

§6.3A: Logistic Growth

“I WILL ...

...apply application to define logistic growth problems.”

I. Logistic Growth

A. Logistics Differential Equation: \_\_\_\_\_ or \_\_\_\_\_

B. Logical Growth Model: \_\_\_\_\_

1.  $L$  = Carrying Capacity (Upper Horizontal Limit)

2.  $K$  = Proportionality Constant

3.  $B$  = Beginning Amount, use the equation: \_\_\_\_\_ where  $P_0$  is the initial population

C. Point of Inflection: \_\_\_\_\_

Ex 1: Given  $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$  identify the  $k$  and  $L$  of the equation.

Your Turn: Given  $\frac{dP}{dt} = 0.01P(100 - P)$  identify the  $k$  and  $L$  of the equation.

Ex 2: Using the equation,  $y = \frac{4}{1+2e^{-3t}}$  identify the point of inflection and sketch a graph.

Your Turn: Using the equation,  $y = \frac{8}{1+2e^{-2t}}$  identify the point of inflection and sketch a graph.

Ex 3: A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the environment can support no more than 4000 elk. The growth rate of the elk population,  $p$ , is:  $\frac{dP}{dt} = kP \left(1 - \frac{P}{4000}\right)$ ,  $40 \leq p \leq 4000$  where  $t$  is the number of years.

(a) Write a model for the elk population in terms of  $t$ .

(b) Estimate the elk population in 15 years.

(c) Find the limit of the model as  $t \rightarrow \infty$ .

Ex 4: Suppose the population of bears in a national park grows according to the logistic differential equation,  $\frac{dP}{dt} = 5P - 0.002P^2$ , where  $P$  is the number of bears at time  $t$  in years. Find the limit of the model as  $t \rightarrow \infty$  where  $P(0) = 100$ .

Your Turn: Ten grizzly bears were introduced to a national park 10 years ago. There are 23 bears in the park at the present time. The park can support a maximum of 100 bears. Assuming a logistic growth model, when will the bear population reach 50? 75? 100?

Ex 5: In a particular town of 100,000 residents, 20,000 watched about a viral video on the Internet. The rate of growth of the spread of information was jointly proportional to the amount of people who had not watched it. If 50% watched it after one hour, how long does 80% of the population watched the viral video?