3.5: Function Operations
"I WILL…
Determine the sum, difference, product, and quotient of functions.
Determine domains of functions and composite functions."

I. Operations

A. To add/subtract functions you must _____ _____ _____.

B. To multiply functions and divide functions, _____ properly with _____.

C. The domain for $+,-,\times$ will be the same but division may be different.

D. To determine the domain you may recall:

A. In a fraction, _____.

B. In a square root, _____.

C. Otherwise it is all real numbers. $(-\infty,\infty)$

II. Composite Functions

A. Determine what is substituted
B. Take the _____ function and replace it
C. Take the _____ function and bring it down
D. _____ with the leftover variable
E. Simplify the expression
F. **Notation:** They may give you $f(g(x))$ or $(f \circ g)$. They mean the same thing.

III. Model Problems

Ex 1: If given $f(x)= 3x^2 + x$ and $g(x)= 4x - 2$, solve $(f + g)(x)$	Your Turn: If given $f(x)= 3x^2 + x$ and $g(x)= 4x - 2$, solve $(f - g)(x)$

Ex 2: If given $f(x) = \sqrt{2x}$ and $g(x) = \sqrt{x^2 - 4}$, solve $(fg)(x)$		Your Turn: If given $f(x) = \sqrt{2x}$ and $g(x) = \sqrt{x^2 - 4}$, solve $(f/g)(x)$
Ex 3: If given $f(x) = 4x$ and $g(x) = 2 - x$, solve $f(g(x))$	Ex 4: If given $f(x) = 4x$ and $g(x) = 2 - x$, solve $g(f(x))$	Ex 5: If given $f(x) = 4x$ and $g(x) = 2 - x$, solve $f(f(x))$
Your Turn: If given $f(x) = 4x$ and $g(x) = 2 - x$, solve $g(f(2))$		Ex 6: If given $f(x) = 4x$ and $g(x) = 2 - x$, solve $g(g(3))$

Ex 7: If given $f(x) = x^2 + 2x - 1$ and $g(x) = \frac{1}{x-3}$, solve $(g \circ f)(-2)$	Ex 8: If given $f(x) = x^2 + 2x - 1$ and $g(x) = \frac{1}{x-3}$, solve $(g \circ f)(x)$	Your Turn: If given $f(x) = x^2 + 2x - 1$ and $g(x) = \frac{1}{x-3}$, solve $(f \circ g)(x)$

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1-29 EOO
1, 9, 21 no need to label domain

Page 196: 1-29 EOO, 1, 9, 21 no need to label domain

Exercises 3.5

In Exercises 1–4, find $(f+g)(x)$, $(f-g)(x)$, $(g-f)(x)$, and their domains.

1. $f(x) = -3x + 2 \quad g(x) = x^3$
2. $f(x) = x^2 + 2 \quad g(x) = -4x + 7$
3. $f(x) = \frac{1}{x} \quad g(x) = x^2 + 2x - 5$
4. $f(x) = \sqrt{x} \quad g(x) = x^2 + 1$

In Exercises 5–7, find $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, and $\left(\frac{g}{f}\right)(x)$.

5. $f(x) = -3x + 2 \quad g(x) = x^3$
6. $f(x) = 4x^2 + x^4 \quad g(x) = \sqrt{x^2 + 4}$
7. $f(x) = \sqrt{x^2 - 1} \quad g(x) = \sqrt{x - 1}$

In Exercises 8–11, find the domains of fg and $\frac{f}{g}$.

8. $f(x) = x^2 + 1 \quad g(x) = \frac{1}{x}$
9. $f(x) = x + 2 \quad g(x) = \frac{1}{x + 2}$
10. $f(x) = \sqrt{4 - x^2} \quad g(x) = \sqrt{3x + 4}$
11. $f(x) = 3x^2 + x^4 + 2 \quad g(x) = 4x - 3$

In Exercises 12–14, find $(g \circ f)(3)$, $(f \circ g)(1)$, and $(f \circ f)(0)$.

12. $f(x) = 3x - 2 \quad g(x) = x^2$
13. $f(x) = |x + 2| \quad g(x) = -x^2$
14. $f(x) = x^2 - 1 \quad g(x) = \sqrt{x}$

In Exercises 15–18, find the indicated values, where $g(t) = t^2 - t$ and $f(x) = 1 + x$.

15. $g(f(0)) + f(g(0))$
16. $(f \circ g)(3) - 2f(1)$
17. $g(f(2) + 3)$
18. $f(2g(1))$

In Exercises 19–22, find the rule of the function $g \circ f$ and its domain and the rule of $f \circ g$ and its domain.

19. $f(x) = x^2 \quad g(x) = x + 3$
20. $f(x) = -3x + 2 \quad g(x) = x^3$
21. $f(x) = \frac{1}{x} \quad g(x) = \sqrt{x}$
22. $f(x) = \frac{1}{2x + 1} \quad g(x) = x^2 - 1$

In Exercises 23–26, find the rules of the functions ff and $f \circ f$.

23. $f(x) = x^3$
24. $f(x) = (x - 1)^2$
25. $f(x) = \frac{1}{x}$
26. $f(x) = \frac{1}{x - 1}$

In Exercises 27–30, verify that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ for the given functions f and g.

27. $f(x) = 9x + 2 \quad g(x) = \frac{x - 2}{9}$
28. $f(x) = \sqrt[3]{x - 1} \quad g(x) = x^3 + 1$
29. $f(x) = \sqrt[3]{x} + 2 \quad g(x) = (x - 2)^3$
30. $f(x) = 2x^3 - 5 \quad g(x) = \sqrt[3]{\frac{x + 5}{2}}$

In Exercises 31–36, write the given function as the composite of two functions, neither of which is the identity function, $f(x) = x$. (There may be more than one possible answer.)

31. $f(x) = \sqrt[3]{x^2 + 2}$
32. $g(x) = \sqrt{x + 3} - \sqrt[3]{x + 3}$
33. $h(x) = (7x^3 - 10x + 17)^7$