3.7: Rate of Change
"I WILL …
Find the average rate of change of a function."

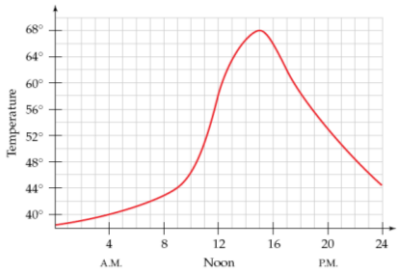
I. Rate of Change
A. Average Rate of Change of $f(x)$ from $x =$ _____ to $x =$ _____ .
B. Average rate of change is like slope.
C. Instead of $\frac{\Delta y}{\Delta x}$ it will be $\bar{X} =$ ——————— .
D. _____ means average slope
E. The data points are $(a, f(a))$ and _____
F. Plug your a and b in to get the corresponding $f(a)$ and $f(b)$.
G. Plug into the formula and reduce.
H. Average Rate of Change uses only endpoints of the interval. It does not reflect any fluctuations within the interval.

II. Difference Quotient
A. Average Rate of Change of $f(x)$ from $x =$ _____ to $x =$ _____ .
Examples of Rate of Change from 4 to 4.01, 4 to 4.001, 4 to 4.0001, etc…
B. The difference quotient finds a formula for the average rate of change,
C. Once found, it is much easier to use than calculating the slope of the secant line over and over.

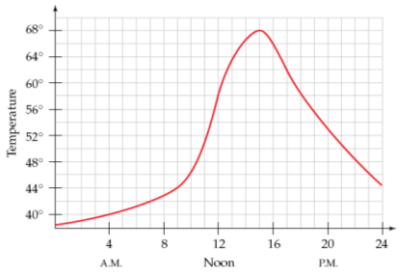
III. Model Problems

Ex 1: A rock that is dropped travels a distance given by $d(t) = 16t^2$ where d is the distance in feet and t is time in seconds. Find the distance and the average speed of the falling rock from $t = 1$ to $t = 3$ seconds.	Ex 2: A rock that is dropped travels a distance given by $d(t) = 16t^2$ where d is the distance in feet and t is time in seconds. Find the average speed of the falling rock from $t = 2$ to $t = 4.5$ seconds.	Your Turn: A balloon is being filled with water. Its approximate volume in gallons is $V(x) = \frac{x^3}{55}$, where v is the volume in gallons and x is radius of the balloons in inches. Find the average rate of change of the volume of the balloon as the radius increases from 5 to 10 inches. Round answer to 3 decimals.

Ex 3: The graph of the temperature function f during a particular day is given below. The temperature at x hours after midnight if $f(x)$. What is the average rate of change of the temperature from 4am to noon?



Your Turn: The graph of the temperature function f during a particular day is given below. The temperature at x hours after midnight if $f(x)$. What is the average rate of change of the temperature from Noon to 5pm?



Ex 4: Find a formula for the average speed of a falling rock from time x to time $x + h$. Use the formula to find the average speed from 3 to 3.1 seconds, using the equation $d(t) = 16t^2$.

Ex 5: Find a formula for the average speed of a falling rock from time x to time $x + h$. Use the formula to find the average speed from 3 to 3.01 seconds, using the equation $d(t) = 16t^2$ where distance is in feet.

Ex 6: Find a formula for the average speed of a falling rock from time x to time $x + h$. Use the formula to find the average speed from 3 to 3.001 seconds, using the equation $d(t) = 16t^2$.

Your Turn: Find a formula for the average speed of a falling rock from time x to time $x + h$. Use the formula to find the average speed in meters from a) 2.8 to 3 seconds and b) 2.8 to 4 seconds, using the equation $d(t) = 4.9t^2$.

Pg. 220: 1, 3, 5A, 5C, 5E, 9, 11, 15-21 odd, 23, 25 a-d

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Exercises 3.7

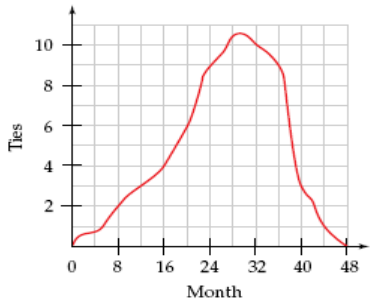
1. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table below. Find the average speed of the car over each interval.
a. 0 to 10 seconds
b. 10 to 20 seconds
c. 20 to 30 seconds
d. 15 to 30 seconds

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	20	140	400	680	1400	1800

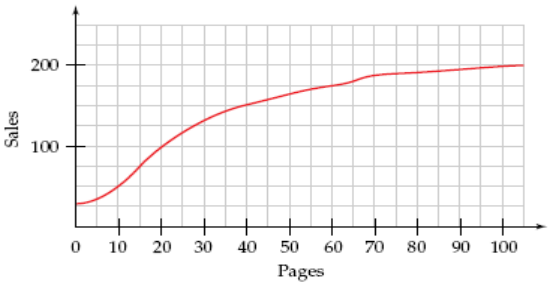
3. Find the average rate of change of the volume of the balloon in Example 2 as the radius increases
a. from 2 to 5 inches. **b.** from 4 to 8 inches.

4. Find the average rate of change of cost for the company in Example 3 when production increases from
a. 5 to 25 desks. **b.** 0 to 40 desks.

5. The graph in the figure shows the monthly sales of floral pattern ties (in thousands of ties) made by a company over a 48-month period. Sales are very low when the ties are first introduced; then they increase significantly, hold steady for a while, and then drop off as the ties go out of fashion. Find the average rate of change of sales (in ties per month) over the given interval.
a. 0 to 12 **b.** 8 to 24
c. 12 to 24 **d.** 20 to 28
e. 28 to 36 **f.** 32 to 44
g. 36 to 40 **h.** 40 to 48



6. A certain company has found that its sales are related to the amount of advertising it does in trade magazines. The graph in the figure shows the sales (in thousands of dollars) as a function of the amount of advertising (in number of magazine ad pages). Find the average rate of change of sales when the number of ad pages increases from
a. 10 to 20. **b.** 20 to 60.
c. 60 to 100. **d.** 0 to 100.
e. Is it worthwhile to buy more than 70 pages of ads if the cost of a one-page ad is $2000? if the cost is $5000? if the cost is $8000?



7. When blood flows through an artery (which can be thought of as a cylindrical tube) its velocity is greatest at the center of the artery. Because of friction along the walls of the tube, the blood's velocity decreases as the distance r from the center of the artery increases, finally becoming 0 at the wall of the artery. The velocity (in centimeters per second) is given by the function

$$v = 18{,}500(0.000065 - r^2)$$

where r is measured in centimeters. Find the average rate of change of the velocity as the distance from the center changes from
a. $r = 0.001$ to $r = 0.002$.
b. $r = 0.002$ to $r = 0.003$.
c. $r = 0$ to $r = 0.025$.

8. A car is stopped at a traffic light and begins to move forward along a straight road when the light turns green. The distance (in feet) traveled by the car in t seconds is given by $s(t) = 2t^2$ for $0 \le t \le 30$ What is the average speed of the car from
a. $t = 0$ to $t = 5$? **b.** $t = 5$ to $t = 10$?
c. $t = 10$ to $t = 30$? **d.** $t = 10$ to $t = 10.1$?

In Exercises 9–14, find the average rate of change of the function f over the given interval.

9. $f(x) = 2 - x^2$
from $x = 0$ to $x = 2$

10. $f(x) = 0.25x^4 - x^2 - 2x + 4$
from $x = -1$ to $x = 4$

11. $f(x) = x^3 - 3x^2 - 2x + 6$
from $x = -1$ to $x = 3$

12. $f(x) = -\sqrt{2x^2 - x + 4}$
from $x = 0$ to $x = 3$

In Exercises 15–22, compute the difference quotient of the function.

15. $f(x) = x + 5$

16. $f(x) = 7x + 2$

17. $f(x) = x^2 + 3$

18. $f(x) = x^2 + 3x - 1$

19. $f(t) = 160{,}000 - 8000t + t^2$

20. $V(x) = x^3$

21. $A(r) = \pi r^2$

22. $V(p) = \dfrac{5}{p}$

23. Water is draining from a large tank. After t minutes there are $160{,}000 - 8000t + t^2$ gallons of water in the tank.

a. Use the results of Exercise 19 to find the average rate at which the water runs out in the interval from 10 to 10.1 minutes.

b. Do the same for the interval from 10 to 10.01 minutes.

c. Estimate the rate at which the water runs out after exactly 10 minutes.

24. Use the results of Exercise 20 to find the average rate of change of the volume of a cube whose side has length x as x changes from

a. 4 to 4.1. **b.** 4 to 4.01. **c.** 4 to 4.001.

d. Estimate the rate of change of the volume at the instant when $x = 4$.

25. Use the results of Exercise 21 to find the average rate of change of the area of a circle of radius r as r changes from

a. 3 to 3.5. **b.** 3 to 3.2. **c.** 3 to 3.1.

d. Estimate the rate of change at the instant when $r = 3$.

e. How is your answer in part **d** related to the circumference of a circle of radius 3?