10.6: Applications of Vectors
"I WILL…
…apply vectors to real-life scenarios."

I. Unit Vectors

A. Have a magnitude of 1

II. Steps

A. Solve ||*v*||

B. To find a unit vector *u* that has the same direction as vector *v*: $u = \frac{v}{\|v\|} = \left\langle \frac{x_1}{\|v\|}, \frac{y_1}{\|v\|} \right\rangle$

E*x* 1: Find the unit vector *u* with the same direction as the vector *v* = <3, 4>.	E*x* 2: Find the unit vector *u* with the same direction as the vector *v* = <5, 12>.
Your Turn: Find the unit vector *u* with the same direction as the vector *v* = <–8, 6>.	

III. Magnitude and Vector are given

A. Solve ||*v*||

B. To find a unit vector *u* that has the same direction as vector *v*: $u = \frac{v}{\|v\|} = \left\langle \frac{x_1}{\|v\|}, \frac{y_1}{\|v\|} \right\rangle$

C. Multiply the vector with the magnitude

Ex 3: Find the vector *v* with the given magnitude and same direction as *u* where ||*v*|| = 3 and u = <4, –4>.	Your Turn: Find the vector *v* with the given magnitude and same direction as *u* where ||*v*|| = 2 and *u* = <20, 21>.

IV. Linear Combination/Alternate Notation

A. Standard Unit Vectors: $i = \langle 1, 0 \rangle$ and $j = \langle 0, 1 \rangle$

B. Vectors $v = \langle v_1, v_2 \rangle$ is also represented as $v = i + j$

C. i and j are considered component vectors whereas $xi + yj$ is a linear combination of i and j

Ex 4: Let $v = \langle -5, 3 \rangle$. Write v as a linear combination of the standard unit vectors of i and j.	Ex 5: The initial point of a vector is (–6, 4) and the terminal point is (0, 1). Write a linear combination of the standard vector.
Ex 6: Find the component form of w and sketch the specified vector operations geometrically where $u = 2i - j$, $v = i + 2j$, and $w = -u + 2v$	Ex 7: Find the component form of w where $u = i - j$, $v = 3i + j$, and $w = 3u - 2v$
Your Turn: Find the component form of w where $u = i - j$, $v = 3i + j$, and $w = u - v$	

V. Direction Angles

A. Direction Angle: measured counterclockwise from the x–axis to terminal point of u

B. $u = \langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle = \cos\theta i + \sin\theta j$

C. $v = \|v\| \langle \cos\theta, \sin\theta \rangle = \|v\| \cos\theta i + \|v\| \sin\theta j$

D. Since $v = ai + bj$, $\tan\theta = b/a$ and use reference angles and Unit Circle when possible

Ex 8: Find the magnitude and direction angle of $u = 3i + 3j$	Ex 9: Find the magnitude and direction angle of $u = 3i - 4j$
Your Turn: Find the magnitude and direction angle of $v = -4i - 5j$	

VI. More Resultant Force Steps

A. Identify the two forces and then add them

B. $u = ai + bj$ represents the vector where $u = \|v\|\cos\theta\, i + \|v\|\sin\theta\, j$

C. Identify the magnitude and direction using the appropriate equations

Ex 10: A Boeing 727 airplane, flying due east at 500 mph in still air, encounters a 70–mph tail wind acting in the direction of 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?	Your Turn: An airplane is traveling at a speed of 500 mph with a bearing of 330°. The airplane encounters a wind blowing 70 mph in the direction N 45° E. What are the resultant speed and direction of the airplane?

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Exercises 10.6

In Exercises 1–5, find u + v, u − v, and 3u − 2v.

1. $\mathbf{u} = \mathbf{i} - \mathbf{j}, \mathbf{v} = 2\mathbf{i} + \mathbf{j}$
2. $\mathbf{u} = 8\mathbf{i}, \mathbf{v} = 2(3\mathbf{i} - 2\mathbf{j})$
3. $\mathbf{u} = -4(-\mathbf{i} + \mathbf{j}), \mathbf{v} = -3\mathbf{i}$
4. $\mathbf{u} = -\left(2\mathbf{i} + \frac{3}{2}\mathbf{j}\right), \mathbf{v} = \frac{3}{4}\mathbf{i}$
5. $\mathbf{u} = \sqrt{2}\mathbf{j}, \mathbf{v} = \sqrt{3}\mathbf{i}$

In Exercises 6–11, find the components of the given vector, where u = i − 2j, v = 3i + j, and w = −4i + j.

6. $\mathbf{u} + 2\mathbf{w}$
7. $\frac{1}{2}(3\mathbf{v} + \mathbf{w})$
8. $\frac{1}{2}\mathbf{w}$
9. $-2\mathbf{u} + 3\mathbf{v}$
10. $\frac{1}{4}(8\mathbf{u} + 4\mathbf{v} - \mathbf{w})$
11. $3(\mathbf{u} - 2\mathbf{v}) - 6\mathbf{w}$

In Exercises 12–19, find the component form of the vector v whose magnitude and direction angle θ are given.

12. $\|\mathbf{v}\| = 4, \theta = 0°$
13. $\|\mathbf{v}\| = 5, \theta = 30°$
14. $\|\mathbf{v}\| = 10, \theta = 225°$
15. $\|\mathbf{v}\| = 20, \theta = 120°$
16. $\|\mathbf{v}\| = 6, \theta = 40°$
17. $\|\mathbf{v}\| = 8, \theta = 160°$
18. $\|\mathbf{v}\| = \frac{1}{2}, \theta = 250°$
19. $\|\mathbf{v}\| = 3, \theta = 310°$

In Exercises 20–27, find the magnitude and direction angle of the vector v.

20. $\mathbf{v} = \langle 4, 4 \rangle$
21. $\mathbf{v} = \langle 5, 5\sqrt{3} \rangle$
22. $\mathbf{v} = \langle -8, 0 \rangle$
23. $\mathbf{v} = \langle 4, 5 \rangle$
24. $\mathbf{v} = 6\mathbf{j}$
25. $\mathbf{v} = 4\mathbf{i} - 8\mathbf{j}$
26. $\mathbf{v} = -2\mathbf{i} + 8\mathbf{j}$
27. $\mathbf{v} = -15\mathbf{i} - 10\mathbf{j}$

In Exercises 28–31, find a unit vector that has the same direction as the given vector.

28. $\langle 4, -5 \rangle$
29. $-7\mathbf{i} + 8\mathbf{j}$
30. $5\mathbf{i} + 10\mathbf{j}$
31. $-3\mathbf{i} - 9\mathbf{j}$

41. A force of 500 pounds is needed to pull a cart up a ramp that makes a 15° angle with the ground. Assuming that no friction is involved, find the weight of the cart. *Hint:* Draw a picture similar to Figure 10.6-6; the 500-pound force is parallel to the ramp.