3}$

2) f has Absolute Minimum at $\left(\frac{\pi}{2}, 0\right)$ and Absolute Maximum at $(0, 2)$

3) EVT is closed interval with endpoints, MVT is Slope of secant line is slope of tangent line (derivative)

4) $c = 4/3$

5) f has a Relative Minimum at $x = -1$ and $x = 2$ when f' changes signs from negative to positive. f has a Relative Maximum at $x = 0$ when f' changes signs from positive to negative.

6) f has a relative maximum at $x = 0$ when $f' = 0$ and $f'' < 0$. f has a relative minimum at $x = \pm 2$ when $f' = 0$ and $f'' > 0$

7a) x −int: DNE, y −int: $\left(0, -\frac{1}{4}\right)$, 7b) $HA: y = 0$, $VA: x = \pm 2$, 7c) f is increasing at $I(-\infty, -2) \cup (-2, 0]$ when $f' > 0$. f is decreasing at $I[0,2) \cup (2, \infty)$ when $f' < 0$.

8a) f concaves up at $I\left(-\frac{\sqrt{2}}{2}, 0\right) \cup \left(\frac{\sqrt{2}}{2}, \infty\right)$ when $f'' > 0$. f concaves down at $I\left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(0, \frac{\sqrt{2}}{2}\right)$ when $f'' < 0$.

9) f has a Relative Maximum at $(0,9)$ when f' change signs from positive to negative. f does not have a relative minimum because f' does not change signs from negative to positive

10a) $s'(t) = v(t) = 15$, 11b) $s''(t) = v'(t) = 6t - 12$, 11c) The particle is at rest at $t = 0$, $t = \pm 4$ seconds when $s'(t) = v(t) = 0$, 11d) The particle is slowing down at $I[2,4]$ when $v(t) < 0$ and $a(t) > 0$

11a) The particle is moving the right at $I(0,1) \cup (4, 6)$ when $v(t) < 0$, 11b) The particle is moving to the left at $I(2,3)$ when $v(t) > 0$, 11c) The particle is standing still $(1, 2) \cup (3, 4)$ when $s'(t) = 0$

12) The particle is moving to the right at $I\left(0, \frac{1}{3}\right) \cup \left(\frac{5}{2}, \infty\right)$ when $s'(t) > 0$

13) $d = \dfrac{\sqrt{5}}{2}$

14) $(\pm 7, \pm 7)$

15) 23 m × 23m

16) 36 cm × 54 cm

17) 25 ft × 100/3 ft (1 side of the dimension)

Chapter 4: Integration

1a) $\frac{2}{3}(3 - 2x)^{3/2} + C$ 1b) $-\frac{1}{2}\sin(3 - 2x) + C$

2) $y = \frac{x^4}{12} + 6x + 3$

3) Left: 2.43, Right: 4.77, Trapezoidal: 3.6

4) RHS: 23.625, f is an overestimation since f is increasing.

5) $\frac{49\pi}{4}$

6a) 36, 6b) -18π, 6c) $\frac{81}{2} - 18\pi$

7a) 5, 7b) 35, 7c) DNE

8a) $\frac{(x^2+1)^3}{3} + C$ 8b) $\frac{2}{9}(x^3 + 25)^{3/2} + C$ 8c) $\sin(5x) + C$ 8d) $\frac{1}{9}\sin^3(3x) + C$

8e) $\frac{(x^2+1)^4}{8} + C$

9a) 225, 9b) $\frac{\pi}{2} - 1$, 9c) 2.5, 9d) $\frac{4\sqrt{5}}{3} - \frac{2}{3}$, 9e) $\frac{1}{3}$, 9f) $\frac{16}{3}$

10) $c = \pm\sqrt{3}$

11) 16

12a) $\sin x$, 12b) $3x^2\sin(x^3)$

13a) 257.5 people/year, 13b) 3383.4 people, 13c) 256 people/year, 13d) 3633.33 people, Pine Grove's population per year from 0 to 10 years.

Chapter 5: Logarithms

1) $\ln\frac{x}{(x^2+1)^2}$

2a) $\frac{x}{x^2-7}$, 2b) $-\frac{x^2-10}{x(x^2-10)}$, 2c) $e^{x^4}(4x^8+5x^4)$, 2d) $7t^6 5^{7t}(t\ln 5 + 1)$, 2e) $\frac{12x^2}{(\ln 6)(x^2-3)(x^3-7)}$

3) $f^{-1}(x) = 3(x+5)^{\frac{2}{3}}$

4) –1/4

5) –1

6) $x = \sqrt{1 - 14x^{10}}$

7) $\arctan(x) + \frac{x}{x^2+1}$

8) $\frac{e^{x^3+1}}{3} + C$

9) $\arctan(x-1) + C$

10) $\frac{5^{(x+1)^2}}{2\ln 5} + C$

11) $\frac{1}{2}\arcsin(x^2) + C$

Chapter 6: Differential Equations

1a) $2\sin\left(\frac{x}{2}\right) + C$, 1b) $y = \pm\sqrt{x^2 + C}$, 1c) $y = 18x - \left(\frac{x^2}{2}\right) + C$, 1d) $y = \pm C\sqrt{x^2+4}$

2) $\frac{t^2}{8} + 12$

3a) 37.2525, 3b) $y = 115e^{(0.5636)(-2)}$, 3c) 10,442.8705, 3d) 10.3829 hours

4a) $P = \frac{40}{1+9e^{-0.0838t}}$, 4b) $t = 42.7773$ hours, 4c) $t = 26.2288$ hours

5) 2

6) 1.5391

FRQ AB/BC3-2014 Q1)

(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$	1 : answer
(b) $g'(x) = f(x)$ The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$
(c) $h'(x) = \frac{5xg'(x) - g(x)5}{(5x)^2} = \frac{5xg'(x) - 5g(x)}{25x^2}$ $h'(3) = \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2} = \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3}$	$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$
(d) $p'(x) = f'\left(x^2 - x\right)(2x - 1)$ $p'(-1) = f'(2)(-3) = (-2)(-3) = 6$	$3 : \begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

AB4-2013 Q2)

(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$.
$W(32) \approx 0.5(32 - 30) + 125 = 126$

1 : answer

(b) $\int_0^{30} W'(t)\, dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$

$W(0) = W(30) - \int_0^{30} W'(t)\, dt = 125 - 22.4 = 102.6$

3 : { 1 : left Riemann sum; 1 : approximation; 1 : answer }

(c) W' is differentiable $\Rightarrow$ W' is continuous.

$W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$

By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.

2 : explanation

(d) $\frac{dA}{dt} = (0.3)\frac{2}{3}W^{-1/3} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}} \cdot \frac{dW}{dt}$

$\left.\frac{dA}{dt}\right|_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$

3 : { 2 : $\frac{dA}{dt}$; 1 : answer }

FRQ Q3)

(a) $H'(0) = -\frac{1}{4}(91 - 27) = -16$
$H(0) = 91$

An equation for the tangent line is $y = 91 - 16t$.

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

3 : { 1 : slope; 1 : tangent line; 1 : approximation }

(b) $\frac{d^2H}{dt^2} = -\frac{1}{4}\frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$

$H > 27$ for $t > 0$ $\Rightarrow$ $\frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0$ for $t > 0$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

1 : underestimate with reason

(c) $\frac{dG}{(G - 27)^{2/3}} = -dt$

$\int \frac{dG}{(G - 27)^{2/3}} = \int (-1)\, dt$

$3(G - 27)^{1/3} = -t + C$

$3(91 - 27)^{1/3} = 0 + C \Rightarrow C = 12$

$3(G - 27)^{1/3} = 12 - t$

$G(t) = 27 + \left(\frac{12 - t}{3}\right)^3$ for $0 \le t < 10$

The internal temperature of the potato at time $t = 3$ minutes is $27 + \left(\frac{12 - 3}{3}\right)^3 = 54$ degrees Celsius.

5 : { 1 : separation of variables; 1 : antiderivatives; 1 : constant of integration and uses initial condition; 1 : equation involving G and t; 1 : $G(t)$ and $G(3)$ }

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

FRQ Q4)

(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$

$h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$