# NATURAL BASE DERIVATIVES

Section 5.4

Calculus AP/Dual, Revised ©2017

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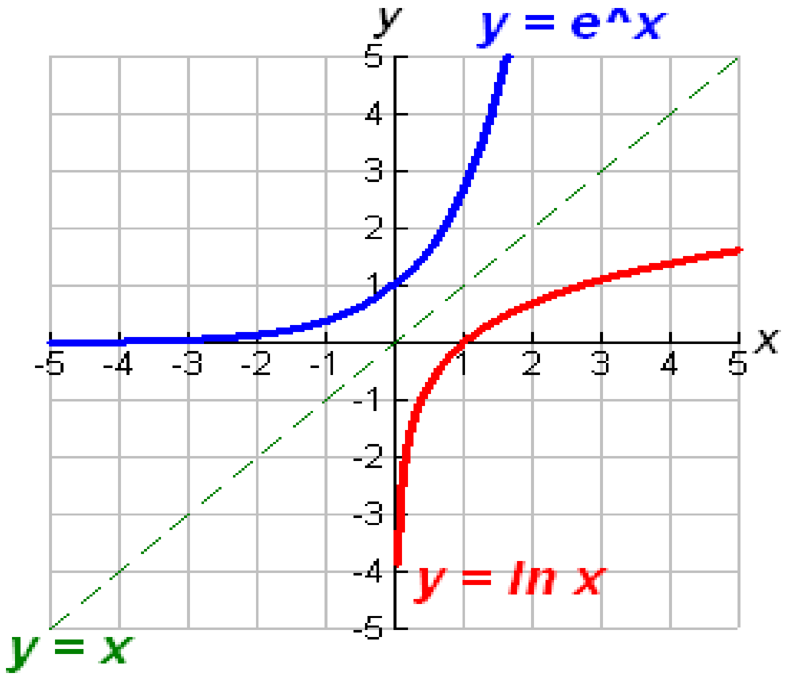
# Review

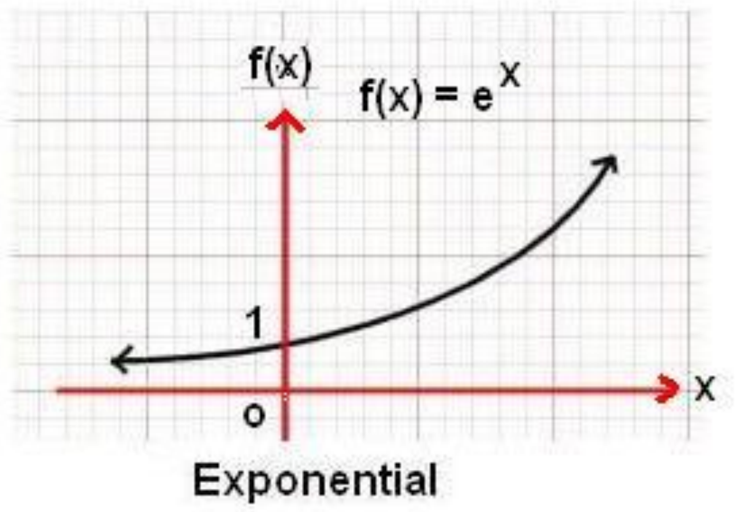
A. Forms
   1. Exponential Form: $y = e^x$
   2. Logarithmic Form: $\log_e y = x$
   3. $e$ and $\ln$ are inverses
   4. $y = \log_b x$ means $x = b^y$ where $b$ and $x > 0$
   5. $y = \ln x$ means $x = e^y$ where $x > 0$



# Review

**B. Properties of Exponents**

1. **Multiplying Exponents:** $e^a\, e^b = e^{a+b}$ **(Base, Base, Add)**
2. **Dividing Exponents:** $\frac{e^a}{e^b} = e^{a-b}$ **(When we divide, we subtract)**
3. **Domain:** $(-\infty, \infty)$
4. **Range:** $(0, \infty)$



# REVIEW

$$e^u = b$$

$$\log_e b = u$$

$$\ln b = u$$

# Example 1

**Convert $\ln\frac{2}{5} = -0.916$ to exponential form**

$$\log_e x = y$$

$$e^y = x$$

$$e^{-0.916} = \frac{2}{5}$$

# Example 2

**Solve $-6+3e^{2x}=9$**

$$-6+3e^{2x}=9$$

$$3e^{2x}=15$$

$$e^{2x}=5$$

$$\ln 5=2x$$

$$\left\{\frac{\ln 5}{2}\right\}$$

# YOUR TURN

**Convert $e^{2x} = 3$ to logarithmic form**

$$\ln 3 = 2x$$

# GUESS THE RULE

| Equation | Derivative |
|---|---|
| 1) $\frac{d}{dx}\left[e^{4x}\right]$ | $4e^{4x}$ |
| 2) $\frac{d}{dx}\left[e^{4x^2+7}\right]$ | $8xe^{4x^2+7}$ |
| 3) $\frac{d}{dx}\left[e^{(4x^3+5)^2}\right]$ | $2\left(4x^3+5\right)\left(12x^2\right)e^{\left(4x^3+5\right)^2}$ |
| 4) $\frac{d}{dx}\left[e^{u}\right]$ | $e^u u'$ |

# Natural Log Derivative Rules

A. $\frac{d}{dx}[e^u] = e^u \cdot u'$

# Derivative of Natural Base

http://www.math.uri.edu/~bkaskosz/flashmo/derplot/

# Example 3

**Solve for the derivative of $y = e^{-2x}$**

$$y = e^{-2x}$$

$$y' = e^{u}u'$$

$$y' = e^{-2x}\frac{d}{dx}\left(-2x\right)$$

$$y' = -2e^{-2x}$$

# EXAMPLE 4

**Solve for the derivative of $y = 3e^{2x+2}$**

$$y' = 6e^{2x+2}$$

# Your Turn

**Solve for the derivative of $y = e^{3x^2}$**

$$y' = 6xe^{3x^2}$$

# Example 5

**Solve for the derivative of $y = x^2 e^{-x}$**

$$y' = x^2 \frac{d}{dx}\left(e^{-x}\right) + e^{-x}\frac{d}{dx}\left(x^2\right)$$

$$y' = x^2\left(e^u u'\right) + e^{-x}\left(2x\right)$$

$$y' = x^2\left(e^{-x}\right)\left(-1\right) + e^{-x}\left(2x\right)$$

$$y' = -x^2\left(e^{-x}\right) + 2x\left(e^{-x}\right)$$

$$y' = e^{-x}\left(-x^2 + 2x\right)$$

# EXAMPLE 6

**Solve for the derivative of $y = \ln\left(4 + e^{3x}\right)$**

$$y' = \frac{3e^{3x}}{4 + e^{3x}}$$

# EXAMPLE 7

**Solve the derivative of $f(x) = \pi x^5 + e^{5x}$**

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$$f'(x) = \frac{d}{dx}\left[\pi x^5\right] + \frac{d}{dx}\left[e^{5x}\right]$$

$$f'(x) = \pi \frac{d}{dx}\left[x^5\right] + \frac{d}{dx}\left[e^{5x}\right]$$

$$f'(x) = 5\pi x^4 + 5e^{5x}$$

# YOUR TURN

**Solve for the derivative of** $\boldsymbol{y = \ln\left(e^{x^3}\right)}$

$$y' = 3x^2$$

# Example 8

**Solve for the derivative of $y = \ln\left(\frac{1-e^x}{1+e^x}\right)$**

$$\ln\left(1-e^x\right)-\ln\left(1+e^x\right)$$

$$y' = \frac{u'}{u} - \frac{u'}{u}$$

$$y' = \frac{-e^x}{1-e^x} - \frac{e^x}{1+e^x}$$

$$y' = \frac{-e^x\left(1+e^x\right)}{\left(1-e^x\right)\left(1+e^x\right)} - \frac{e^x\left(1-e^x\right)}{\left(1+e^x\right)\left(1-e^x\right)}$$

# EXAMPLE 8

**Solve for the derivative of $y = \ln\left(\frac{1-e^x}{1+e^x}\right)$**

$$y' = \frac{-e^x\left(1+e^x\right)}{\left(1-e^x\right)\left(1+e^x\right)} - \frac{e^x\left(1-e^x\right)}{\left(1+e^x\right)\left(1-e^x\right)}$$

$$y' = \frac{-e^x - e^{2x} - e^x + e^{2x}}{\left(1-e^x\right)\left(1+e^x\right)}$$

$$y' = \frac{-2e^x}{\left(1-e^x\right)\left(1+e^x\right)}$$

# Example 9

**Solve for the derivative of $e^{xy} + x^2 - y^2 = 10$**

$$\frac{d}{dx}\left(e^{xy} + x^2 - y^2\right) = \frac{d}{dx}(10)$$

$$\frac{d}{dx}\left(e^{xy}\right) = \left(e^{xy}\right)(xy)'$$

Taking the derivative of $e^{xy}$ using two rules

$$\frac{d}{dx}(xy)e^{xy} + 2x - 2y\frac{dy}{dx} = 0$$

$$\frac{d}{dx}(xy) = (x)(1)\frac{dy}{dx} + (y)$$

# EXAMPLE 9

**Solve for the derivative of $e^{xy} + x^2 - y^2 = 10$**

$$\frac{d}{dx}(xy) = (x)(1)\frac{dy}{dx} + (y)$$

$$e^{xy}\left(x\frac{dy}{dx} + y\right) + 2x - 2y\frac{dy}{dx} = 0$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} + 2x - 2y\frac{dy}{dx} = 0$$

$$xe^{xy}\frac{dy}{dx} - 2y\frac{dy}{dx} = -ye^{xy} - 2x$$

$$\frac{dy}{dx} = \frac{-ye^{xy} - 2x}{xe^{xy} - 2y}$$

# YOUR TURN

**Solve for the derivative of** $e^{x^2y} - x^2 + y^2 = 5$

$$\frac{dy}{dx} = \frac{-2xye^{x^2y} + 2x}{x^2e^{x^2y} + 2y}$$

# Example 11 (2009 Form B)

**Let $f$ be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of $f'$, the derivative of $f$, is shown above. The graph of $f'$ crosses the $x$-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let $g$ be the function given by $g(x) = e^{f(x)}$. Write an equation for the line tangent to the graph of $g$ at $x = 1$.**

$$g(x) = e^{f(x)}$$

$$y = e^{u}$$

$$y' = e^{u}u'$$

$$g'(x) = e^{u} \cdot f'(x)$$

$$g'(1) = e^{f(1)} \cdot f'(1)$$

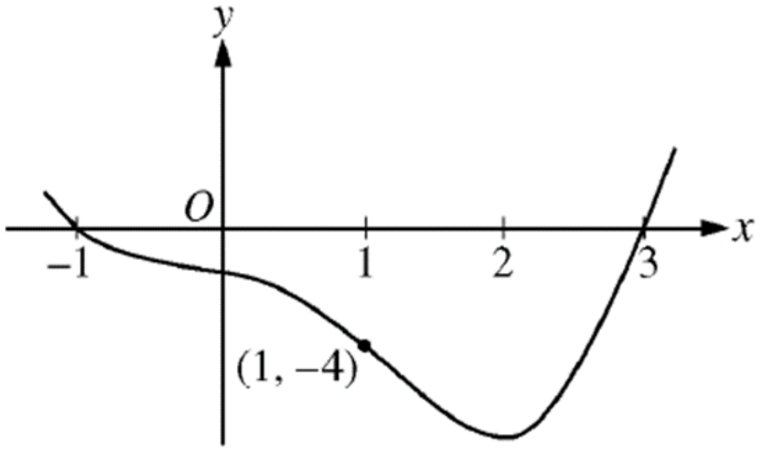
$$g'(1) = e^{2} \cdot (-4) = -4e^{2}$$

Point: $(1, g(1))$

Point: $(1, e^{f(1)})$

Slope: $g'(1)$

$u = f(x)$



Graph of $f'$

$$y - e^{2} = -4e^{2}(x - 1)$$

# AP Multiple Choice Practice Question 1 (Non-Calculator)

**Given $y = xe^x$, Solve for $y'$=**

(A) $y' = xe^x + e^x$

(B) $y' = x^2e^x + 2xe^x$

(C) $y' = xe^x$

(D) $y' = e^x$

# AP Multiple Choice Practice Question 1 (Non-Calculator)

**Given $y = xe^x$, Solve for $y'$=**

| Vocabulary | Process and Connections | Answer and Justifications |
|---|---|---|
| Derivative<br>Natural Base<br>Product Rule | $y' = fg' + gf'$<br>$y' = (x)(e^x) + (e^x)(1)$<br>$y' = (e^x)(x+1)$ | $A$ |

# ASSIGNMENT

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**25-28 all, 33-49 odd, 55-59 odd, 63, 65**