Bases other than Natural Base

Section 5.5

Calculus AP/Dual, Revised ©2017

viet.dang@humbleisd.net

REVIEW

A. To Rewrite Logarithms

1. $a^u = b$ is $\log_a b = u$
2. a : Base
3. u : Power
4. b : Value/Argument

B. Change of Base Formula

1. $\log_a x = \frac{\log x}{\log a} or \frac{\ln x}{\ln a}$

REVIEW

$$a^u = b$$

$$\Downarrow$$

$$\log_a b = u$$

Example 1

Rewrite $2^4 = 16$ into $\log$ form and evaluate

$$2^4 = 16$$

$$\log_2 16 = 4$$

EXAMPLE 2

Rewrite $x^2 - x = \log_3 9$ into log form and evaluate

$$x^2 - x = \log_3 9$$

$$x^2 - x = \frac{\log 9}{\log 3}$$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$\{-1, 2\}$$

YOUR TURN

Rewrite $3x + 5 = \log_2 64$ into $\log$ form and evaluate

$$\left\{\frac{1}{3}\right\}$$

REVIEW EXPONENT RULES

A. Product: $a^n a^m = a^{\boxed{n+m}}$

B. Quotient: $\dfrac{a^n}{a^m} = a^{\boxed{n-m}}$

C. Power: $\left(a^n\right)^m = a^{\boxed{n*m}}$

LOG PROPERTIES

A. Product: $\log_b(x)+\log_b(y)=\log_b(xy)$

B. Quotient: $\log_b x-\log_b y=\log_b\left(\dfrac{x}{y}\right)$

C. Power: $\log_b\left(a^p\right)=p\log_b(a)$

Guess the Rule

Solve for y'

1) $y = \log_3(3x)$	$y' = \dfrac{3}{3x(\ln 3)} = \dfrac{1}{x(\ln 3)}$
2) $y = \log_{12}(5x)$	$y' = \dfrac{5}{5x(\ln 12)} = \dfrac{1}{x(\ln 12)}$
3) $y = \log_{11}(x^3 - x)$	$y' = \dfrac{3x^2 - 1}{x^3 - x(\ln 11)}$
4) $y = \log_a u$	$y' = \dfrac{u'}{u(\ln a)}$

DERIVATIVE OF A LOGARITHMIC FUNCTION

Change of Base Formula: $\log_a x = \dfrac{\ln x}{\ln a}$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx}\left(\log_a x\right) = \left(\frac{1}{\ln a}\right)\frac{d}{dx}\left(\ln x\right)$$

$$\frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a}\cdot\frac{d}{dx}\left(\ln x\right)$$

$$\frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a}\cdot\frac{1}{x}$$

$$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$$

EXAMPLE 3

Solve for the derivative of $y = \log_7 x$

$$\log_7 x = \frac{\ln x}{\ln 7}$$

$$\frac{d}{dx}\log_7 x = \frac{u'}{u \ln a}$$

$$= \frac{1}{\ln 7} \cdot \frac{1}{x}$$

$$y' = \frac{1}{x \ln 7}$$

Example 4

Solve for the derivative of $y = \log_{10}(x^3 + x)$

$$\log_{10} x^3 + x = \frac{\ln x^3 + x}{\ln 10}$$

$$\frac{d}{dx}\left[\log_{10}\left(x^3 + x\right)\right] = \frac{d}{dx}\frac{\ln\left(x^3 + x\right)}{\ln 10}$$

$$\frac{1}{\ln 10}\cdot\frac{d}{dx}\left(\ln x^3 + x\right)$$

$$\frac{1}{\ln 10}\cdot\frac{3x^2 + 1}{x^3 + x}$$

$$y' = \frac{3x^2 + 1}{\left(x^3 + x\right)\ln 10}$$

EXAMPLE 5

Solve for the derivative of $y = \log_3(\sin t)$

$$y' = \frac{\cot t}{\ln 3}$$

YOUR TURN

Solve for the derivative of $y = \log_3\left(x^2 + 1\right)$

$$y' = \frac{2x}{\left(x^2+1\right)\ln 3}$$

GUESS THE RULE

Solve for y'

1) $y = 5^x$	$y' = \left(5^x\right)(\ln 5)(1)$
2) $y = 4^{2x}$	$y' = \left(4^{2x}\right)(\ln 4)(2)$
3) $y = \left(\frac{1}{7}\right)^x$	$y' = \left(\frac{1}{7}\right)^x \left(\ln \frac{1}{7}\right)(1)$
4) $y = 8^{5x-7}$	$y' = \left(8^{5x-7}\right)(\ln 8)(5)$
5) $y = a^u$	$y' = a^u (\ln a)(u')$

Log Derivative Rules

A. $\frac{d}{dx}[a^u] = a^u(\ln a) \cdot u'$

EXAMPLE 6

Solve for the derivative of $y = 2^x$

$$y = 2^x$$

$$y' = a^u(\ln a)u'$$

$$y' = 2^x(\ln 2)$$

Example 7

Solve for the derivative of $y = 2^{x^3}$

$$y' = a^u \left(\ln a\right) u'$$

$$y' = 2^{x^3} \left(\ln 2\right)\left(x^3\right)\frac{du}{dx}$$

$$y' = 2^{x^3} \left(\ln 2\right)\left(3x^2\right)$$

$$y' = \left(2^{x^3}\right) 3x^2 \left(\ln 2\right)$$

EXAMPLE 8

Solve for the derivative of $y = x(6^{-x})$

$$y' = fg' + gf'$$

$$y' = x\frac{d}{dx}\left(6^{-x}\right) + 6^{-x}\frac{d}{dx}(x)$$

$$y' = x\left((-1)\left(6^{-x}\right)(\ln 6)\right) + \left(6^{-x}\right)(1)$$

$$y' = \frac{-x}{6^x}\ln 6 + \frac{1}{6^x}$$

YOUR TURN

Solve for the derivative of $y = 3^{x-4}$

$$y' = 3^{x-4}(\ln 3)$$

AP Multiple Choice Practice Question 1 (non-calculator)

Find an equation of the tangent line to the graph of $y = 6^{-x}$ at the point $(-1, 6)$

(A) $y = 6 - \frac{(x+1)\ln 6}{6}$

(B) $y = 6\big(1 - \ln(6)(x+1)\big)$

(C) $y = 6\big(1 - \ln(6)(x-1)\big)$

(D) $y = 6\big(1 + \ln(6)(x+1)\big)$

AP Multiple Choice Practice Question 1 (non-calculator)

Find an equation of the tangent line to the graph of $y = 6^{-x}$ at the point $(-1, 6)$

Vocabulary	Process and Connections	Answer and Justifications
Derivative Natural Base Tangent Line Slope	$y - y_1 = y'(x - x_1)$ $y - 6 = y'(x+1)$ $y' = (a^u)\ln a(u')$ $y' = (6^{-x})\ln 6(-1)$ $y = \left[(6^{-x})\ln 6(-1)\right](x+1) + 6$ $y = -6\ln(6)x + 6 - 6\ln 6$ $y = 6(1 - \ln 6)(x+1)$	**B**

Assignment

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