Name ____ Date _________

Exam Date and Time: ____

Read and answer all questions accordingly. All work and problems must be done on your own paper and <u>work must be shown</u>. No work = No Credit = NO EXCEPTIONS. It is worth 1.5 quiz grades.

<u>Note</u>: There are two parts of the exam. The NON–CALC test has 20 questions (50%) and 3 FRQ's (50%).

<u>Chapter 1 – Limits</u>

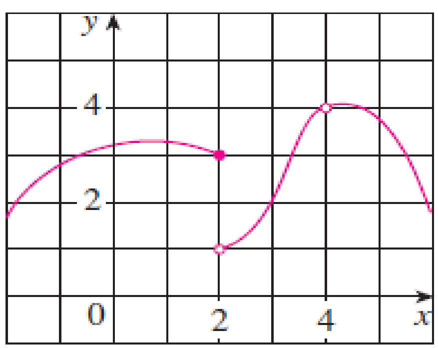
1) Find the limit of the following:

a) $\lim_{x\to 3}\dfrac{x-3}{x^2-9}$ b) $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}$ c) $\lim_{x\to 2^-}\dfrac{|x-2|}{x-2}$

d) $\lim_{x\to 5^-}\dfrac{x-1}{x-5}$ e) $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$ f) $\lim_{x\to 0}\dfrac{\sin^3 5x}{x^3}$

g) $\lim_{x\to \infty}\dfrac{x^2+x-2}{x-1}$ h) $\lim_{x\to 0}\dfrac{x^2}{1-\cos x}$ i) $\lim_{x\to \infty}\dfrac{\sqrt{9x^6-1}}{x^3+3}$

2) Solve. If it does not exist, state "DNE."



a) $\lim_{x\to 2^-} f(x) =$ __________ b) $\lim_{x\to 2^+} f(x) =$ __________

c) $\lim_{x\to 2} f(x) =$ __________

d) $f(2) =$ __________

e) $f(4) =$ __________

3) Explain why $f(x) = x^2 - 7x - 8$ is a guarantee zero in between $[-3, 0]$

4) Define the Intermediate Value Theorem.

5) State the definition of continuity of a function f at which $x = c$.

Given: $\lim_{x\to 3} f(x) = 4$ $\lim_{x\to 3} g(x) = -5$ $\lim_{x\to 3} h(x) = 0$

6) Find the limits as they exist.

a) $\lim_{x\to 3}\left[f(x)+5g(x)\right] =$ ________ b) $\lim_{x\to 3}\left[g(x)\right]^3 =$ ________

c) $\lim_{x\to 3}\sqrt{f(x)} =$ ________ d) $\lim_{x\to 3}\dfrac{g(x)h(x)}{f(x)} =$ ________

7) Given the following function, find k so that the function is continuous at $x = -1$. $f(x)=\begin{cases} kx-3, & x\le -1 \\ x^2+k, & x>-1\end{cases}$

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<u>Chapter 2 – Differentiation</u>

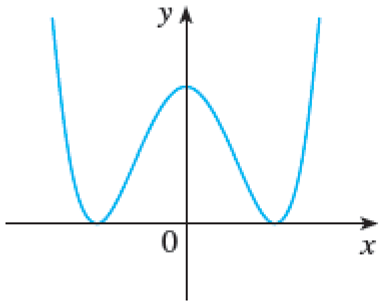
1) Find the equation of the tangent line to the graph, $f(x) = 5x + 6$ at the point $(2, 16)$.

2) Using the difference quotient, find $f'(x)$ if $f(x) = 2x^2 - 5x$. Show all work.

3) Given: $f(x)=\begin{cases} x-5, & x<1 \\ 3, & x=1 \\ x^2, & x>1\end{cases}$

(a) Is f continuous at $x = 1$? Use the definition of continuity to explain answer.
(b) Is f differentiable at $x = 1$? Explain.
(c) Sketch a graph of f.

4) Given the f graph, graph $f'(x)$. Show all work.



5) Find $\frac{dy}{dx}$ of the following:

(a) $f(x)=x^3-3x^2$ (b) $f(x)=x^{1/2}-x^{-1/2}$ (c) $y=\frac{2x^3-1}{x^2}$

(d) $y=\frac{x+1}{x-1}$ (e) $y=e^{x+6}+2$ (f) $y=x^5e^x$

(g) $(x^4+7x^2-3)^4$ (h) $\left(\frac{x^2+3}{x^2-3}\right)^6$ (i) $\sin(5x^4)$ (j) $\sin^2(5x^4)$

6) Suppose that $f(5)=1$, $f'(5)=3$, $g(5)=-4$, and $g'(5)=6$. Find the following values.

a) $(fg)'(5)=$ ______ b) $\left(\frac{f}{g}\right)'(5)=$ ______ c) $\left(\frac{g}{f}\right)'(5)=$ ______

7)

	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
$x=3$	–3	2	5	3
$x=5$	7	–1	–2	4

(a) If $h(x)=f(x)g(x)$, solve $h'(5)$

(b) If $h(x)=3[g(x)]^2$, solve $h'(3)$

8) If $F(x)=f(g(x))$, where $f(-3)=4$, $f'(-3)=6$, $f'(1)=5$, $g(1)=-3$, and $g'(1)=7$, find $F'(1)$.

9) Solve for $f''(x)$: (a) $10x^2-8x$ (b) $\sec x$ (c) $y=\cos^2 x-\sin^2 x$

10) Find $\frac{dy}{dx}$ if (a) $x^3y+xy^3=-10$ (b) $x^2-2xy+3y^2=8$
(c) $e^y\cos x=6+\sin(xy)$

11) The radius of a circle is increasing at a rate of 2 cm/sec. Find the rate of change of the area of the circle when the radius is 5 cm.

12) A 15 ft ladder is leaning against a wall. The bottom of the ladder begins to slide away from the wall at a rate of 3 ft/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 9 feet away from the wall?

13) Water is dripping through the top of a conical shaped cup at the rate of 0.5 cubic inches per minute. If the cup's original size is 4 inches high with a diameter of 4 inches across the top, how fast is the height of water decreasing when the water is two inches deep in the cone?

14) If a tank holds 4500 gallons of water, which drains from the bottom of the tank in 50 minutes, then Toricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V=4500\left(1-\frac{1}{50}t\right)^2 \quad 0\le t\le 50.$$

Find the rate at which water is draining from the tank after the following amounts of time. (Remember that the rate must be negative because the amount of water in the tank is decreasing.)

(a) 5 min (b) 50 minutes
(c) At what time is the water flowing out the fastest?
(d) At what time is the water flowing out the slowest?

Chapter 3 – Applications of Differentiation

1) Given $f(x) = 3x^3 - 4x + 1$, identify all absolute extrema on the interval $[0, 4]$.

2) Find the absolute maximum and absolute minimum values of f on the given interval, $f(t) = 2\cos t + \sin(2t)$ at $I\left[0, \frac{\pi}{2}\right]$

3) Define the Extreme Value Theorem and Mean Value Theorem and identify the difference of the two theorems.

4) What is the number for c which satisfies the conditions of the Mean Value Theorem of differentiable calculus for $f(x) = x^3 - 2x^2$ on $[0, 2]$.

5) Find the relative extrema of $f(x) = 3x^4 - 4x^3 - 12x^2 + 3$ using the First Derivative Test.

6) Use the second derivative test to determine the relative extrema for $f(x) = x^4 - 8x^2$.

7) Determine the $f(x) = \frac{1}{x^2-4}$, determine the following:

(a) x and y–intercepts

(b) horizontal and vertical asymptotes

(c) intervals of increasing and decreasing

8) Determine the open intervals in which the graph $f(x) = 3x^5 - 5x^3 + 3$ is concave up or down.

9) Find the relative extrema of $f(x) = 9 - x^2$ on the $I[-3, 3]$.

10) Let $s(t) = t^3 - 6t^2 + 2$ be the position function of a particle moving along the x–axis.

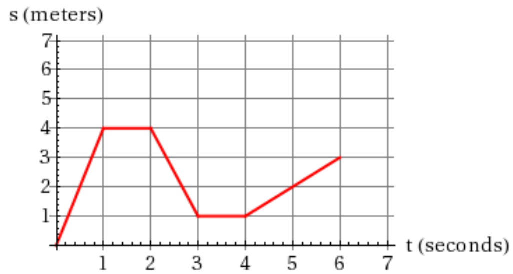
(a) Find the velocity of the object when $t = 5$

(b) Find the acceleration at time t

(c) Establish when the particle is at rest

(d) When is the particle slowing down?

11) A particle starts by moving to the right along a horizontal line; the graph of its position function is shown.



(a) When is the particle moving to the right?

(b) When is the particle moving to the left?

(c) When is the particle standing still?

12) A particle is moving along a horizontal line according to the equation, $s(t) = 2t^3 - \frac{17}{2}t^2 + 5t - 1$where $[0, \infty)$. Determine the interval of time when the particle is moving to the right.

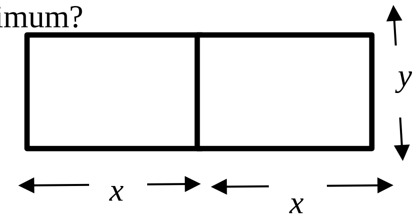
13) Find the shortest distance from $f(x) = x^2$to the point $(2, ½)$.

14) Find two positive numbers whose product is 49 and whose sum is a minimum.

15) Find the dimensions of a rectangle with perimeter 92 m whose area is as large as possible.

16) The top and bottom margins of a poster are each 9 cm and the side margins are each 6 cm. If the area of printed material on the poster is fixed at 864 cm^2, find the dimensions of the poster with the smallest area.

17) A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?



Chapter 4 – Integration

1) Given $\int_0^6 f(x)dx$ for the functions below, use the left, right, and trapezoidal Riemann sum with 3 subintervals.

x	0	1	4	6
$f(x)$	0	0.25	0.84	1

2) If $f(x) = 3x^2 - 2x$, $0 \le x \le 3$, evaluate the Riemann sum with $n = 6$, taking the sample points to be right endpoints. Then, determine if the graph is an underestimation or an overestimation.

3) Given $\int_0^3 f(x)dx = 6$ and $\int_3^9 f(x)dx = -1$, solve for $\int_0^9 f(x)dx$, $\int_0^9 7f(x)dx$, and $\int_9^3 g(x)dx$

4) Accumulation/Riemann's Sum:

Let $y(t)$ represent the population of Sugar Mill over a 10-year period, where y is differentiable function of t. The table shows the population recorded every two years.

t (years)	**0**	**2**	**4**	**6**	**8**	**10**
y (people)	**2500**	**2912**	**3360**	**3815**	**4330**	**4875**

(a) Use the data from the table to find an approximation for $y'(7)$ and explain the meaning of $y'(7)$ in terms of the population of Sugar Mill. Show the computations that lead to the answer.

(b) Use the data from the table to approximate the average population of Sugar Mill over the time interval, $0 \le t \le 10$ by using a **LEFT** Riemann's Sum with five equal intervals. Show all work.

Chapter 5: Logarithms

1) Write the following expression as a log of a single quantity, $\ln x - 2\ln(x^2 + 1)$.

2) Find the derivative:

(a) $y = \ln\sqrt{x^2 - 7}$ (b) $y = \ln\left(\frac{3x}{x^2 + 10}\right)$ (c) $y = x^5 e^{x^4}$

(d) $f(t) = t^7 5^{7t}$ (e) $\log_6 \frac{x^3 - 7}{x^3 - 3}$

3) Find $(f^{-1})' = x^3 - 5$ if $f(x) = x^3 - 5$

4) Given $f(3) = 5$, $f'(3) = 7$, $f(2) = 3$, and $f'(2) = -4$, and f and g are inverses, solve for $g'(3)$.

5) Solve for $g'(4)$:

x	2	4	6	8	10
$f(x)$	4	1	2	0	6
$f'(x)$	–1	3	1/2	4	5

FRQ:

1) AB6-2011

Let f be a function defined by $f(x)=\begin{cases}1-2\sin x & \text{for } x\le 0\\ e^{-4x} & \text{for } x>0\end{cases}$.

(a) Show that f is continuous at $x = 0$

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$

2) AB4-2012

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \leq x \leq 5$.

(a) Find $f'(x)$

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x)\ for -5 \leq x \leq -3 \\ x + 7\ \ for\ -3 < x \leq 5 \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

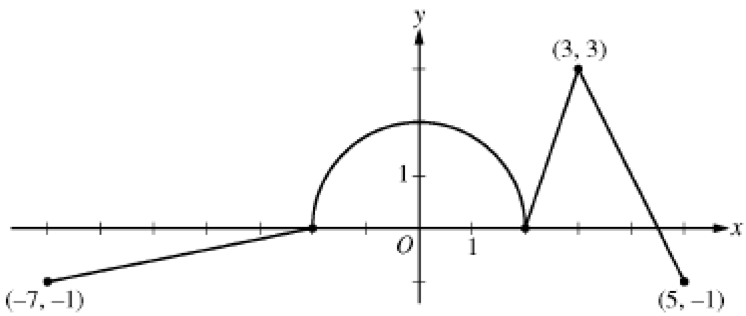
3) AP3-2008 (Modified for non-calc)

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeter per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. Setup but do not solve the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

4) AB5-2010



Graph of g'

The function g is defined and differentiable on the closed interval $[-7,5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three-line segments, as shown in the figure above.

(a) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(b) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.