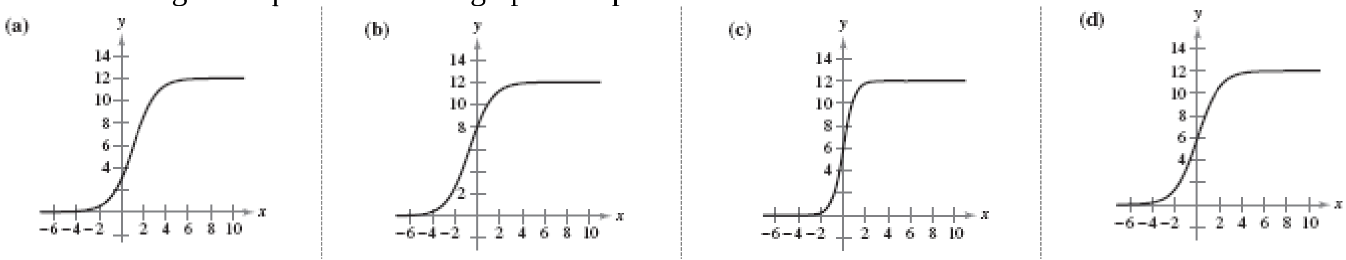
Calculus BC AP/Dual Worksheet Rev 2018 **Name** ________________________________
§6.3A: Logistic Growth **Date** ________________ **Period** _____

Matching.

1) Match the logistic equation with its graph in capital letters.



_______ $y=\dfrac{12}{1+e^{-x}}$ _______ $y=\dfrac{12}{1+3e^{-x}}$ _______ $y=\dfrac{12}{1+\frac{1}{2}e^{-x}}$ _______ $y=\dfrac{12}{1+e^{-2x}}$

Solve these Logistic Differential/Growth questions with the necessary work.

2) Suppose the population of bears in a national park grows according to the logistic differential equation, $\dfrac{dP}{dt}=5P-0.002P^2$, where P is the number of bears at time t in years.

(a) Given $P(0)=100$.

(i) Find $\lim\limits_{t\to\infty} P(t)$.

(ii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iii) Does the solution curve have an inflection point? Justify your answer.

(iv) Use the information you found to sketch the graph of $P(t)$.

(b) Given $P(0)=1500$.

(i) Find $\lim\limits_{t\to\infty} P(t)$.

(ii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iii) Does the solution curve have an inflection point? Justify your answer.

(iv) Use the information you found to sketch the graph of $P(t)$.

(c) How many bears are in the park when the population of bears is growing the fastest?

3) Use the equation of $P(t)=\dfrac{2100}{1+29e^{-0.75t}}$:

(a) Find the value of k

(b) Find the carrying capacity

(c) Find the initial population

(d) Determine when the population will reach 50% of its carrying capacity

(e) Write a logistic differential equation that has the solution

4) Use the equation of $\dfrac{dP}{dt}=3P\left(1-\dfrac{P}{100}\right)$:

(a) Find the value of k

(b) Find the carrying capacity

(c) determine the value of at P which the population growth rate is the greatest.

5) A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t.

(b) Use your answer to (a) and your graphing calculator to find P when $t = 3$ years.

(c) Use your answer to (a) and your graphing calculator to find t when $P = 80$ animals.

6) The rate at which a rumor spreads through a high school of 2,000 students can be modeled by the differential equation, $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

(a) How many students have heard the rumor when it is spreading the fastest? Justify your answer.

(b) If $P(0) = 5$, solve for P as a function of t.

(c) Use your answer to (b) and your graphing calculator to determine how many hours have passed when half the student body has heard the rumor.

(d) Use your answer to (b) and your graphing calculator to determine how many students have heard the rumor after 2 hours.

7) At the time $t = 0$, a bacterial culture weighs 1 gram. Two hours later, the culture weighs 4 grams. The maximum weight of the culture is 20 grams.

(a) Write a logistic equation that models the weight of the bacterial culture.

(b) Find the culture's weight after 5 hours.

(c) When will the culture's weight reach 18 grams?

(d) Write a logistic equation that models the growth rate of the culture's weight. Then, repeat part (b) using **<u>Euler's Method</u>** with a step size of 1. Compare the approximation with the exact answer.

(e) After how many hours is the culture's weight increasing most rapidly? Explain.

_____ 8) The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

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9) A population is modeled by the function P that satisfies the logistic differential equation, $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right)$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by the function Y that satisfies the separable differentiable equation,

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right)$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?