

Stuff You Must Know Cold for AP Test – Calculus BC (Rev 2017-18)

Values of Trig Functions for Common Angles:

0°	$\sin \theta$	$\cos \theta$	$\tan \theta$
0			
$\pi/6$			
$\pi/4$			
$\pi/3$			
$\pi/2$			
π			

Careful with Trig Values: $\tan\left(\frac{3\pi}{4}\right) = -1$ but $\arctan(-1) = -\frac{\pi}{4}$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

Limits

Limits to know:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n =$$

Situations Limits Fail to Exist

- _____
- _____
- _____

Definition of Continuity:

A function is continuous at the point $x = c$ if and only if:

- _____
- _____
- _____

Intermediate Value Theorem

- _____
- _____
- _____
- Must be _____.

Derivatives

FORMAL Definition of Derivative

$$\frac{d}{dx}(f(x)) = \underline{\hspace{2cm}}$$

Alternate Form of Definition of a Derivative

$$\frac{d}{dx}(f(x)) \text{ at } x = c \text{ is } \underline{\hspace{2cm}}$$

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = \underline{\hspace{2cm}}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \underline{\hspace{2cm}}$$

Chain Rule

$$\frac{d}{dx}(f(g(x))) = \underline{\hspace{2cm}}$$

Situations Derivatives Fail to Exist

- _____
- _____
- _____

Derivatives

Where u is a function of x and c is a constant

$$\frac{d}{dx}(\sin u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\csc u) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(\cos u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\sec u) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(\tan u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\cot u) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(e^u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\ln u) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(a^u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\log_a u) = \underline{\hspace{2cm}}$$

$$(f^{-1})'(a) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(\sin^{-1} u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\csc^{-1} u) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(\cos^{-1} u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\sec^{-1} u) = \underline{\hspace{2cm}}$$

$$\frac{d}{dx}(\tan^{-1} u) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\cot^{-1} u) = \underline{\hspace{2cm}}$$

Curve Sketching and Analysis

Critical Values: $\frac{dy}{dx} = \underline{\hspace{2cm}}$ OR _____

Absolute/Global Max or Min: _____ Test

Local/Relative MINIMUM

If $f' = 0$ changes from _____ to _____

$$\text{OR } \frac{d^2y}{dx^2} \boxed{\hspace{1cm}} 0$$

Local/Relative MAXIMUM

If $f' = 0$ changes from _____ to _____

$$\text{OR } \frac{d^2y}{dx^2} \boxed{\hspace{1cm}} 0$$

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Point of Inflection

1) If $f''(c) = \underline{\hspace{2cm}}$ or $f''(c) = \underline{\hspace{2cm}}$ does not exist **AND**

2) if $f''(x)$ changes from $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$ at $x = c$ **OR** if $f'(x)$ changes from $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$ at $x = c$

Extreme Value Theorem

If $f(x)$ is $\underline{\hspace{2cm}}$ on $[a,b]$, then there exists both a $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.

The Mean Value Theorem (derivatives)

Slope of secant line = Slope of tangent line

$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Must be $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$

Particle Motion

Position = $\underline{\hspace{2cm}}$ Velocity = $\underline{\hspace{2cm}}$

Speed = $\underline{\hspace{2cm}}$ Acceleration = $\underline{\hspace{2cm}}$

Displacement = $\underline{\hspace{2cm}}$

Total Distance travelled = $\underline{\hspace{2cm}}$

Speed of object **increasing** when $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$ have $\underline{\hspace{2cm}}$ signs

Speed of object **decreasing** when $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$ have $\underline{\hspace{2cm}}$ signs

Integration

Where u is a function of x and c is a constant

$\int \cos u \, du = \underline{\hspace{2cm}}$ $\int \sin u \, du = \underline{\hspace{2cm}}$

$\int \sec^2 u \, du = \underline{\hspace{2cm}}$ $\int \csc^2 u \, du = \underline{\hspace{2cm}}$

$\int \sec u \tan u \, du = \underline{\hspace{2cm}}$ $\int \csc u \cot u \, du = \underline{\hspace{2cm}}$

$\int \tan u \, du = \underline{\hspace{2cm}}$ $\int \cot u \, du = \underline{\hspace{2cm}}$

$\int \sec u \, du = \underline{\hspace{2cm}}$ $\int \csc u \, du = \underline{\hspace{2cm}}$

$\int \frac{du}{u} = \underline{\hspace{2cm}}$ $\int e^u \, du = \underline{\hspace{2cm}}$

$\int a^u \, du = \underline{\hspace{2cm}}$ $\int \frac{du}{\sqrt{a^2 - u^2}} = \underline{\hspace{2cm}}$

$\int \frac{du}{a^2 + u^2} = \underline{\hspace{2cm}}$ $\int \frac{du}{u\sqrt{u^2 - a^2}} = \underline{\hspace{2cm}}$

Area Under the Curve (Trapezoids)

Riemann's Sum:

$\int_a^b f(x) \, dx = \underline{\hspace{2cm}}$ or $A = bh$

$\lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = \underline{\hspace{2cm}}$ (**Sigma equation**)

Trapezoidal Sum:

$\int_a^b f(x) \, dx = \underline{\hspace{2cm}}$ or $A = \frac{h(b_1 + b_2)}{2}$

1st Fundamental Theorem of Calculus

$\int_a^b f'(x) \, dx = \underline{\hspace{2cm}}$

Average Value of $f(x)$ on $[a,b]$:

$\underline{\hspace{2cm}}$

2nd Fundamental Theorem of Calculus

$\frac{d}{dx} \int_a^x f(t) \, dt = \underline{\hspace{2cm}}$

2nd Fundamental Theorem (Chain Rule):

$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) \, dt = \underline{\hspace{2cm}}$

Euler's Method:

Given that $\frac{dy}{dx} = f(x, y)$ and solution passes

through (x_0, y_0) : $\underline{\hspace{2cm}}$

Logistic Differential Growth Equation:

$\frac{dP}{dt} = \underline{\hspace{2cm}}$

Logistic Differential Growth Equation:

$P = \underline{\hspace{2cm}}$

Solids of Revolution

Area between two curves: $\underline{\hspace{2cm}}$

Disk: $\underline{\hspace{2cm}}$ Washer: $\underline{\hspace{2cm}}$

Volume by Cross Sections: $\underline{\hspace{2cm}}$

Squares:	Isosceles Triangle:
Semicircles:	Equilateral Triangle:

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Arc Length (Perimeter)

$$s = \int_a^b \text{_____}$$

Pythagorean Identities

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

L'Hôpital's Rule

If $\frac{f(a)}{g(b)} = \text{---}$ OR --- ,

then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \text{---}$

Integration by Parts

$$\int u dv = \text{_____}$$

Integration of ln

$$\int \ln x \, dx = x \ln x - x + C$$

In Polar Curves

$x = \text{_____}$ and $y = \text{_____}$

Slope of Polar Curve

$$\frac{dy}{dx} = \text{_____}$$

Area within a Polar Curve

$$A = \text{_____}$$

Slope of a Parametric Equation

$$\frac{dy}{dx} = \text{_____} = \text{_____}$$

Velocity Vector $\langle \text{---}, \text{---} \rangle$

Speed

$$\|v\| = \text{_____}$$

Total Distance

$$\int_{\text{initial time}}^{\text{final time}} |v(t)| \, dt = \text{_____}$$

Geometric Series: $\sum_{n=0}^{\infty} ar^n$

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1} + \dots = \sum_{n=0}^{\infty} ar^n$$

The series CONVERGES if: _____

The series DIVERGES if: _____

Sum: _____

Nth Term Test: $\sum_{n=1}^{\infty} a_n$

It DIVERGES if: _____

If the limit approaches 0, test is _____

(Test cannot be used to show convergence and converse is not true)

Ratio Test: $\sum_{n=0}^{\infty} ar^n$

It CONVERGES if: _____

It DIVERGES if: _____

If the limit equals to 1, test is _____

Root Test: $\sum_{n=1}^{\infty} a_n$

It CONVERGES if: _____

It DIVERGES if: _____

If the limit equals to 1, test is _____

Telescoping Test: $\sum_{n=1}^{\infty} (b_n - b_{n+1})$

It CONVERGES if it is expanded form and terms begin to “_____” out.

(Test cannot be used to show divergence. Infinite sum is the sum of the first few terms that do not subtract out.)

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p-Series Test: $\sum_{n=1}^{\infty} \frac{1}{n^p}$

The series CONVERGES if: _____

The series DIVERGES if: _____

If $p = 1$, then it is _____.

Integral Test: $\sum_{n=1}^{\infty} a_n$ where $a_n = f(n) \geq 0$

It CONVERGES if: _____

It DIVERGES if: _____

Remainder: $0 < R_n < \int_n^{\infty} f(x) dx$

Direct Comparison (a_n, b_n) Test: $\sum_{n=1}^{\infty} a_n$

It CONVERGES if $0 < \underline{\hspace{1cm}} \leq \underline{\hspace{1cm}}$ and

It DIVERGES if $0 < \underline{\hspace{1cm}} \leq \underline{\hspace{1cm}}$ and

(Note: $a_n > 0$ and $b_n > 0$)

Limit Comparison (a_n, b_n) Test: $\sum_{n=1}^{\infty} a_n$

It CONVERGES if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ where _____

It DIVERGES if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ where _____

(Note: $a_n > 0$ and $b_n > 0$ and L must be finite)

Alternating Series Test: $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$

The series CONVERGES if: _____ and

_____ (This test cannot be used to test for divergence)

Alternating Series Remainder

If $S_n = \sum_{k=1}^n (-1)^k a_n$ is the nth partial sum of a convergent alternating series, _____.

LaGrange's Error Bound:

If c equals a number centered on an x equals a value, approximate. There exists a z between x and c , such that:

$|R_n| \leq \underline{\hspace{2cm}} \left(\underline{\hspace{1cm}} \right)$

Find interval of $f^{n+1}(z)$ to find error interval.

Taylor Polynomial

If the function f has n derivatives at $x = c$, then the polynomial's equation for the nth term is:

$P_n(x) = \underline{\hspace{4cm}}$

Taylor Series

If function f is "smooth" at $x = c$, then f can be approximated by the nth degree polynomial

$f(x) = \underline{\hspace{4cm}}$

Maclaurin Series:

A Taylor Series about $x = 0$ is called Maclaurin.

$e^x = \underline{\hspace{4cm}}$

$\cos x = \underline{\hspace{4cm}}$

$\sin x = \underline{\hspace{4cm}}$

$\frac{1}{1-x} \approx \underline{\hspace{4cm}}$

$\ln(x+1) = \underline{\hspace{4cm}}$

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