Name ______________________________________ Date _________

Exam Date and Time: _______________________________________

**Read and answer all questions accordingly. All work and problems must be done on your own paper and <u>work must be shown</u>. No work = No Credit = NO EXCEPTIONS. It is worth 1.5 quiz grades.**

**<u>Note</u>: There are two parts of the exam. The NON–CALC test has 20 questions (50%) and 3 FRQ's (50%).**

## <u>Chapter 1 – Limits</u>

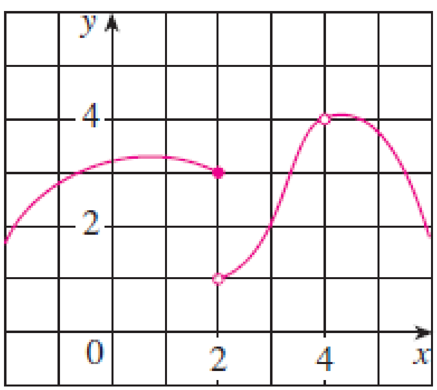
1) Find the limit of the following:

a) $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-9}$ b) $\lim\limits_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}$ c) $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{x-2}$

d) $\lim\limits_{x\to 5^-}\dfrac{x-1}{x-5}$ e) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$ f) $\lim\limits_{x\to 0}\dfrac{\sin^3 5x}{x^3}$

g) $\lim\limits_{x\to\infty}\dfrac{x^2+x-2}{x-1}$ h) $\lim\limits_{x\to 0}\dfrac{x^2}{1-\cos x}$ i) $\lim\limits_{x\to\infty}\dfrac{\sqrt{9x^6-1}}{x^3+3}$

2) Solve. If it does not exist, state "DNE."




a) $\lim\limits_{x\to 2^-} f(x) =$ _________

b) $\lim\limits_{x\to 2^+} f(x) =$ _________

c) $\lim\limits_{x\to 2} f(x) =$ _________

d) $f(2) =$ _________

e) $f(4) =$ _________

3) Explain why $f(x) = x^2 - 7x - 8$ is a guarantee zero in between $[-3, 0]$

4) Define the Intermediate Value Theorem.

5) State the definition of continuity of a function *f* at which $x = c$.

Given: $\lim\limits_{x\to 3} f(x) = 4 \quad \lim\limits_{x\to 3} g(x) = -5 \quad \lim\limits_{x\to 3} h(x) = 0$

6) Find the limits as they exist.

a) $\lim\limits_{x\to 3}\left[f(x)+5g(x)\right] =$ _________ b) $\lim\limits_{x\to 3}\left[g(x)\right]^3 =$ _________

c) $\lim\limits_{x\to 3}\sqrt{f(x)} =$ _________ d) $\lim\limits_{x\to 3}\dfrac{g(x)h(x)}{f(x)} =$ _________

7) Given the following function, find *k* so that the function is continuous at $x = -1$. $f(x) = \begin{cases} kx-3, & x \le -1 \\ x^2+k, & x > -1 \end{cases}$

## <u>Chapter 2 – Differentiation</u>

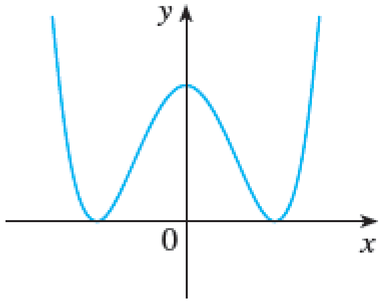
1) Find the equation of the tangent line to the graph, $f(x) = 5x + 6$ at the point $(2, 16)$.

2) Using the difference quotient, find $f'(x)$ . Show all work.

3) Given: $f(x) = \begin{cases} x-5, & x < 1 \\ 3, & x = 1 \\ x^2, & x > 1 \end{cases}$

(a) Is $f$ continuous at $x = 1$? Use the definition of continuity to explain answer.
(b) Is $f$ differentiable at $x = 1$? Explain.
(c) Sketch a graph of $f$.

4) Given the $f$ graph, graph $f'(x)$. Show all work.




5) Find $\frac{dy}{dx}$ of the following:

(a) $f(x) = x^3 - 3x^2$ (b) $f(x) = x^{1/2} - x^{-1/2}$ (c) $y = \frac{2x^3 - 1}{x^2}$

(d) $y = \frac{x+1}{x-1}$ (e) $y = e^{x+6} + 2$ (f) $y = x^5 e^x$

(g) $(x^4 + 7x^2 - 3)^4$ (h) $\left(\frac{x^2+3}{x^2-3}\right)^6$ (i) $\sin(5x^4)$ (j) $\sin^2(5x^4)$

6) Suppose that $f(5) = 1$, $f'(5) = 3$, $g(5) = -4$, and $g'(5) = 6$. Find the following values.

a) $(fg)'(5) =$ ______ b) $\left(\frac{f}{g}\right)'(5) =$ ______ c) $\left(\frac{g}{f}\right)'(5) =$ ______

7)

| | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| **x = 3** | –3 | 2 | 5 | 3 |
| **x = 5** | 7 | –1 | –2 | 4 |

(a) If $h(x) = f(x)g(x)$, solve $h'(5)$

(b) If $h(x) = 3[g(x)]^2$, solve $h'(3)$

8) If $F(x) = f(g(x))$, where $f(-3) = 4$, $f'(-3) = 6$, $f'(1) = 5$, $g(1) = -3$, and $g'(1) = 7$, find $F'(1)$.

9) Solve for $f''(x)$: (a) $10x^2 - 8x$ (b) $\sec x$ (c) $y = \cos^2 x - \sin^2 x$

10) Find $\frac{dy}{dx}$ if (a) $x^3y + xy^3 = -10$ (b) $x^2 - 2xy + 3y^2 = 8$
(c) $e^y \cos x = 6 + \sin(xy)$

11) The radius of a circle is increasing at a rate of 2 cm/sec. Find the rate of change of the area of the circle when the radius is 5 cm.

12) A 15 ft ladder is leaning against a wall. The bottom of the ladder begins to slide away from the wall at a rate of 3 ft/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 9 feet away from the wall?

13) Water is dripping through the top of a conical shaped cup at the rate of 0.5 cubic inches per minute. If the cup's original size is 4 inches high with a diameter of 4 inches across the top, how fast is the height of water decreasing when the water is two inches deep in the cone?

14) If a tank holds 4500 gallons of water, which drains from the bottom of the tank in 50 minutes, then Toricelli's Law gives the volume $V$ of water remaining in the tank after $t$ minutes as

$$V = 4500\left(1 - \frac{1}{50}t\right)^2 \quad 0 \le t \le 50.$$

Find the rate at which water is draining from the tank after the following amounts of time. (Remember that the rate must be negative because the amount of water in the tank is decreasing.)

(a) 5 min (b) 50 minutes
(c) At what time is the water flowing out the fastest?
(d) At what time is the water flowing out the slowest?

**Chapter 3 – Applications of Differentiation**

1) Given $f(x) = 3x^3 - 4x + 1$, identify all absolute extrema on the interval $[0, 4]$.

2) Find the absolute maximum and absolute minimum values of $f$ on the given interval, $f(t) = 2\cos t + \sin(2t)$ at $I\left[0, \frac{\pi}{2}\right]$

3) Define the Extreme Value Theorem and Mean Value Theorem and identify the difference of the two theorems.

4) What is the number for $c$ which satisfies the conditions of the Mean Value Theorem of differentiable calculus for $f(x) = x^3 - 2x^2$ on $[0, 2]$.

5) Find the relative extrema of $f(x) = 3x^4 - 4x^3 - 12x^2 + 3$ using the First Derivative Test.

6) Use the second derivative test to determine the relative extrema for $f(x) = x^4 - 8x^2$.

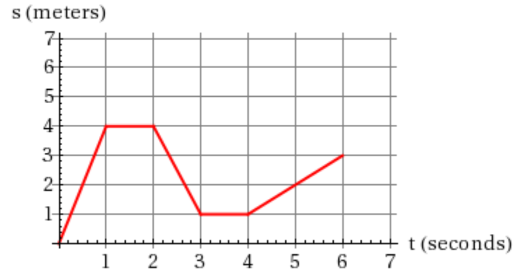
7) Determine the $f(x) = \frac{1}{x^2-4}$, determine the following:
(a) $x$ and $y$–intercepts
(b) horizontal and vertical asymptotes
(c) intervals of increasing and decreasing

8) Determine the open intervals in which the graph $f(x) = 3x^5 - 5x^3 + 3$ is concave up or down.

9) Find the relative extrema of $f(x) = 9 - x^2$ on the $I[-3, 3]$.

10) Let $s(t) = t^3 - 6t^2 + 2$ be the position function of a particle moving along the $x$–axis.
(a) Find the velocity of the object when $t = 5$
(b) Find the acceleration at time $t$
(c) Establish when the particle is at rest
(d) When is the particle slowing down?

11) A particle starts by moving to the right along a horizontal line; the graph of its position function is shown.




(a) When is the particle moving to the right?
(b) When is the particle moving to the left?
(c) When is the particle standing still?

12) A particle is moving along a horizontal line according to the equation, $s(t) = 2t^3 - \frac{17}{2}t^2 + 5t - 1$where $[0, \infty)$. Determine the interval of time when the particle is moving to the right.

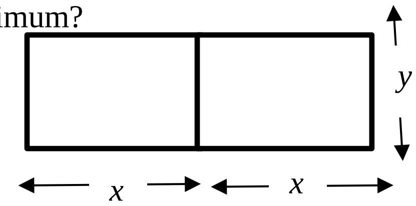
13) Find the shortest distance from $f(x) = x^2$to the point $(2, ½)$.

14) Find two positive numbers whose product is 49 and whose sum is a minimum.

15) Find the dimensions of a rectangle with perimeter 92 m whose area is as large as possible.

16) The top and bottom margins of a poster are each 9 cm and the side margins are each 6 cm. If the area of printed material on the poster is fixed at 864 cm$^2$, find the dimensions of the poster with the smallest area.

17) A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?




## Chapter 4 – Integration

1) Solve for these indefinite integrals.

(a) $\int -2\sqrt{3-2x}\, dx$ (b) $\int \cos(3-2x)\, dx$

2) Find $y = f(x)$ if $f''(x) = x^2$, $f'(0) = 6$, and $f(0) = 3$.

3) Given $\int_0^6 f(x)dx$ for the functions below, use the left, right, and trapezoidal Riemann sum with 3 subintervals.

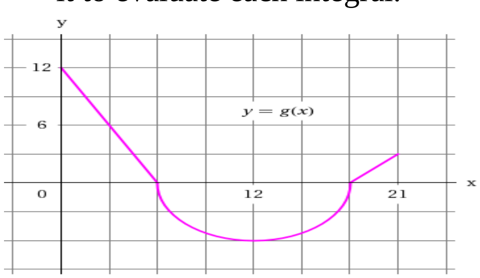
| $x$ | 0 | 1 | 4 | 6 |
|---|---|---|---|---|
| $f(x)$ | 0 | 0.25 | 0.84 | 1 |

4) If $f(x) = 3x^2 - 2x$, $0 \le x \le 3$, evaluate the Riemann sum with $n = 6$, taking the sample points to be right endpoints. Then, determine if the graph is an underestimation or an overestimation.

5) Sketch the region whose is area is given by the definite integral and then use a geometric formula to evaluate the integral of

$\int_0^7 \left(\sqrt{49-x^2}\right) dx$

6) The graph of $g$ consists of two straight lines and a semicircle. Use it to evaluate each integral.




A) $\int_0^6 g(x)\, dx$

B) $\int_6^{18} g(x)\, dx$

C) $\int_0^{21} g(x)\, dx$

7) Given $\int_0^3 f(x)\, dx = 6$ and $\int_3^9 f(x)\, dx = -1$, solve for $\int_0^9 f(x)\, dx$, $\int_0^9 7f(x)\, dx$, and $\int_9^3 g(x)\, dx$

8) Evaluate:

(a) $\int (x^2+1)^2(2x)\, dx$ (b) $\int x^2\sqrt{x^3+25}\, dx$ (c) $\int 5\cos(5x)\, dx$

(d) $\int \sin^2(3x)\cos(3x)\, dx$ (e) $\int x(x^2+1)^3\, dx$

9) Use the Fundamental Theorem of Calculus to solve:

(a) $\int_{-10}^{5} (-2x+10)\, dx$ (b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2-\csc^2 x)\, dx$ (c) $\int_0^2 |2x-1|\, dx$

(d) $\int_1^3 \frac{x}{\sqrt{2x-1}}\, dx$ (e) $\int_0^1 x\sqrt{1-x^2}\, dx$ (f) $\int_1^5 \frac{x}{\sqrt{2x-1}}\, dx$

10) Use the Mean Value Theorem of Integration to identify $c$ of $f(x) = 9 - x^2$, $[-3, 3]$

11) Find the average value of $f(x) = 3x^2 - 2x$ on $[1, 4]$

12) Solve (a) $\frac{d}{dx}\int_0^x \sin t\, dt$ and (b) $\frac{d}{dx}\int_0^{x^3} \sin t\, dt$

13) Accumulation/Riemann's Sum:

Let $y(t)$ represent the population of Sugar Mill over a 10-year period, where $y$ is differentiable function of $t$. The table shows the population recorded every two years.

| **$t$ (years)** | **0** | **2** | **4** | **6** | **8** | **10** |
|---|---|---|---|---|---|---|
| **$y$ (people)** | **2500** | **2912** | **3360** | **3815** | **4330** | **4875** |

(a) Use the data from the table to find an approximation for $y'(7)$ and explain the meaning of $y'(7)$ in terms of the population of Sugar Mill. Show the computations that lead to the answer.

(b) Use the data from the table to approximate the average population of Sugar Mill over the time interval, $0 \le t \le 10$ by using a **LEFT** Riemann's Sum with five equal intervals. Show all work.

(c) A model for the population of another town, Pine Grove, over the same 10-year period is given by the function $P(t) = (2t+50)^2$ where $t$ is measured in years and $P(t)$ is measured in people. Use the model to find the value of $P'(7)$.

(d) Use the model given in part (c) to find the value of $\frac{1}{10}\int_0^{10} P(t)\,dt$. Explain the meaning of this integral expression in terms of the population of Pine Grove.

**Chapter 5: Logarithms**

1) Write the following expression as a log of a single quantity, $\ln x - 2\ln(x^2+1)$.

2) Find the derivative:

(a) $y = \ln\sqrt{x^2-7}$ (b) $y = \ln\left(\frac{3x}{x^2+10}\right)$ (c) $y = x^5 e^{x^4}$

(d) $f(t) = t^7 5^{7t}$ (e) $\log_6 \frac{x^3-7}{x^3-3}$

3) Find $(f^{-1})' = x^3 - 5$ if $f(x) = x^3 - 5$

4) Given $f(3)=5$, $f'(3)=7$, $f(2)=3$, and $f'(2)=-4$, and $f$ and $g$ are inverses, solve for $g'(3)$.

5) Solve for $g'(4)$:

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 1 | 2 | 0 | 6 |
| $f'(x)$ | –1 | 3 | 1/2 | 4 | 5 |

6) Solve $\cos\left(\arcsin\left(7x^5\right)\right)$ 7) $\frac{d}{dx} x\tan^{-1}(x)$ 8) $\int x^2 e^{x^3+1}dx$

9) $\int \frac{dx}{x^2-2x+2}$ 10) $\int (x+1)5^{(x+1)^2}dx$ 11) $\int \frac{x}{\sqrt{1-x^4}}dx$

**Chapter 6- Differential Equations**

1) Solve for the differential equation:

(a) $\frac{dy}{dx} = \cos\frac{x}{2}$ (b) $\frac{dy}{dx} = \frac{x}{y}$

(c) $y' = 18 - x$ (d) $\left(x^2+4\right)\frac{dy}{dx} = xy$

2) Write the function $y = f(t)$ passing through the point $(0, 12)$ with the given first derivative of $\frac{dy}{dt} = \frac{1}{4}t$

3) The number of bacteria in a culture is increasing according to the law of exponential growth. There are 115 bacteria in the culture after 2 hours and 355 bacteria after 4 hours. (calc)

(a) Find the initial population.
(b) Write an exponential growth model for the bacteria population. Let t represent time in hours.
(c) Use the model to determine the number of bacteria after 8 hours.
(d) After how many hours will the bacteria count be 40,000?

4) At time $t = 0$, bacterium weighs 4 grams. Three hours later, the culture weighs 5 grams. The maximum weight of the culture is 40 grams. (calc.)

(a) Write a logistic equation that models the weight of the bacterial culture.

(b) When will the culture's weight reach 32 grams?

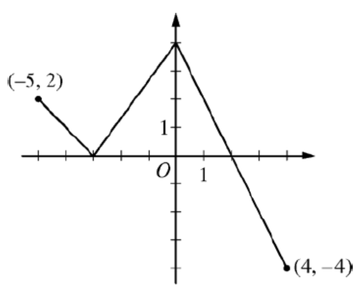
(c) At what time is the culture's weight increasing most rapidly?

5) In a town of population 100,000 twenty thousand residents heard a radio announcement about a local political scandal. The rate of growth of the spread of information was jointly proportional to the amount of people who had not heard it. If 50% heard the scandal after one hour, how long until 80% of the population has heard the rumor? (calc)

6) Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$ Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

**Free Response Questions**

1) AB/BC3-2014




Graph of $f$

The function $f$ is defined on the closed interval $[-5, 4]$. The graph of $f$ consists of three line segments and is shown in the figure above.

Let $g$ be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of $g$ both increasing and concave down? Give a reason for your answer.

(c) The function $h$ is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

(d) The function $p$ is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of $p$ at the point where $x = -1$.

2) AB4-2013

| $t$ (days) | 0 | 10 | 22 | 30 |
|---|---|---|---|---|
| $W'(t)$ (GL per day) | 0.6 | 0.7 | 1.0 | 0.5 |

The twice-differentiable function $W$ models the volume of water in a reservoir at time $t$, where $W(t)$ is measured in gigaliters (GL) and $t$ is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to $W$ at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time $t$, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area $A$, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of $A$, in square kilometers per day, with respect to $t$ when $t = 30$ days.

3) AB/BC5-2016

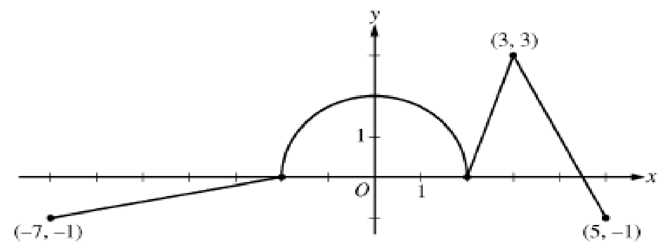
At time $t = 0$ , boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius $\left({}^{\circ}C\right)$ at time $t=0$, and the internal temperature of the potato is greater than $27^{\circ}C$ for all times $t>0$ . The internal temperature of the potato at time $t$ minutes can be modeled by the function $H$ that satisfies the differential equation $\frac{dH}{dt}=-\frac{1}{4}(H-27)$, where $H(t)$ is measured in degrees Celsius and $H(0)=91$.

(a) Write an equation for the line tangent to the graph of $H$ at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t<10$, an alternate model for the internal temperature of the potato at time t minutes is the function $G$ that satisfies the differential equation $\frac{dG}{dt}=-(G-27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0)=91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$ ?

4) AB5-2010



Graph of $g'$

The function $g$ is defined and differentiable on the closed interval $[-7,5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of $g$, consists of a semicircle and three-line segments, as shown in the figure above.

(a) Find the $x$-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(b) The function $h$ is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the $x$-coordinate of each critical point of $h$, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

5) List 3 ways you will do to help you study for the midterm exam.