Solving Trigonometric Equations

Section 5.3

Precalculus PreAP/Dual, Revised ©2017

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Steps

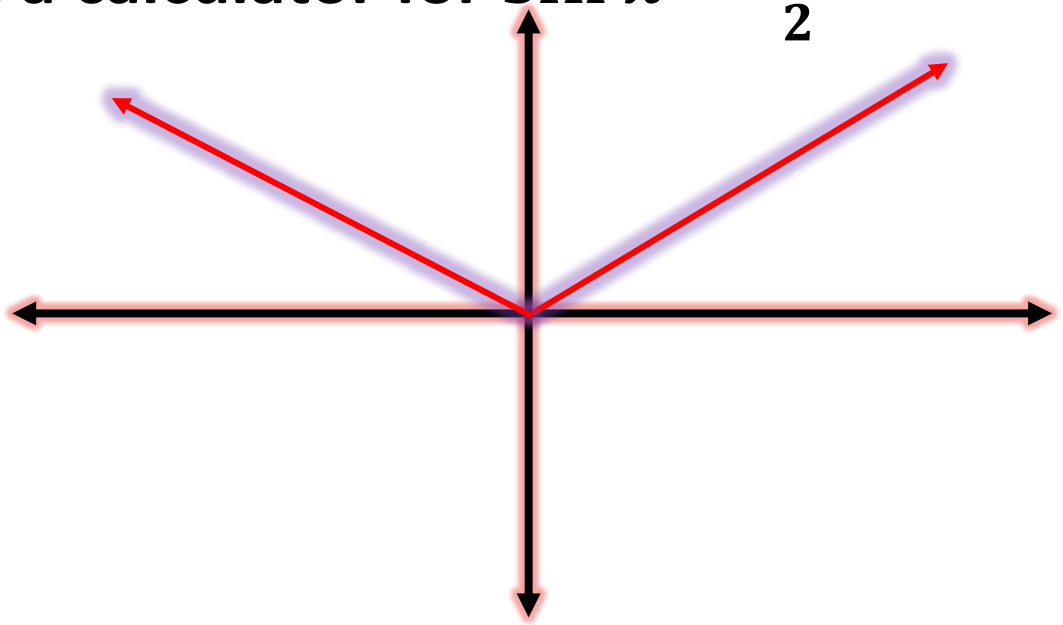
A. Isolate the trig function. [ex. $2\sin x = 1$]
B. Make sure to apply the inverse restrictions, if necessary. I
C. Give all answers [with a $+2\pi n$ or $+\pi n$ on all answers]
D. $+\pi n$ will come from tan or cot

EXAMPLE 1

Find all solutions and solve without a calculator for $\sin x = \frac{\sqrt{2}}{2}$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$\sin(?) = \frac{\sqrt{2}}{2}$$



$$\left\{\frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n\right\}$$

EXAMPLE 2

Find all solutions and solve without a calculator for $\sqrt{3}\csc x - 2 = 0$

$$\left\{\frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n\right\}$$

EXAMPLE 3

Find all solutions and solve without a calculator for $\sin x + \sqrt{2} = -\sin x$

$$\sin x + \sqrt{2} = -\sin x$$

$$\sin x + \sin x + \sqrt{2} = 0$$

$$2\sin x + \sqrt{2} = 0$$

$$2\sin x = -\sqrt{2}$$

EXAMPLE 3

Find all solutions and solve without a calculator for $\sin x + \sqrt{2} = -\sin x$

$$2\sin x = -\sqrt{2}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$\left\{\frac{5\pi}{4} + 2\pi n, \frac{7\pi}{4} + 2\pi n\right\}$$

YOUR TURN

Find all solutions and solve without a calculator for $\cos x - \sqrt{3} = -\cos x$

$$\left\{\frac{\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n\right\}$$

EXAMPLE 4

Find all solutions and solve without a calculator for $3\tan^2 x - 1 = 0$

$$3(\tan x)^2 - 1 = 0$$

$$3(\tan x)^2 = 1$$

$$(\tan x)^2 = \frac{1}{3}$$

$$\sqrt{(\tan x)^2} = \sqrt{\frac{1}{3}}$$

EXAMPLE 4

Find all solutions and solve without a calculator for $3\tan^2 x - 1 = 0$

$$\sqrt{(\tan x)^2} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm\frac{1}{\sqrt{3}}$$

$$\tan x = \pm\frac{\sqrt{3}}{3}$$

$$\left\{\frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n\right\}$$

EXAMPLE 5

Find all solutions and solve without a calculator for $\cot x \cos^2 x = 2 \cot x$

$$\cot x \cos^2 x - 2\cot x = 0$$

$$\cot x\left(\cos^2 x - 2\right) = 0$$

$$\cot x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2 x = 2$$

$$\cos x = \pm\sqrt{2}$$

$$\left\{\frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n\right\}$$

EXAMPLE 6

Find all solutions and solve without a calculator for $2\cos^2 x = \cos x$

$$\left\{\frac{\pi}{2}+\pi n, \frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2\pi n\right\}$$

YOUR TURN

Find all solutions and solve without a calculator for $\sin^2 x = 2 \sin x$

$$\{\pi + 2\pi n, 2\pi + 2\pi n\}$$

U-Substitution for Trigonometry

A. Try to get into one trig function
B. As a form of substitution, plug the u in for the variable with trig functions
C. Factor the equation with the substitution
D. Replacing the u back to the original trig function and solve

EXAMPLE 7

Find all solutions and solve without a calculator for

$$\mathbf{2\sin^2 x + 3\cos x - 3 = 0}$$

$$2\sin^2 x + 3\cos x - 3 = 0$$

$\sin^2 x + \cos^2 x = 1$ $\qquad \sin^2 x = 1 - \cos^2 x$

$$2\left(1 - \cos^2 x\right) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$-2\cos^2 x + 3\cos x - 1 = 0$$

EXAMPLE 7

Find all solutions and solve without a calculator for

$$\mathbf{2\sin^2 x + 3\cos x - 3 = 0}$$

$$-2\cos^2 x + 3\cos x - 1 = 0$$

$$-\left(2\cos^2 x - 3\cos x + 1\right) = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$2u^2 - 3u + 1 = 0 \qquad TS: -3, TP: +2$$

$$2u^2 - 1u - 2u + 1 = 0$$

$$\left(2u - 1\right)\left(u - 1\right) = 0$$

EXAMPLE 7

Find all solutions and solve without a calculator for

$$\mathbf{2\sin^2 x + 3\cos x - 3 = 0}$$

$$(2u-1)(u-1)=0$$

$$2u-1=0$$

$$2\cos x-1=0$$

$$2\cos x=1$$

$$\cos x=\frac{1}{2}$$

$$x=\frac{\pi}{3},\frac{5\pi}{3}$$

$$u-1=0$$

$$\cos x-1=0$$

$$\cos x=1$$

$$x=0$$

$$\left\{\frac{\pi}{3}+2\pi n,\frac{5\pi}{3}+2\pi n,0+2\pi n\right\}$$

EXAMPLE 8

Find all solutions and solve without a calculator for $\cot^2 x + \csc x - 5 = 0$

$$\left\{\frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n\right\}$$

YOUR TURN

Find all solutions and solve without a calculator for $3\sec^2 x - 2\tan^2 x - 4 = 0$

$$\left\{\frac{\pi}{4} + \pi n, \frac{3\pi}{4} + \pi n\right\}$$

EXAMPLE 9

Find all solutions and solve without a calculator for $\sin 2x = -\frac{1}{2}$

$$u = 2x$$

$$\sin u = -\frac{1}{2}$$

$$u = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

EXAMPLE 9

Find all solutions and solve without a calculator for $\sin 2x = -\frac{1}{2}$

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{7\pi}{6}\left(\frac{1}{2}\right), \frac{11\pi}{6}\left(\frac{1}{2}\right)$$

$$\left\{\frac{7\pi}{12} + \pi n, \frac{11\pi}{12} + \pi n\right\}$$

EXAMPLE 10

Find all solutions and solve without a calculator for $2\cos 3t - 1 = 0$

$$\left\{\frac{\pi}{9} + \frac{2\pi}{3}n, \frac{5\pi}{9} + \frac{2\pi}{3}n\right\}$$

EXAMPLE 11

Find all solutions and solve without a calculator for $3\tan\frac{x}{2}+3=0$ from $[0,\pi)$

$$3\tan u=-3$$

$$\tan u=-1$$

$$\tan^{-1}(-1)=u$$

EXAMPLE 11

Find all solutions and solve without a calculator for $3\tan\frac{x}{2}+3=0$ from $[0,\pi)$

$$\tan^{-1}(-1)=\frac{x}{2}$$

$$\frac{3\pi}{4}=\frac{x}{2}$$

$$\left\{\frac{3\pi}{2}+2\pi n\right\}$$

YOUR TURN

Find all solutions and solve without a calculator for $\csc 2x - \frac{2}{\sqrt{3}} = 0$

$$\left\{\frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n\right\}$$

ASSIGNMENT

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