3.6: Inverse Functions
"I WILL …
Find the inverse of a function and determine if it is an inverse.
Verify inverses using composition of function.

I. Inverse Function

A. The result of exchanging the input and output value of a relation is an Inverse Function
B. An inverse "undoes" the function. It switches (x, y) to (y, x)
C. Interchange the x and the y. (make $y \rightarrow x$ and make $x \rightarrow y$)
D. Resolve for y.
E. You may need to do the "SAT trick" $x = \frac{3}{y}$ becomes $y = \frac{3}{x}$
F. Write in function notation $f^{-1}(x)$

II. One-to-One functions

A. One-to-One Functions are functions that each x has only one y-value and each y has only one x-value. One-to-one functions pass both the vertical and _________________ line test.
B. If a function is one-to-one, its inverse is also a function.
C. ___________________________________ determines if the function is one-to-one.

III. Verify Using Composition of Functions.

A. Composite $f(g(x))$. Take the $g(x)$ function and substitute this into the f-function and simplify. For $g(f(x))$ take $f(x)$ function and substitute this into the g-function and simplify.
B. Notation $f(g(x))$ is also $(f \circ g)(x)$ and $(g(f(x))$ is $(g \circ f)(x)$
C. If 2 functions are inverses then $f(g(x)) = x$ and $g(f(x)) = x$.

IV. Model Problems

Ex 1: Determine the inverse of this relation, {(0, –3), (2, 1), and (6, 3)}	Ex 2: Determine the inverse of $y = 3x - 2$
Ex 3: Determine the inverse of $f(x) = \frac{5}{x-2}$	Your Turn: Determine the inverse of $f(x) = \frac{4x-3}{2}$

Ex 4: Determine the inverse of $y = 4x^2$

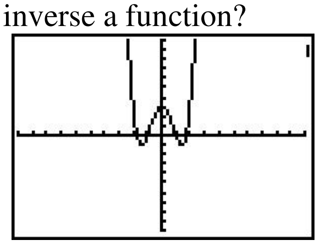
Your Turn: Determine the inverse of $y = 3x^2 - 5$

Ex 5: Determine the inverse of $y = \sqrt[5]{\dfrac{3x-1}{x-2}}$

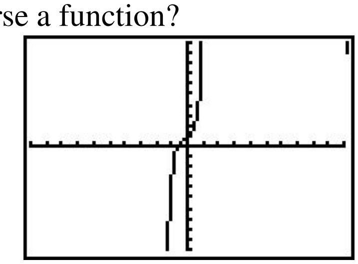
Your Turn: Determine the inverse of $y = \dfrac{x+1}{x-2}$

Your Turn: Determine the inverse of $f(x) = \dfrac{8x-4}{2x+6}$

Ex 6: Use a calculator to determine if the relation is a function, $f(x) = x^4 - 4x^2 + 3$. If so, is the inverse a function?



Your Turn: Use a calculator to determine if the relation is a function, $f(x) = 7x^5 + 3x^4 - 2x^3 + 2x + 1$. If so, is the inverse a function?



Ex 7: Prove that $f(x) = 2x - 6$ and $g(x) = \frac{x}{2} + 3$ are inverses through a composition.	Ex 8: Prove that $f(x) = \frac{x-2}{5}$ and $g(x) = 5x + 2$ are inverses through a composition.

Your Turn: Prove that $f(x) = 2x - 1$ and $g(x) = \frac{1}{2}x + \frac{1}{2}$ are inverses through a composition.

Assignment: Page 212: 1, 3, 9-29 EOO (25 and 29 use Graphing Calculator), 31 (identify the inverse only), 45, 47, 49

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Exercises 3.6

In Exercises 1 and 2, write a table that represents the inverse of the function given by the table.

1.

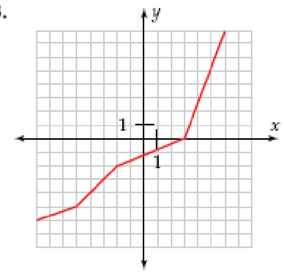
x	$f(x)$
1	4
2	2
3	3
4	6
5	1

2.

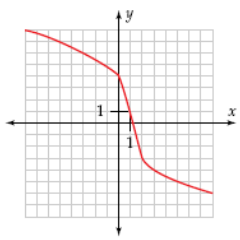
x	$g(x)$
−1	4
0	3
1	4
2	1
3	5

In Exercises 3 and 4, the graph of a function f is given. Sketch the graph of the inverse function of f and give the coordinates of three points on the inverse.

3.



4.



In Exercises 5–8, graph f and its inverse in parametric mode (see Example 2).

5. $f(x) = x^3 - 3x^2 + 2$ **6.** $f(x) = \sqrt[3]{x^2 - 1}$

7. $f(x) = x^4 - 3x^2$ **8.** $f(x) = \sqrt{x^2 + 1}$

In Exercises 9–22, find the rule for the inverse of the given function. Solve your answers for y and, if possible, write in function notation (see Examples 3 and 4).

9. $f(x) = -x$ **10.** $f(x) = -x + 1$

11. $f(x) = 5x^2 - 4$ **12.** $f(x) = -3x^2 + 5$

13. $f(x) = 5 - 2x^3$ **14.** $f(x) = (x^5 + 1)^3$

15. $f(x) = \sqrt{4x - 7}$ **16.** $f(x) = 5 + \sqrt{3x - 2}$

17. $f(x) = \frac{1}{x}$ **18.** $f(x) = \frac{1}{\sqrt{x}}$

19. $f(x) = \frac{1}{2x^2 + 1}$ **20.** $f(x) = \frac{x}{x^2 + 1}$

21. $f(x) = \frac{x^3 - 1}{x^3 + 5}$ **22.** $f(x) = \sqrt[5]{\frac{3x - 1}{x - 2}}$

In Exercises 23–30, use a calculator and the Horizontal Line Test to determine whether the function f is one-to-one.

23. $f(x) = x^4 - 4x^2 + 3$

24. $f(x) = x^4 - 4x + 3$

25. $f(x) = x^3 + x - 5$

26. $f(x) = \begin{cases} x - 3 & \text{for } x \le 3 \\ 2x - 6 & \text{for } x > 3 \end{cases}$

27. $f(x) = x^5 + 2x^4 - x^2 + 4x - 5$

29. $f(x) = 0.1x^3 - 0.1x^2 - 0.005x + 1$

30. $f(x) = 0.1x^3 + 0.005x + 1$

In Exercises 31–36, each given function has an inverse function. Sketch the graph of the inverse function.

31. $f(x) = \sqrt{x + 3}$

32. $f(x) = \sqrt{3x - 2}$

33. $f(x) = 0.3x^5 + 2$

34. $f(x) = \sqrt[3]{x + 3}$

35. $f(x) = \sqrt[5]{x^3 + x - 2}$

36. $f(x) = \begin{cases} x^2 - 1 & \text{for } x \le 0 \\ -0.5x - 1 & \text{for } x > 0 \end{cases}$

In Exercises 37–44, none of the functions is one-to-one. State at least one way of restricting the domain of the function so that the restricted function has an inverse that is a function. Then find the rule of the inverse function (see Example 6).

37. $f(x) = |x|$ **38.** $f(x) = |x - 3|$

39. $f(x) = -x^2$ **40.** $f(x) = x^2 + 4$

41. $f(x) = \frac{x^2 + 6}{2}$ **42.** $f(x) = \sqrt{4 - x^2}$

43. $f(x) = \frac{1}{x^2 + 1}$

44. $f(x) = 3(x + 5)^2 + 2$

In Exercises 45–50, use composition to show that f and g are inverses of each other (see Example 7).

45. $f(x) = x + 1$ $g(x) = x - 1$

46. $f(x) = 2x - 6$ $g(x) = \frac{x}{2} + 3$

47. $f(x) = \frac{1}{x + 1}$ $g(x) = \frac{1 - x}{x}$

48. $f(x) = \frac{-3}{2x + 5}$ $g(x) = \frac{-3 - 5x}{2x}$

49. $f(x) = x^5$ $g(x) = \sqrt[5]{x}$

50. $f(x) = x^3 - 1$ $g(x) = \sqrt[3]{x + 1}$