

BASES OTHER THAN NATURAL BASE

Section 5.5

Calculus AP/Dual, Revised ©2017

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REVIEW

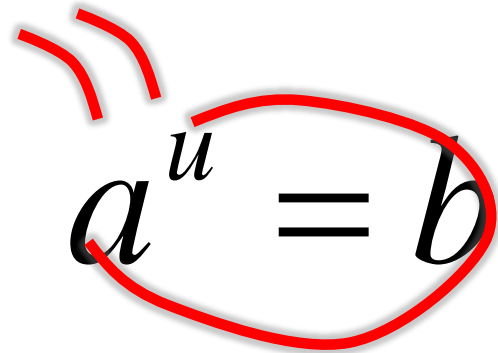
A. To Rewrite Logarithms

1. $a^u = b$ is $\log_a b = u$
2. a : Base
3. u : Power
4. b : Value/Argument

B. Change of Base Formula

1. $\log_a x = \frac{\log x}{\log a}$ *or* $\frac{\ln x}{\ln a}$

REVIEW

$$a^u = b$$




$$\log_a b = u$$

EXAMPLE 1

Rewrite $2^4 = 16$ into log form and evaluate

$$2^4 = 16$$

$$\log_2 16 = 4$$

EXAMPLE 2

Rewrite $x^2 - x = \log_3 9$ into log form and evaluate

$$x^2 - x = \log_3 9$$

$$x^2 - x = \frac{\log 9}{\log 3}$$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$\{-1, 2\}$$

YOUR TURN

Rewrite $3x + 5 = \log_2 64$ into log form and evaluate

$$\left\{ \frac{1}{3} \right\}$$

REVIEW EXPONENT RULES

A. Product: $a^n a^m = a^{\boxed{n + m}}$

B. Quotient: $\frac{a^n}{a^m} = a^{\boxed{n - m}}$

C. Power: $\left(a^n\right)^m = a^{\boxed{n * m}}$

LOG PROPERTIES

A. Product: $\log_b (x) + \log_b (y) = \log_b (xy)$

B. Quotient: $\log_b x - \log_b y = \log_b \left(\frac{x}{y} \right)$

C. Power: $\log_b (a^p) = p \log_b (a)$

GUESS THE RULE

Solve for y'

1) $y = \log_3(3x)$	$y' = \frac{3}{3x(\ln 3)} = \frac{1}{x(\ln 3)}$
2) $y = \log_{12}(5x)$	$y' = \frac{5}{5x(\ln 12)} = \frac{1}{x(\ln 12)}$
3) $y = \log_{11}(x^3 - x)$	$y' = \frac{3x^2 - 1}{x^3 - x(\ln 11)}$
4) $y = \log_a u$	$y' = \frac{u'}{u(\ln a)}$

DERIVATIVE OF A LOGARITHMIC FUNCTION

Change of Base Formula: $\log_a x = \frac{\ln x}{\ln a}$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx}(\log_a x) = \left(\frac{1}{\ln a}\right) \frac{d}{dx}(\ln x)$$

$$\frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a} \cdot \frac{d}{dx}(\ln x)$$

$$\frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

EXAMPLE 3

Solve for the derivative of $y = \log_7 x$

$$\log_7 x = \frac{\ln x}{\ln 7}$$

$$\frac{d}{dx} \log_7 x = \frac{u'}{u \ln a}$$

$$= \frac{1}{\ln 7} \cdot \frac{1}{x}$$

$$y' = \frac{1}{x \ln 7}$$

EXAMPLE 4

Solve for the derivative of $y = \log_{10}(x^3 + x)$

$$\log_{10} x^3 + x = \frac{\ln x^3 + x}{\ln 10}$$

$$\frac{d}{dx} \left[\log_{10}(x^3 + x) \right] = \frac{d}{dx} \frac{\ln(x^3 + x)}{\ln 10}$$

$$\frac{1}{\ln 10} \cdot \frac{d}{dx} (\ln x^3 + x)$$

$$\frac{1}{\ln 10} \cdot \frac{3x^2 + 1}{x^3 + x}$$

$$y' = \frac{3x^2 + 1}{(x^3 + x)\ln 10}$$

EXAMPLE 5

Solve for the derivative of $y = \log_3(\sin t)$

$$y' = \frac{\cot t}{\ln 3}$$

YOUR TURN

Solve for the derivative of $y = \log_3(x^2 + 1)$

$$y' = \frac{2x}{(x^2 + 1)\ln 3}$$

GUESS THE RULE

Solve for y'

1) $y = 5^x$	$y' = (5^x)(\ln 5)(1)$
2) $y = 4^{2x}$	$y' = (4^{2x})(\ln 4)(2)$
3) $y = \left(\frac{1}{7}\right)^x$	$y' = \left(\frac{1}{7}\right)^x \left(\ln \frac{1}{7}\right)(1)$
4) $y = 8^{5x-7}$	$y' = (8^{5x-7})(\ln 8)(5)$
5) $y = a^u$	$y' = a^u (\ln a)(u')$

LOG DERIVATIVE RULES

A. $\frac{d}{dx} [a^u] = a^u (\ln a) \cdot u'$

EXAMPLE 6

Solve for the derivative of $y = 2^x$

$$y = 2^x$$

$$y' = a^u (\ln a) u'$$

$$y' = 2^x (\ln 2)$$

EXAMPLE 7

Solve for the derivative of $y = 2^{x^3}$

$$y' = a^u (\ln a) u'$$

$$y' = 2^{x^3} (\ln 2) (x^3) \frac{du}{dx}$$

$$y' = 2^{x^3} (\ln 2) (3x^2)$$

$$y' = \left(2^{x^3}\right) 3x^2 (\ln 2)$$

EXAMPLE 8

Solve for the derivative of $y = x(6^{-x})$

$$y' = fg' + gf'$$

$$y' = x \frac{d}{dx} (6^{-x}) + 6^{-x} \frac{d}{dx} (x)$$

$$y' = x \left((-1) (6^{-x}) (\ln 6) \right) + (6^{-x}) (1)$$

$$y' = \frac{-x}{6^x} \ln 6 + \frac{1}{6^x}$$

YOUR TURN

Solve for the derivative of $y = 3^{x-4}$

$$y' = 3^{x-4} (\ln 3)$$

AP MULTIPLE CHOICE PRACTICE QUESTION 1

(NON-CALCULATOR)

Find an equation of the tangent line to the graph of $y = 6^{-x}$ at the point $(-1, 6)$

(A) $y = 6 - \frac{(x+1)\ln 6}{6}$

(B) $y = 6(1 - \ln(6)(x + 1))$

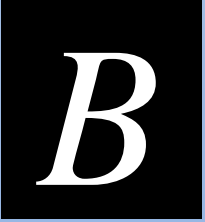
(C) $y = 6(1 - \ln(6)(x - 1))$

(D) $y = 6(1 + \ln(6)(x + 1))$

AP MULTIPLE CHOICE PRACTICE QUESTION 1

(NON-CALCULATOR)

Find an equation of the tangent line to the graph of $y = 6^{-x}$ at the point $(-1, 6)$

Vocabulary	Process and Connections	Answer and Justifications
Derivative Natural Base Tangent Line Slope	$y - y_1 = y'(x - x_1)$ $y - 6 = y'(x + 1)$ $y' = (a^u) \ln a (u')$ $y' = (6^{-x}) \ln 6 (-1)$ $y = \left[(6^{-x}) \ln 6 (-1) \right] (x + 1) + 6$ $y = -6 \ln(6)x + 6 - 6 \ln 6$ $y = 6(1 - \ln 6)(x + 1)$	

ASSIGNMENT

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