10.6A: Dot Product
"I WILL…
…operate using Dot Product"

I. Definitions

A. The dot product (also called inner product) of two vectors

B. The product of u $<u_1, u_2>$ and v $<v_1, v_2>$ *is* $u \bullet v = u_1v_1 + u_2v_2$ as it yields to a scalar (known as a number)

C. The product of u $<u_1, u_2>$, v $<v_1, v_2>$, and w $<w_1, w_2>$ is $u \bullet v \bullet w$ yields to a vector

Ex 1: Find the dot product of $u \bullet v$ where $u = <4, 5>$ and $v = <2, 3>$	Ex 2: Find the dot product of $u \bullet v$ where $u = 8i - 2j$ and $v = 4i - 3j$
Your Turn: Find the dot product of $u \bullet v$ where $u = 2i - 3j$ and $v = i + 5j$	Ex 3: Find the dot product of $u \bullet (v + w)$ where $u = <2, 5>$, $v = <-4, 3>$, and $w = <2, -1>$
Your Turn: Find the dot product of $u \bullet 2v$ where $u = <-2, 5>$ and $v = <1, 2>$	

II. The Angle between Two Vectors

A. If θ is the angle between two nonzero vectors u and v: $\cos\theta = \dfrac{uv}{\|u\|\|v\|}$

Ex 4: Find the angle between $u = <4, 3>$ and $v = <3, 5>$	Ex 5: Find the angle between $u = 2i - 3j$ and $v = i - 2j$

Your Turn: Find the angle between $u = i - 3j$ and $v = 2j$

III. Definition of Vectors

A. Parallel: If the slopes are the same (y/x)

B. Orthogonal: If the slopes are perpendicular (dot product = 0)

C. Neither: the slopes are neither parallel or perpendicular

IV. Steps

A. Find the dot product first to determine whether it is orthogonal (perpendicular)

B. If not, apply the equations, $v = ku$ and solve for k.

1. If the k on both equations are equal, the slope is parallel
2. If the k on both equations are not equal, the slope is neither.

Ex 6: Are the vectors orthogonal, parallel or neither: $u = \langle 2, -3 \rangle$ and $v = \langle 6, 4 \rangle$	Ex 7: Are the vectors orthogonal, parallel or neither: $u = \langle 6, 3 \rangle$ and $v = \langle 8, 4 \rangle$
Ex 8: Are the vectors orthogonal, parallel or neither: $u = \langle 4, 7 \rangle$ and $v = \langle 5, 1 \rangle$	Your Turn: Are the vectors orthogonal, parallel or neither: $u = \langle 3, -7 \rangle$ and $v = \langle -9, 21 \rangle$
Ex 9: Solve for k so that vectors are orthogonal: $u = 3i - 2j$ and $v = 4i + kj$	Your Turn: Solve for k so that vectors are orthogonal: $u = -4i + 5j$ and $v = 2i + 2kj$

Assignment: Page 679: 3-11 EOO, 13-27 odd

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Exercises 10.6.A

In Exercises 1–6, find u · v, u · u, and v · v.

1. $u = \langle 3, 4\rangle$, $v = \langle -5, 2\rangle$
2. $u = \langle -1, 6\rangle$, $v = \left\langle -4, \frac{1}{3}\right\rangle$
3. $u = 2i + j$, $v = 3i$
4. $u = i - j$, $v = 5j$
5. $u = 3i + 2j$, $v = 2i + 3j$
6. $u = 4i - j$, $v = -i + 2j$

In Exercises 7–12, find the dot product when $u = \langle 2, 5\rangle$, $v = \langle -4, 3\rangle$, and $w = \langle 2, -1\rangle$.

7. $u \cdot (v + w)$
8. $u \cdot (v - w)$
9. $(u + v) \cdot (v + w)$
10. $(u + v) \cdot (u - v)$
11. $(3u + v) \cdot 2w$
12. $(u + 4v) \cdot (2u + w)$

In Exercises 13–18, find the angle between vectors u and v.

13. $u = \langle 4, -3\rangle$, $v = \langle 1, 2\rangle$
14. $u = \langle 2, 4\rangle$, $v = \langle 0, -5\rangle$
15. $u = 2i - 3j$, $v = -i$
16. $u = 2j$, $v = 4i + j$
17. $u = \sqrt{2}\,i + \sqrt{2}\,j$, $v = i - j$
18. $u = 3i - 5j$, $v = -2i + 3j$

In Exercises 19–24, determine whether the vectors u and v are parallel, orthogonal, or neither.

19. $u = \langle 2, 6\rangle$, $v = \langle 3, -1\rangle$
20. $u = \langle -5, 3\rangle$, $v = \langle 2, 6\rangle$
21. $u = \langle 9, -6\rangle$, $v = \langle -6, 4\rangle$
22. $u = -i + 2j$, $v = 2i - 4j$
23. $u = 2i - 2j$, $v = 5i + 8j$
24. $u = 6i - 4j$, $v = 2i + 3j$

In Exercises 25–28, find a real number k such that vectors u and v are orthogonal.

25. $u = 2i + 3j$, $v = 3i - kj$
26. $u = -3i + j$, $v = 2ki - 4j$
27. $u = i - j$, $v = ki + \sqrt{2}\,j$
28. $u = -4i + 5j$, $v = 2i + 2kj$

In Exercises 29–32, find $\text{proj}_u v$ and $\text{proj}_v u$.

29. $u = 3i - 5j$, $v = 6i + 2j$
30. $u = 2i - 3j$, $v = i + 2j$
31. $u = i + j$, $v = i - j$
32. $u = 5i + j$, $v = -2i + 3j$

In Exercises 33–36, find $\text{comp}_v u$.

33. $u = 10i + 4j$, $v = 3i - 2j$
34. $u = i - 2j$, $v = 3i + j$
35. $u = 3i + 2j$, $v = -i + 3j$
36. $u = i + j$, $v = -3i - 2j$

In Exercises 37–39, let $u = \langle a, b\rangle$, $v = \langle c, d\rangle$ and $w = \langle r, s\rangle$. Verify that the given property of dot products is valid by calculating the quantities on each side of the equal sign.

37. $u \cdot (v + w) = u \cdot v + u \cdot w$
38. $ku \cdot v = k(u \cdot v) = u \cdot kv$
39. $0 \cdot u = 0$
40. Suppose $\mathbf{u} = \langle a, b\rangle$ and $\mathbf{v} = \langle c, d\rangle$ are nonzero parallel vectors.
    a. If $c \neq 0$, show that **u** and **v** lie on the same nonvertical straight line through the origin.
    b. If $a \neq 0$, show that $\mathbf{v} = \frac{c}{a}\mathbf{u}$ (that is, **v** is a scalar multiple of **u**). *Hint:* The equation of the line on which **u** and **v** lie is $y = mx$ for some constant m (why?), which implies that $b = ma$ and $d = mc$.
    c. If $c = 0$, show that **v** is a scalar multiple of **u**.