§5.3: Inverse Functions

"I WILL …

…solve and take the derivative of Inverse Functions."

I. Review

A. The result of exchanging the input and output value of a relation is an Inverse Function
B. An inverse "undoes" the function. It switches $(x, y)$ to $(y, x)$
C. Interchange the $x$ and the $y$. (make $y$ $x$ and make $x$ $y$)
D. Resolve for $y$.
E. Written in function notation as $f^{-1}(x)$
F. Vertical Line Test: $y$ is a function of $x$ if and only if no vertical line intersects the graph at more than one point
G. Horizontal Line Test: to see if it has an inverse "one to one" must pass the test to have an inverse

| Review Ex 1: Determine the inverse of this relation, {(0, –3), (2, 1), and (6, 3)} | Review Ex 2: Determine the inverse of $y = 3x - 2$ |
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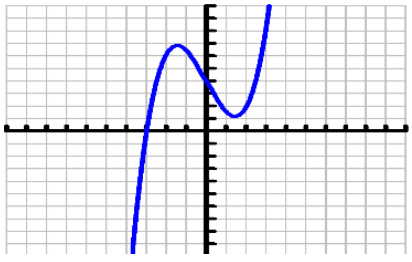
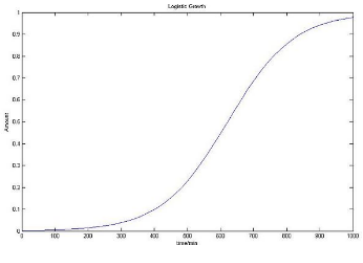
II. Review

A. The function $g$ is the inverse function of the function $f$ if $f(g(x)) = x$ and $g(f(x)) = x$ to verify inverses
B. If $g$ has an inverse of $f$, then $f$ is the inverse of $g$
C. A function has an inverses if:
D. One-to-One Function
E. If $f$ is strictly "monotonic" (strictly increasing or decreasing over the entire interval)
F. Pass the Horizontal Line Test

| Review Ex 3: Is this graph a function? Does it have an inverse? Is it one-to-one function and monotonic?  | Review Ex 4: Is this graph a function? Does it have an inverse? Is it one-to-one function and monotonic?  |
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Review Ex 5: Prove that $f(x) = 2x + 6$ and $f^{-1}(x) = \frac{1}{2}x + 3$ are inverses through a composition.

III. Derivative of an Inverse Function

A. Let $f$ be a function that is differentiable on an interval $I$. If $f$ has an inverse function $g$, then $g$ is differentiable at any $x$ for which $(f^{-1})'(a) =$ _____ OR $g'(x) =$ _____

B. Slopes of the original functions and its derivative are _____ slopes of each other.

IV. Steps

A. State the given by listing as points
B. Make the equation equal to the inverse's derivative, equate to zero, and solve for $x$.
C. Take the derivative of the _____ function and plug in what is given.
D. Take the reciprocal of the answer to establish the $f^{-1}$.

| | |
|---|---|
| Ex 1: Let $f$ be a function defined by $f(x) = x^2$ for which $x \geq 0$, $g(x) = f^{-1}(x)$ and $g(4) = 2$. Solve for $g'(4)$. | Ex 2: Let $f$ be a function defined by $f(x) = x^3 + x$, $g(x) = f^{-1}(x)$ and $g(2) = 1$. Solve for $g'(2)$. |
| Ex 3: If $f(x) = x^3 + 4$. Solve for $g'(5)$ where $g(x) = f^{-1}(x)$ | Your Turn: If $f(x) = \sqrt{x - 4}$. Solve $(f^{-1})'(5)$. |

Ex 4: Given $f(3) = 5$, $f'(3) = 7$, $f(2) = 3$, and $f'(2) = -4$ and $f$ and $g$ are inverses, solve for $g'(3)$.

Your Turn: Given $f(4) = 5$, $f'(4) = 3$, $f(5) = 4$, and $f'(5) = -2$ and $f$ and $g$ are inverses, solve for $g'(5)$.

Ex 5: Given the table below. Solve for $g'(2)$ where and $f$ and $g$ are inverses.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 1 | 2 | 0 | 6 |
| $f'(x)$ | $-1$ | 3 | $\frac{1}{2}$ | 4 | 5 |

Ex 6: Given the table below. Solve for $g'(6)$ where and $f$ and $g$ are inverses.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 1 | 2 | 0 | 6 |
| $f'(x)$ | $-1$ | 3 | $\frac{1}{2}$ | 4 | 5 |

Your Turn: Given the table below. Solve for $g'(6)$ where and $f$ and $g$ are inverses.

| $x$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 0 | 2 | 3 | $-1$ | 1 |
| $f'(x)$ | 3 | $-1$ | $-2$ | 0 | 1 |

AP 1) Given the table below. Solve for $g'(1)$ and $f$ and $g$ are inverses.

| $x$ | 3 | 4 | 7 | 9 |
|---|---|---|---|---|
| $f(x)$ | 1 | 3 | 4 | 8 |
| $f'(x)$ | $\frac{1}{4}$ | $\frac{3}{7}$ | 2 | $\frac{7}{8}$ |

(A) 1 (B) $\frac{1}{4}$ (C) 4 (D) $\frac{1}{3}$

| Vocabulary | Process and Connections | Answer and Justifications |
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Assignment: Worksheet