6.3A: Coterminal angles and Arc Lengths
"I WILL…
convert between degree and radian measure.
find co-terminal angle measures.
find the arc length of a sector."

I. Co-Terminal Angles

A. Coterminal angles are angles in standard position with the same terminal side
B. To determine the coterminal angles, add and/or subtract 360° by rotating counter clockwise for a positive rotation
C. Initial Ray is the positive x-axis
D. Terminal Ray is the location where the ray ends
E. Coterminal Angles can be negative

Ex 1: Find two co-terminal rays (1 positive and 1 negative) of 40°	Ex 2: Find one positive and one negative co-terminal ray of $\frac{7\pi}{3}$.
Ex 3: Find one positive and one negative co-terminal ray of $\frac{29\pi}{6}$.	Your Turn: Find one positive and one negative co-terminal ray of $-\frac{7\pi}{9}$

II. Arc Lengths

A. Sector is a region of the circle that bounded by two radii and an arc of a circle
B. The Central Angle of a sector is the angle formed by the two radii
C. Arc Length equation: $s = r\theta$
D. Degrees must be converted to Radians
E. Do NOT forget the units

Ex 4: Determine the Arc Length with the given radius of $r = 4$ inches and $\theta = \pi/6$.	Ex 5: The second hand on a clock is 6 inches long. How far does the tip of the second hand move in 15 seconds? 1 full rotation is 60 seconds. Round to 4 decimal places.

Ex 6: The radius is 36 cm. Find the angle when the arc length is 5 meters. Leave answer in Radian mode.	Your Turn: The second hand on a clock is 6 inches long. How far does the tip of the second hand move in 1 minute and 10 seconds? Hint: Keep the answer in seconds.

III. Linear and Angular Speed

A. Angular Speed applies to any object or particle that turns; angle through with the point *rotates* over time (also known as angle rotation)

B. Angular Speed Equation: $\frac{Angle}{Time} = \omega = \frac{\theta}{t}$

C. Linear speed applies to any object or particle that moves; distance that the point *travels* over time (distance)

D. Linear Speed Equation: $\frac{Arc\ Length}{Time} = V = r\omega$ where $\frac{\theta}{t}$ is the angular speed

E. Therefore, Linear Speed is also known as (Radius) * (Angular Speed)

F. The angular speed of an object traveling in a circular path is the same, regardless of its distance from the center of the circle. When the angular speed of the object stays the same, the linear speed increases as the object moves farther from the center

Ex 7: A merry go round makes 8 revolutions feet per minute. What is the angular speed?	Ex 8: A merry go round makes 8 revolutions per minute. The horse 12 feet from the center is traveling with a radius of 12. How fast is the horse going per hour?
Ex 9: An earth satellite in circular orbit 1200 km high makes one complete revolution every 90 min. What is its linear speed? Use 6400 km for the length of a radius of the earth.	Your Turn: A circular mobile above a baby's crib makes 5 revolutions per minute. What is the angular speed of the mobile in radians per minute and how fast is a toy 9 inches from the center traveling in feet per minute?

Page 441: 35-63 odd; 35-41: Disregard the 0 to 2π. List one positive and one negative coterminal angle.

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Exercises 6.3

In Exercises 1–10, find the degree and radian measure of the angle in standard position formed by rotating the terminal side by the given amount.

1. $\frac{1}{9}$ of a circle
2. $\frac{1}{24}$ of a circle
3. $\frac{1}{18}$ of a circle
4. $\frac{1}{72}$ of a circle
5. $\frac{1}{36}$ of a circle
6. $\frac{1}{5}$ of a circle
7. $\frac{2}{3}$ of a circle
8. $\frac{7}{12}$ of a circle
9. $\frac{4}{5}$ of a circle
10. $\frac{5}{36}$ of a circle

In Exercises 11–22, convert the given radian measure to degrees.

11. $\frac{\pi}{5}$
12. $-\frac{\pi}{6}$
13. $-\frac{\pi}{10}$
14. $\frac{2\pi}{5}$
15. $\frac{3\pi}{4}$
16. $-\frac{5\pi}{3}$
17. $\frac{\pi}{45}$
18. $-\frac{\pi}{60}$
19. $-\frac{5\pi}{12}$
20. $\frac{7\pi}{15}$
21. $\frac{27\pi}{5}$
22. $-\frac{41\pi}{6}$

In Exercises 23–34, convert the given degree measure to radians. Write your answer in terms of π.

23. 6°
24. −10°
25. −12°
26. 36°
27. 75°
28. −105°
29. 135°
30. −165°
31. −225°
32. 252°
33. 930°
34. −585°

In Exercises 35–42, state the radian measure of an angle in standard position between 0 and 2π that is coterminal with the given angle in standard position.

35. $-\frac{\pi}{3}$
36. $-\frac{3\pi}{4}$
37. $\frac{19\pi}{4}$
38. $\frac{16\pi}{3}$
39. $-\frac{7\pi}{5}$
40. $\frac{45\pi}{8}$
41. 7
42. 18.5

In Exercises 43–46, find the radian measure of four angles in standard position that are coterminal with the given angle in standard position.

43. $\frac{\pi}{4}$
44. $\frac{7\pi}{5}$
45. $-\frac{\pi}{6}$
46. $-\frac{9\pi}{7}$

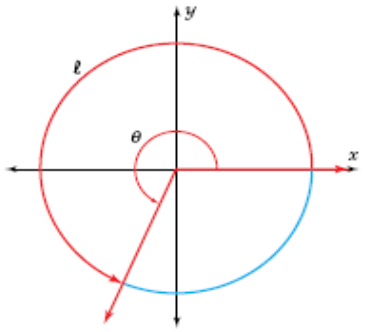
In Exercises 47–52, determine the positive radian measure of the angle that the second hand of a clock travels through in the given time.

47. 40 seconds
48. 50 seconds
49. 35 seconds
50. 2 minutes 15 seconds
51. 3 minutes 25 seconds
52. 1 minute 55 seconds

53. The second hand on a clock is 6 cm long. How far does its tip travel in 40 seconds?

54. The second hand on a clock is 5 cm long. How far does its tip travel in 2 minutes and 15 seconds?

55. If the radius of the circle in the figure is 20 cm and $\ell = 85$ cm, what is the radian measure of the angle θ?



56. Find the radian measure of the angle θ in the preceding figure if the diameter of the circle is 150 cm and $\ell = 360$ cm.

In Exercises 57–60, assume that a wheel on a car has radius 36 cm. Find the angle (in radians) that the wheel turns while the car travels the given distance.

57. 2 meters (200 cm)
58. 5 meters
59. 720 meters
60. 1 kilometer (1000 meters)

In Exercises 61–64, find the length of the circular arc with the central angle whose radian measure is given. Assume that the circle has diameter 10.

61. 1 radian
62. 2 radians
63. 1.75 radians
64. 2.2 radians