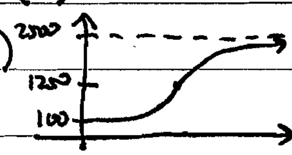
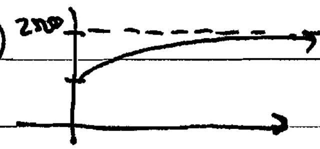
6.3A Key Logistic Growth

1. D, A, B, C
2. A) (i) 2500 (ii) 480 (iii) 1250 bears

   (iv)


   B) (i) 2500 (ii) 1500 (iii) Inflection: 1250 bears, NO Inf. Point

   (iv)


   c) Population is greatest at 1250 bears. POI defined at $\frac{d^2y}{dt^2}$
3. (a) $K=-0.75$, (b) $L=2100$, (c) 70, (d) $t=4.4897$ years,

   (e) $\frac{dP}{dt}=0.75P\left(1-\frac{P}{2100}\right)$
4. (a) $K=3$, (b) $L=100$, (c) 50
5. (a) $P=\frac{100}{1+4e^{-t}}$, (b) 83 animals, (c) $t=2.7726$ years,
6. (a) $\frac{dP}{dt}=6P(2000-P)$, (b) $P=\frac{2000}{1+399e^{-6t}}$, (c) $t=0.9982$ hours

   (d) ≈ 1995.1089 students
7. (a) $P=\frac{20}{1+19e^{-0.7790t}}$, (b) $P\approx 14.4246$ hours, (c) $t\approx 6.6054$ hrs
8. E
9. (a) 12, 12

   (b) 6

   (c) $Y=3e^{\frac{1}{5}t-t^2/120}$

   (d) 0

1. $\boxed{D}$ $Y = \frac{12}{1+e^{-x}}$ 1

$Y = \frac{12}{1+e^{-0}}$ $Y = \frac{12}{1+1} =$ $Y = \frac{12}{2} = Y = 6$

$1+e^{-x} = 0$ $Y = \frac{12}{1+e^{-2}} =$

$-1 \quad -1$

$\boxed{A}$ $Y = \frac{12}{1+3e^{-x}}$ $\frac{12}{1+3e^{-0}} = \frac{12}{4} = 3$

$\boxed{B}$ $Y = \frac{12}{1+\frac{1}{2}e^{-x}} = \frac{12}{1+\frac{1}{2}e^{0}} = \frac{12}{3/2} = 12 \cdot \frac{2}{3} = \frac{24}{3} = 8$

$\boxed{C}$ $Y = \frac{12}{1+e^{2x}}$

$0.002P\left(1 - \frac{P}{2500}\right)$ $0.002P - \underline{0.002P}$

2. $\frac{dP}{dt} = 5P - 0.002P^2$ $\frac{dP}{dt} = KP(\cancel{L}-P)$ or $KP\left(1 - \frac{P}{L}\right)$

$0.002P\left(\frac{5}{0.002} - P\right)$

$\boxed{\frac{dP}{dT} = 0.002P(2500-P)}$

(a) $P(0) = 100$

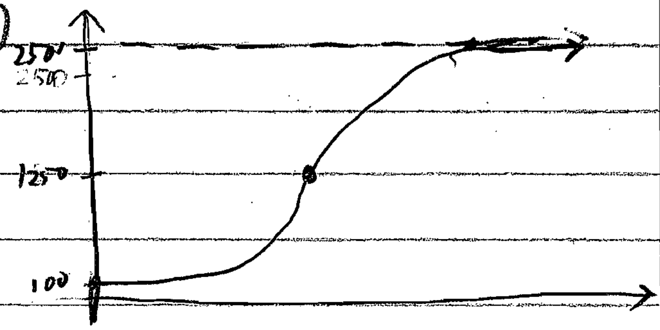
(i) $\lim_{t \to \infty} P(t) = 2500$ Bears

(ii) $\frac{dP}{dt} = 0.002(100)[2500-100]$

$0.2(2400) = \boxed{480 \text{ bears}}$

(iii) P.O.I. $= \frac{2500}{2} = 1250$ bears

(iv)



(2B) $P(0) = 1500$

(i) $\lim_{t \to \infty} P(t) = 0.002P(2500 - P)$

$\boxed{= 2500 \text{ Bears}}$

(ii) $\frac{dP}{dt} = 0.002P(2500 - P)$

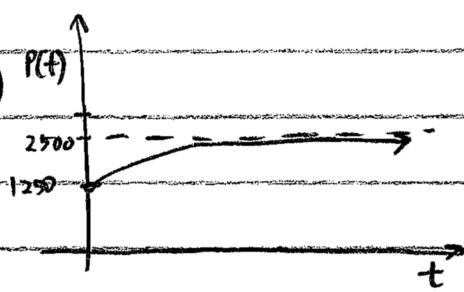
$= 0.002(1500)[2500 - 1500]$

$= 3(500) = \boxed{1500 \text{ bears}}$

(iii) POI: $\frac{\lim P(t)_{t \to \infty}}{2} = \frac{2500}{2} = 1250$ bears

If $P(0) = 1500$ ad POI is 1250, ∴ no Inf. Point

(iv)



(c) Population is greatest at 1250 bears. It is the Point-of-Inflection which is defined as $\frac{d^2y}{dt^2}$ of Max point

(3) $P(t) = \frac{2100}{1 + 29e^{-0.75t}}$ $\quad P(t) = \frac{L}{1 + Pe^{-Kt}}$

(a) $K = -0.75$

(b) $L = 2,100$

(c)

$P(0) = \frac{2100}{1 + 29e^{-0.75(0)}} = \frac{2100}{30} = 70$

(D) $P(t) = \frac{2100}{1+29e^{-0.75t}}$

$1050 = \frac{2100}{1+29e^{-0.75t}}$

$\frac{1050(1+29e^{-0.75t})}{1050} = \frac{2100}{1050}$

$1+29e^{-0.75t} = 2$

$-1 \qquad -1$

$29e^{-0.75t} = 1$

$e^{-0.75t} = \frac{1}{29}$

$\ln e^{0.75t} = \ln\frac{1}{29}$

$\frac{0.75t}{0.75} = \frac{\ln\frac{1}{29}}{0.75}$

$\boxed{t \approx 4.4897 \text{ years}}$

(E) $\frac{dP}{dt} = KP\left(1-\frac{P}{L}\right)$

$\boxed{\frac{dP}{dt} = 0.75P\left(1-\frac{P}{2100}\right)}$

$L = 2100$
$K = 0.75$

$\frac{0.75}{2100}P(2100-P)$

$\boxed{\text{OR } \frac{1}{2800}P(2100-P)}$

4) $\frac{dP}{dt} = 3P\left(1-\frac{P}{100}\right) \qquad KP\left(1-\frac{P}{L}\right)$

(A) $K = 3$ (B) $L = 100$

(C) Greatest is 50 through POI

$\frac{P}{2} = \frac{100}{2} = 50$ (POI)

5) $\frac{dP}{dt} = 0.01P(100-P)$ $\boxed{\frac{dP}{dt} = 1P\left(1-\frac{P}{100}\right)}$

$L = 100$
$B = ?$
$K = 1$

$(100)\,0.01P\left(\frac{100}{100} - \frac{P}{100}\right)$

$\boxed{P = \frac{100}{1+4e^{-t}}}$

(a) $P(0) = 20$

$P = \frac{L}{1+Be^{-Kt}} \qquad 20 = \frac{100}{1+Be^{-K(0)}} = 20 = \frac{100}{1+B}$

$20(1+B) = 100$

$1+B = 5$

$\boxed{B = 4}$

(B) $P = \frac{100}{1+4e^{-3}} = \boxed{83.3933 \text{ animals or } 83 \text{ animals}}$

(C) Graphing Calculator

$80 = \frac{100}{(1+4e^{-t})}$

$80(1+4e^{-t}) = 100$

$1+4e^{-t} = \frac{5}{4}$

$\frac{4e^{-t}}{4} = \frac{\frac{1}{4}}{4}$

$e^{-t} = \frac{1}{16}$

$\ln e^{-t} = \ln \frac{1}{16}$

$\frac{-t}{-1} = \frac{\ln \frac{1}{16}}{-1}$

$t = -\ln\left(\frac{1}{16}\right)$

$\boxed{t = 2.7726 \text{ years}}$

⑥ $\frac{dP}{dT} = 0.003P(2000-P)$

$(2000)\,0.003P\left(1 - \frac{P}{2000}\right)$

(A) $\boxed{\frac{dP}{dt} = 6P(2000-P)}$

POI: $\frac{L}{2} = \frac{2000}{2} = \boxed{1000}$

(B) $P(0) = 5$

$\frac{dP}{dt} = 0.003P(2000-P)$

$K = \cancel{0.003}\ 6$ $\quad B = ?$

$L = 2000$

$P = \frac{L}{1+Be^{-kt}} = P = \frac{2000}{1+Be^{-0.03(t)}}$

$5 = \frac{2000}{1+B}$

$5(1+B) = 2000$

$1+B = 400$

$\boxed{B = 399}$

$\boxed{P = \frac{2000}{1+399e^{-6t}}}$

(C) $1000 = \frac{2000}{1+399e^{-6t}}$

$1+399e^{-6t} = 2$

$399e^{-6t} = 1$

$e^{-6t} = \frac{1}{399}$

$\frac{1}{e^{-6t}} = \frac{1}{399}$

$\ln e^{6t} = \ln 399$

$\frac{6t}{6} = \frac{5.98896}{6}$

$\boxed{t = 0.9982 \text{ hours}}$

(d) $P = \frac{2000}{1+399e^{-6(2)}}$ $\boxed{= 1995.1089 \text{ students}}$

7a $\frac{dP}{dt} = KP\left(1-\frac{P}{L}\right)$

(0,1) B = 19
(2,4) L = 20
(–, 20) K = ?
T = ?

$P = \frac{L}{1+Be^{-Kt}}$

$1 + Be^{-Kt} = 20$
$1 + B = 20$
$B = 19$

$1 = \frac{20}{1+19e^{-Kt}}$

$4 = \frac{20}{1+19e^{-K(2)}}$

$4(1+19e^{-2K}) = 20$

$1+19e^{-2K} = 5$

$\frac{19e^{-2K}}{19} = \frac{4}{19}$

$e^{-2K} = \frac{4}{19}$

$\frac{1}{e^{2K}} = \frac{4}{19}$

$\frac{4e^{2K}}{4} = \frac{19}{4}$

$e^{2K} = \frac{19}{4}$

$\ln e^{2K} = \ln \frac{19}{4}$

$\frac{2K}{2} = \frac{\ln 19/4}{2}$

$K \approx \frac{\ln \frac{19}{4}}{2}$

$\boxed{K \approx 0.7790}$

$\boxed{P = \frac{20}{1+19e^{-0.7790t}}}$

7b $P = \frac{20}{1+19e^{-0.7790(5)}}$ $\boxed{P \approx 14.4246 \text{ hours}}$

7c $18 = \frac{20}{1+19e^{-0.7790t}}$

$\frac{18(1+19e^{-0.7790t})}{18} = \frac{20}{18}$

$1+19e^{-0.7790t} = \frac{10}{9} - 1$
-1

$\frac{19e^{-0.07790t}}{19} = \frac{1}{9}\left(\frac{19}{1}\right)$

$e^{-0.7790t} = 0.005847$

$\log_e 0.05847 = -0.779$

$\frac{\ln 0.05847}{-0.7790} = \frac{-0.7790t}{-0.7790}$

$\boxed{t \approx 6.6054 \text{ hours}}$

8 E) 10,000 $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)^{(5000)}$

OR

$\frac{dP}{dt} = \frac{1}{5000}P\,(2(5000) - P))$

$\frac{dP}{dt} = P\left(\frac{2}{2} - \frac{P}{5000}\left(\frac{1}{2}\right)\right)$

$(10{,}000)\frac{1}{5000}P\left(\frac{10000}{10000} - \frac{P}{10000}\right)$

$\frac{dP}{dt} = P\left(1 - \frac{P}{10{,}000}\right)$

$dP/dt = 2P\left(1 - \frac{P}{10000}\right) \leftarrow \boxed{L = 10{,}000}$

9 (A) at $P(0) = 3$ $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{\boxed{12}}\right)$

$\lim_{t\to\infty} P(t) = \boxed{12}$

at $P(0) = 20$

$\lim_{t\to\infty} P(t) = \boxed{12}$

(B) POI: $\frac{L}{2}$ $\frac{12}{2} = \boxed{6}$ Population growth is at $\frac{1}{2}$ carrying capacity.

(C) $\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right)$, Find $Y(t)$ at $Y(0) = 3$

$\int \frac{dY}{Y} = \int \frac{1}{5}\left(1 - \frac{t}{12}\right)dt$

$\ln Y = \frac{1}{5}\int 1 - \frac{t}{12}\,dt$

$\ln Y = \frac{1}{5}\left(t - \frac{t^2}{12(2)}\right) + $

$\ln Y = \frac{1}{5}\left(t - \frac{t^2}{24}\right)$

$e^{\ln Y} = e^{\frac{1}{5}t - \frac{t^2}{120}}$

$Y = Ce^{1/5 t - t^2/120}$

$Y = Ce^{1/5(0) - (0^2)/120}$

$3 = C$

$\boxed{Y = 3e^{1/5 t - t^2/120}}$