May 21, 2018
TO: Next Year's AP Calculus BC/Dual Students
FROM: Mr. Dang, AP Calculus BC Teacher

Congratulations on your wisdom in taking the Calculus BC course. This course is primarily concerned with developing your understanding of the concepts of calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. To be successful in this course, you need the proper foundation (i.e. knowledge of algebra, geometry, trigonometry, analytic geometry, and elementary functions). You should be very familiar with the primary families of functions, and all their representations, to be successful in your study of calculus. The concept of functions underlies everything that calculus considers. You will also need to be able to carry out certain computational tasks (i.e. algebra skills) with efficiency and accuracy.

As you probably know, the students who take AP Calculus AB and pass the Advanced Placement Test will place out of **one** semester of college Calculus; those who take AP Calculus BC and pass the AP Test will place out of **two** semesters of college Calculus. To have enough time to learn the material of two college-level courses, we will start learning Calculus as soon as possible when school begins and will not be able to spend time reviewing the material you learned in Algebra, Geometry, and Precalculus.

This homework packet is due BLOCK DAY, AUGUST 29/30. It will be turned in for six formative grades for an accuracy grade. A quiz, over the material, will be presented on BLOCK DAY. (1.5 quiz grades)

My recommendation is that you start the packet in Mid-July. We want these skills to be fresh in your mind at the beginning of the school year. Also, do not wait to do them until the last minute as it presents a lack of time management. Tutorials will be available on **Tuesday, August 28**. Do not wait until the very last minute to solve these questions. Your time management skills will be tested in this course. I will not spend any class time going over this packet.

All the AP Calculus AB and BC classes at KPHS will be using the **TI-84 graphing calculators**. If you do not have one, we will have a class set for you to use.This class will help you prepare yourself for the rigorous coursework for college. If you lose this packet, download it at dangmath.com under Calculus BC.

Also, please add yourself to the Remind Text Messaging service. Please send an SMS text to the number: **81010** with the text of **@kphscalbc**.

This is a highly demanding class. It is unlike any other high school math course you have ever taken. If you are up to the challenge, I would like to welcome you to Calculus. It is a place where we will be experiencing differencing rate of change, every day. To be better, we must do better.

Sincerely,
Mr. Jim Dang,
viet.dang@humbleisd.net

CALCULUS AB
SUMMER PACKET – NO GRAPHING CALCULATOR

Work these problems on notebook paper. All work must be shown to earn full credit.

Topic I: Fractional and Negative Exponents.

Assistance: http://www.purplemath.com/modules/exponent5.htm

Simplify using only positive exponents.

1) $-3x^3$

2) $\left(27x^6\right)^{\frac{5}{3}}$

3) $x^4\left(-2x\right)^3\left(6x^0\right)^{-2}$

4) $\dfrac{\left(x^{-\frac{1}{2}}y^4\right)^{1/4}}{x^{\frac{2}{3}}y^{\frac{3}{2}}\cdot x^{-\frac{3}{2}}y^{\frac{1}{2}}}$

5) $5^x\cdot 5^{\left(\frac{7}{2}\right)x}$

6) $27^{1/3}\left(8\right)^{5/3}$

Topic II: Long Division.

Assistance: http://www.purplemath.com/modules/polydiv2.htm

Solve using Long Division. Show all steps.

1) $f\left(x\right)=\dfrac{x^3-12x^2+47x-60}{x-3}$

2) $f\left(x\right)=\dfrac{9x^5-9x^4-x^3-12x^2+x-11}{3x-5}$

3) $f\left(x\right)=\dfrac{6x^3+7x^2-1}{2x+1}$

4) Find all the roots using long division and sketch the graph of: $f\left(x\right)=\dfrac{3x^3-7x^2-22x+8}{3x-1}$

Topic III: Complex Fractions.

Assistance: http://www.purplemath.com/modules/compfrac.htm

Simplify using only positive exponents.

1) $\dfrac{x}{x-\frac{1}{2}}$

2) $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3) $\dfrac{x-\frac{1}{x}}{\frac{1}{x}+x}$

4) $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

5) $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}}$

6) $\dfrac{\frac{16}{x-3}-\frac{4}{x-4}}{\frac{16}{x^2}-\frac{x-4}{x-3}}$

Topic IV: Factoring and solving.

Assistance: http://www.purplemath.com/modules/factquad.htm
http://www.purplemath.com/modules/simpfact3.htm
http://www.purplemath.com/modules/solvquad3.htm

1) $10x^2 - 29x + 10$ 2) $9x^2y + 73xy + 70x$ 3) $x^3 + 8$

4) $2x^4 + 5x^3 - 3x^2$ 5) $2\sqrt{x} - 6x^{3/2}$ 6) $ac + cd - ab - bd$

Solve using Completing the Square. Show all steps. No calculator.

7) $x^2 + 7x + 6 = 0$ 8) $x^2 + 4x = 1$ 9) $10x^2 + 4x + 77 = 9$

Topic V: Compositions of Functions and Function Notation.

Assistance: http://www.purplemath.com/modules/fcncomp3.htm

Given: $f(x) = x^2 + 3$, $g(x) = 2x - 1$, and $h(x) = 2^x$, solve for the following three questions:

1) $f(g(2))$ 2) $g(f(1))$ 3) $f(h(-1))$

4) Find $f(1) - f(5)$ given $f(x) = |x - 3| - 7$

5) Find $f(x+2) - f(2)$ given $f(x) = x^2 - 3x + 4$

6) Find $f(x+h)$ given $f(x) = x^2 + 2x + 1$

7) Find $\dfrac{f(x+2) - f(x)}{h}$ given $f(x) = 3x^2 + 1$

Topic VI: Function Transformations.

Assistance: http://www.purplemath.com/modules/fcntrans.htm
http://www.purplemath.com/modules/graphrad.htm

If $f(x) = x^2 - 1$, describe in words what the following would do the graph of $f(x)$:

1) $f(x) + 3$ 2) $f(x+4)$ 3) $-f(x+2)$

Find the zeros (x-intercepts), domain and range from $(-\infty, \infty)$, and sketch the graph.

4) $y = x + 3$

5) $y = 2 - x^2$

6) $y = \sqrt{9 - x^2}$

7) $y = (x+2)^2$

8) $y = e^x$

9) $y = \ln x$

10) $f(x) = |x|$

11) $f(x) = \dfrac{1}{x}$

12) $f(x) = \dfrac{1}{x^2 + 1}$

13) $f(x) = \sqrt{x}$

15) $y = \begin{cases} x^2 + 1, & if\ x > 0 \\ -2x + 2, & if\ x \le 0 \end{cases}$

16) $y = \begin{cases} -1, & if\ x \le -1 \\ 3x + 2, & if\ -1 < x \le 1 \\ 7 - 2x, & if\ x > 1 \end{cases}$

Topic VII: Asymptotes and Intercepts.

Assistance: http://www.purplemath.com/modules/grphrtnl.htm

Find the asymptotes (horizontal, vertical, and slant/oblique), *x* and *y*-intercepts, domain, and sketch the graph. Do not use your graphing calculator on these.

1) $y=\dfrac{1}{x-1}$

2) $y=\dfrac{2\left(x^2-9\right)}{x^2-4}$

3) $y=\dfrac{x^2-5x-6}{x^2-3x-18}$

4) $y=\dfrac{1}{\left(x+2\right)^2}$

5) $y=\dfrac{x^2-4}{2x+4}$

6) $y=\sqrt{x^2-5x-14}$

Topic VIII: Even or Odd Functions.

Assistance: http://www.purplemath.com/modules/grphrtnl.htm

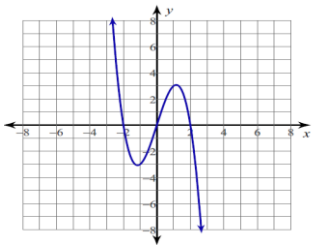
Show all your work to determine if the relation is even, odd, or neither through algebra (on questions 1-3)

1) $f\left(x\right)=x^2-7$

2) $f\left(x\right)=-4x^3-2x$

3) $y=-5x^4-4x^3+3x^2-1$

4)



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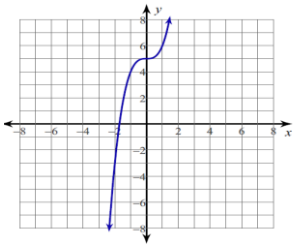

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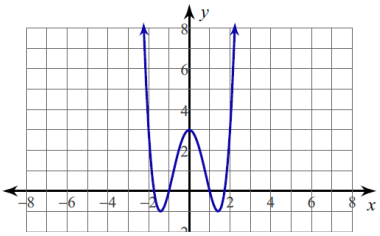


Topic IX: Trigonometry:

If the point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture.

1) $P\left(-2,4\right)$

2) $P\left(\sqrt{5},-2\right)$

3) If $\cos\theta=-\dfrac{5}{13}$, θ is in Quadrant II, find $\sin\theta$ and $\tan\theta$

4) If $\cot\theta=3$, θ is in Quadrant III, find $\sin\theta$ and $\cos\theta$

Identify zeros, domain and range, and sketch the graph of the following Trig Functions. No calculator.

1) $y=\sin x,\ -2\pi\le x\le 2\pi$

2) $y=\cos x,\ -2\pi\le x\le 2\pi$

3) $y=\tan x,\ -\pi\le x\le \pi$

4) $y=\csc x,\ -2\pi\le x\le 2\pi$

5) $y=\sec x,\ -2\pi\le x\le 2\pi$

6) $y=\cot x,\ -\pi\le x\le \pi$

Solve. Give exact answers in radians, $[0,2\pi)$. No calculator. KNOW THE UNIT CIRCLE.

1) $\cos\dfrac{5\pi}{6}$

2) $\sin\dfrac{3\pi}{2}$

3) $\tan\dfrac{5\pi}{4}$

4) $\sin\dfrac{7\pi}{4}$

5) $\cos\pi$

6) $\tan\dfrac{2\pi}{3}$

7) $\sec\dfrac{4\pi}{3}$

8) $\csc\dfrac{\pi}{4}$

9) $\cot\dfrac{2\pi}{3}$

10) $\tan\dfrac{\pi}{2}$

Solve. Give exact answers in radians, $[0, 2\pi)$. Remember the techniques such as U-Substitution for Trigonometry, Trigonometry Identities, and double and half angle formulas. No calculator.

1) $2\cos^2 x + 3\cos x - 2 = 0$

2) $\sin(2x) = \cos x$

3) $2\csc^2 x + 3\csc x - 2 = 0$

4) $2\cos\left(\frac{x}{3}\right) - \sqrt{3} = 0$

5) $2\sin^2 x - \cos x = 1$

6) $\tan^2 x - \sec x = 1$

7) $\tan(2x) = -\sqrt{3}$

8) $2\sin^2 x - \cos x = 1$

Topic X: Logarithms.

Condense each expression to a single logarithm.

1) $\frac{2\log 7}{3}$

2) $2\ln(x-3) + \ln(x+2) - 6\ln x$

3) $\frac{1}{3}\left[\log_2(x) + \log_2(x-4)\right]$

Expand each of the following with the law of logs.

4) $\log\frac{x}{y^5}$

5) $\log 5x^2y^4$

6) $\log_2\sqrt[3]{6x}$

GRAPHING CALCULATOR ALLOWED ON THIS SECTION

Solve. Use your calculator to solve. List any extraneous solutions.

1) $3^{1-2x} = 243$

2) $16^{x-7} + 5 = 24$

3) $e^{2x+3} = 37$

4) $e^{2x} - e^x - 12 = 0$

5) $\ln(5x-1) = 3$

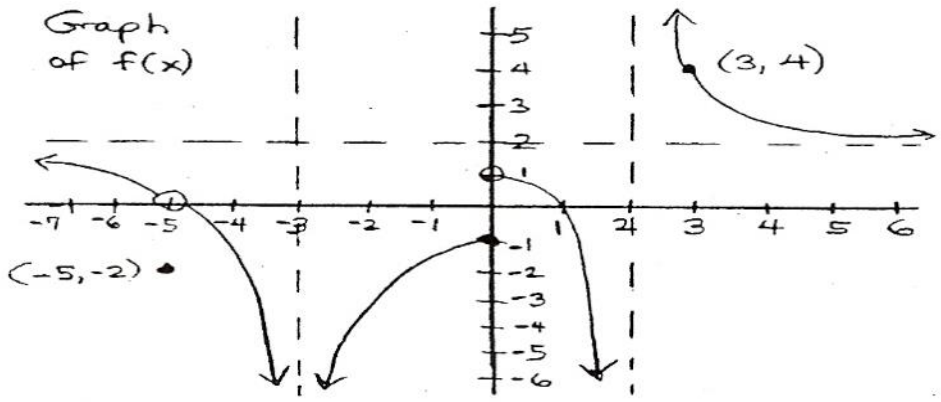
6) $\log_8(x+5) - \log_8(x-2) = 1$

7) $\log_2(x+3) + \log_2(x-1) = \log_2 12$

8) $\log_4(x^2 - 3x) = 1$

Topic XI: Limits.

Use the graph to solve. If the limit fails to exist, explain why.



1) $\lim_{x\to 3} f(x)$ 2) $\lim_{x\to 2^+} f(x)$ 3) $\lim_{x\to -\infty} f(x)$

4) $\lim_{x\to \infty} f(x)$ 5) $\lim_{x\to 0} f(x)$ 6) $\lim_{x\to -5} f(x)$

Evaluate. Show supporting work for each problem (algebraic steps or sketch). No calculator. If the limit fails to exist, explain why.

1) $\lim_{x\to -3} \dfrac{x^2+x-6}{x+3}$ 2) $\lim_{x\to -6} \dfrac{x+6}{x^2+3x-18}$ 3) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

4) $\lim_{x\to 3^+} x-1$ 5) $\lim_{x\to 3^-} x-1$ 6) $\lim_{x\to 3} x-1$

7) $\lim_{x\to 3^+} \dfrac{1}{x-3}$ 8) $\lim_{x\to 3^-} \dfrac{1}{x-3}$ 9) $\lim_{x\to 3} \dfrac{1}{x-3}$

10) $\lim_{x\to 5^+} \dfrac{3x-5}{5-x}$ 11) $\lim_{x\to \infty} \dfrac{6x^2-x+1}{-2x^2+4x+7}$ 12) $\lim_{x\to -\infty} \dfrac{2x-1}{|x|}$

13) $f(x)=\begin{cases} 1-x, & x\le 1 \\ x^2, & x>1 \end{cases}$

(a) $\lim_{x\to 1^-} f(x)$

(b) $\lim_{x\to 1^+} f(x)$

(c) $\lim_{x\to 1} f(x)$

14) If given f is a function that is differentiable on the open $I(1,10)$. If $f(2)=-5$, $f(5)=5$, and $f(9)=-5$, then explain why f has at least two roots. Justify with the appropriate definitions and terms.

Use the definition of the derivative to find the derivative. No calculator.

$$f'(x)=\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$ (You must **know** this formula.)

1) $f(x)=x^2-8x$ 2) $f(x)=\sqrt{x+9}$ 3) $f(x)=x^3+2x^2-x+4$