

6.4: Reference Angles

“I WILL...

Identify reference angles of a given angle

Apply the six trig functions on the coordinate plane from a point.”

I. Definitions

- A. Angle of rotation is formed by two rays with a common endpoint, also called the vertex.
- B. One ray is called the _____ side.
- C. The other ray is called the _____ side.
- D. The measure of the angle is determined by the amount and direction of rotation from the initial side to the terminal side.

Ex 1: Draw 210° with the given measure in standard position. Then determine in which quadrant the terminal side lies.

Ex 2: Draw 510° with the given measure in standard position. Then determine in which quadrant the terminal side lies.

Your Turn: Draw -45° with the given measure in standard position. Then determine in which quadrant the terminal side lies.

II. Reference Angles

A. Reference angle is a positive acute angle formed by the terminal side of θ and the _____-axis. They are viewed as linear pairs. (Think: *REFER* to the ____-axis)

B. No reference trigonometric values of measure are greater than 90° or less than 0°

1. Quadrant I: _____

2. Quadrant II: _____

3. Quadrant III: _____

4. Quadrant IV: _____

<p>Ex 3: Determine the reference angle for $\theta = 135^\circ$</p>	<p>Ex 4: Determine the reference angle for $\theta = \frac{4\pi}{3}$</p>
<p>Ex 5: Determine the reference angle for $\theta = \frac{11\pi}{6}$</p>	<p>Your Turn: Determine the reference angle for $\theta = -\frac{\pi}{7}$</p>

Exercises 6.4

Note: Unless stated otherwise, all angles are in standard position.

In Exercises 1–6, find $\sin t$, $\cos t$, and $\tan t$ when the terminal side of an angle of t radians passes through the given point.

1. $(2, 7)$ 2. $(-3, 2)$ 3. $(-5, -6)$
4. $(4, -3)$ 5. $(\sqrt{3}, -10)$ 6. $(-\pi, 2)$

In Exercises 7–10, find $\sin t$, $\cos t$, and $\tan t$ when the terminal side of an angle of t radians passes through the given point on the unit circle.

7. $\left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ 8. $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$
9. $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ 10. $(0.6, -0.8)$

In Exercises 11–14, identify an angle $0 \leq t' \leq \pi$ that is coterminal with the given angle, and find the sine and cosine of the given angle.

11. $\frac{13\pi}{6}$ 12. $\frac{9\pi}{2}$ 13. 16π 14. $\frac{7\pi}{4}$

In Exercises 15–23,

- a. Use a calculator in radian mode to find the sine, cosine, and tangent of each number. Round your answers to four decimal places.
b. Use the signs of the functions to identify the quadrant of the terminal side of an angle of t radians. If the terminal side lies on an axis, identify which axis and whether it is on the positive or negative side of the axis. Explain your reasoning.

15. $\frac{7\pi}{5}$ 16. 11 17. $-\frac{14\pi}{9}$
18. -23π 19. $\frac{10\pi}{3}$ 20. 6.4π
21. 9.5π 22. $\frac{\pi}{17}$ 23. -17

In Exercises 24–29, sketch each angle whose radian measure is given and find its reference angle.

24. $\frac{7\pi}{3}$ 25. $\frac{17\pi}{6}$ 26. $\frac{6\pi}{5}$

27. 1.75π 28. $-\frac{3\pi}{4}$ 29. $-\frac{\pi}{7}$

In Exercises 30–47, find the exact value of the sine, cosine, and tangent of the number without using a calculator.

30. $\frac{7\pi}{6}$ 31. $\frac{7\pi}{3}$ 32. $\frac{17\pi}{3}$ 33. $\frac{11\pi}{4}$
34. $\frac{5\pi}{4}$ 35. $-\frac{3\pi}{2}$ 36. 3π 37. $-\frac{23\pi}{6}$
38. $\frac{11\pi}{6}$ 39. $-\frac{19\pi}{3}$ 40. $-\frac{10\pi}{3}$ 41. $-\frac{15\pi}{4}$
42. $-\frac{25\pi}{4}$ 43. $\frac{5\pi}{6}$ 44. $-\frac{17\pi}{2}$ 45. $\frac{9\pi}{2}$
46. $-\pi$ 47. 4π

In Exercises 48–53, write the expression as a single real number. Do not use decimal approximations.

48. $\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{2}\right)$
49. $\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{4}\right)$
50. $\cos\left(\frac{2\pi}{3}\right)\cos\pi + \sin\left(\frac{2\pi}{3}\right)\sin\pi$
51. $\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) - \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{5\pi}{6}\right)$
52. $\sin\left(\frac{-7\pi}{3}\right)\cos\left(\frac{5\pi}{4}\right) + \cos\left(\frac{-7\pi}{3}\right)\sin\left(\frac{5\pi}{4}\right)$
53. $\sin\left(\frac{\pi}{3}\right)\cos\pi + \sin\pi\cos\left(\frac{\pi}{3}\right)$

In Exercises 54–59, the terminal side of an angle of t radians lies in the given quadrant on the given line. Find $\sin t$, $\cos t$, and $\tan t$. (*Hint:* Find a point on the terminal side of the angle.)

54. Quadrant III; line $2y - 4x = 0$
55. Quadrant IV; line through $(-3, 5)$ and $(-9, 15)$
56. Quadrant III; line through the origin parallel to $7x - 2y = -6$