Calculus AB Review Sheet KEY – Fall Semester Rev 2018

Chapter 1: Limits

1a) $\frac{1}{6}$ 1b) 4, 1c) –1, 1d) $-\infty$, 1e) $\frac{1}{4}$, 1f) 125, 1g) DNE, 1h) 2, 1i) 3

2a) 3, 2b) 1, 2c) DNE, 2d) 3, 2e) DNE

3) f is continuous, $f(-3) \neq f(0)$, and $f(-3) < 0$ and $f(0) > 0$ f guarantees a zero in $I[-3,0]$

4) f is continuous, $f(a) \neq f(b)$, and $f(c) = k$ where c is in between a and b and $f(c)$ is in between $f(a)$ and $f(b)$

5) $f(c)$ is defined, limit of $f(x)$ where x approaches to c exists, and limit of $f(x)$ where it approaches to c equals $f(c)$

6a) –21, 6b) –125, 6c) 2, 6d) 0

7) $k = -2$

Chapter 2: Differentiation

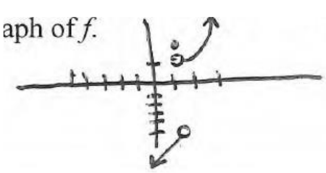
1) $y - 16 = 5(x - 2)$

2)
$$\lim_{h\to 0} \frac{2(x+h)^2-5(x+h)-(2x^2-5x)}{h}$$
$$\lim_{h\to 0} \frac{2x^2+4xh+2h^2-5h}{h}$$
$$\lim_{h\to 0} 4x+2h-5 = \boxed{4x-5}$$

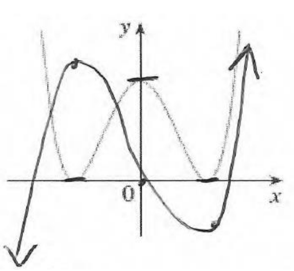
3a) No. $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

3b) No, differentiation implies continuity and the piecewise function is not continuous

3c)



4)



5a) $3x^2 - 6x$, 5b) $\frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$, 5c) $\frac{2x^3+2}{x^3}$,

5d) $-\frac{2}{(x-1)^2}$, 5e) e^{x+6}, 5f) $e^x x^5 + 5x^4 e^x$,

5g) $4\left(x^4+7x^2-3\right)^3\left(4x^3+14x\right)$,

5h) $\frac{-72x(x^2+3)^5}{(x^2-3)^7}$, 5i) $20x^3\cos(5x^4)$, 5j) $40x^3\sin\left(5x^4\right)\cos\left(5x^4\right)$

6a) –6, 6b) –9/8, 6c) 18

7a) 30, 7b) 90

8) 42

9a) 20, 9b) $\sec x\left(\sec^2 x + \tan^2 x\right)$, 9c) $4\left(\sin^2 x - \cos^2 x\right)$

10a) $\frac{-3x^2y - y^3}{x^3 + 3xy^2}$, 10b) $\frac{y-x}{3y-x}$, 10c) $\frac{e^y \sin x + y\cos(xy)}{e^y \cos x - x\cos(xy)}$

11) $\frac{dA}{dt} = 20\pi\ cm^2/\sec$

12) $\frac{dy}{dt} = -\frac{9}{4}\ ft/\sec$; The ladder is changing at a rate of $-\frac{9}{4} ft/sec$

13) Water is decreasing at a rate of $\frac{1}{2\pi} in/\min$

14a) –162 gal/min, 14b) 0 gal/min, 14c) $t = 0$ min, 14d) $t = 50$ min

<u>Chapter 3: Applications</u>

1) f has Absolute Maximum at $x = 4$ and Absolute Minimum at $x = \frac{2}{3}$

2) f has Absolute Minimum at $\left(\frac{\pi}{2}, 0\right)$ and Absolute Maximum at $(0, 2)$

3) EVT is closed interval with endpoints, MVT is Slope of secant line is slope of tangent line (derivative)

4) $c = 4/3$

5) f has a Relative Minimum at $x = -1$ and $x = 2$ when f' changes signs from negative to positive. f has a Relative Maximum at $x = 0$ when f' changes signs from positive to negative.

6) f has a relative maximum at $x = 0$ when $f' = 0$ and $f'' < 0$. f has a relative minimum at $x = \pm 2$ when $f' = 0$ and $f'' > 0$

7a) x −int: DNE, y −int: $\left(0, -\frac{1}{4}\right)$, 7b) $HA: y = 0$, $VA: x = \pm 2$, 7c) f is increasing at $I(-\infty, -2) \cup (-2, 0]$ when $f' > 0$. f is decreasing at $I[0,2) \cup (2, \infty)$ when $f' < 0$.

8a) f concaves up at $I\left(-\frac{\sqrt{2}}{2}, 0\right) \cup \left(\frac{\sqrt{2}}{2}, \infty\right)$ when $f'' > 0$. f concaves down at $I\left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(0, \frac{\sqrt{2}}{2}\right)$ when $f'' < 0$.

9) f has a Relative Maximum at $(0,9)$ when f' change signs from positive to negative. f does not have a relative minimum because f' does not change signs from negative to positive

10a) $s'(t) = v(t) = 15$, 11b) $s''(t) = v'(t) = 6t - 12$, 11c) The particle is at rest at $t = 0$, $t = \pm 4$ seconds when $s'(t) = v(t) = 0$, 11d) The particle is slowing down at $I[2,4]$ when $v(t) < 0$ and $a(t) > 0$

11a) The particle is moving the right at $I(0,1) \cup (4,6)$ when $v(t) < 0$, 11b) The particle is moving to the left at $I(2,3)$ when $v(t) > 0$, 11c) The particle is standing still $(1,2) \cup (3,4)$ when $s'(t) = 0$

12) The particle is moving to the right at $I\left(0, \frac{1}{3}\right) \cup \left(\frac{5}{2}, \infty\right)$ when $s'(t) > 0$

13) $d = \dfrac{\sqrt{5}}{2}$

14) $(\pm 7, \pm 7)$

15) 23 m × 23m

16) 36 cm × 54 cm

17) 25 ft × 100/3 ft (1 side of the dimension)

Chapter 4: Integration

1) Left: 2.43, Right: 4.77, Trapezoidal: 3.6

2) RHS: 23.625, f is an overestimation since f is increasing.

3a) 36, 3b) -18π, 3c) $\frac{81}{2} - 18\pi$

4a) 257.5 people/year, 4b) 3383.4 people, 4c) 256 people/year, 4d) 3633.33 people, Pine Grove's population per year from 0 to 10 years.

Chapter 5: Logarithms

1) $\ln \frac{x}{(x^2+1)^2}$

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2a) $\frac{x}{x^2-7}$, 2b) $-\frac{x^2-10}{x\left(x^2-10\right)}$, 2c) $e^{x^4}\left(4x^8+5x^4\right)$, 2d) $7t^6 5^{7t}(t\ln 5+1)$, 2e) $\frac{12x^2}{(\ln 6)\left(x^2-3\right)\left(x^3-7\right)}$

3) $f^{-1}(x) = 3(x+5)^{\frac{2}{3}}$

4) –1/4

5) –1

Key:

Question 1:

(a) $\lim_{x\to 0^-}(1-2\sin x) = 1$ $\lim_{x\to 0^+} e^{-4x} = 1$ $f(0) = 1$ So, $\lim_{x\to 0} f(x) = f(0)$. Therefore f is continuous at $x = 0$.	2 : analysis
(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$ $-2\cos x \neq -3$ for all values of $x < 0$. $-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$. Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.	$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

Question 2:

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

$2: f'(x)$

(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

$2: \begin{cases} 1: f'(-3) \\ 1: \text{answer} \end{cases}$

(c) $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sqrt{25 - x^2} = 4$

$\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4$

Therefore, $\lim_{x \to -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim_{x \to -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

$2: \begin{cases} 1: \text{considers one-sided limits} \\ 1: \text{answer with explanation} \end{cases}$

Question 3:

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000\ \text{cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5$ cm/min.

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = 0.038$ or 0.039 cm/min

$4: \begin{cases} 1: \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2: \text{expression for } \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$

(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$.

This occurs when $t = 25$ minutes.

Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.

$3: \begin{cases} 1: R(t) = 2000 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

Question 4:

(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$

$h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$