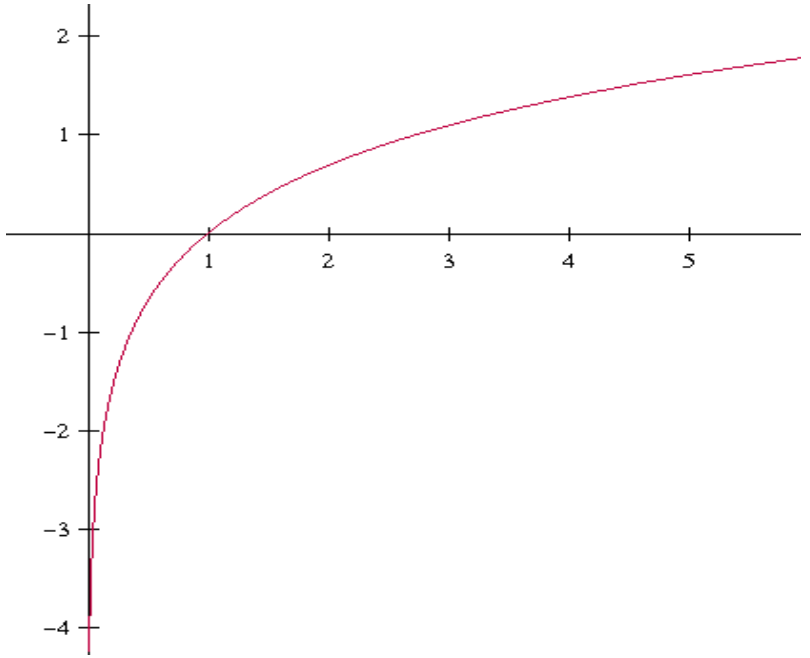
Derivatives of Natural Log Functions

Section 5.1

Calculus AP/Dual, Revised ©2017

viet.dang@humbleisd.net

Natural Logarithm Graph



A. **Domain:** $(0, \infty)$
B. **Range:** $(-\infty, \infty)$
C. **Continuous Function**
D. **One-to-One Function**
E. **Concaves Down**

Log Properties

A. $\ln(1) = 0$

B. $\ln(ab) = \ln a + \ln b$

C. $\ln(a^n) = n \ln a$

D. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Review Example 1

Expand $\sqrt[3]{\ln\left(\frac{x^3-2}{x^4}\right)}$

$$\frac{1}{3}\left[\ln\left(x^3-2\right)-4\ln x\right]$$

REVIEW EXAMPLE 2

Condense 2 $[\ln x - (\ln(x+1) + \ln(x-1))]$

$$\ln\left(\frac{x}{x^2-1}\right)^2$$

Guess The Rule

Equation	Derivative
1) $\frac{d}{dx}[\ln(4x)]$	$\frac{4}{4x}=\frac{1}{x}$
2) $\frac{d}{dx}[\ln(2x^2)]$	$\frac{4x}{2x^2}=\frac{2}{x}$
3) $\frac{d}{dx}[\ln(\sqrt{x})]$	$\frac{\frac{1}{2}x^{-1/2}}{x^{1/2}}=\frac{1}{2x}$
4) $\frac{d}{dx}\left[\ln(1+x)(1+x^2)^2(1+x^3)^3\right]$	$\frac{d}{dx}\left[\ln(1+x)+2\ln(1+x^2)+3\ln(1+x^3)\right]=$ $\frac{1}{1+x}+\frac{4x}{1+x^2}+\frac{9x^2}{1+x^3}$
5) $\frac{d}{dx}[\ln(u)]$	$\frac{u'}{u}$

Log Derivative Rules

A. $\frac{d}{dx}[\ln(u)] = \frac{u'}{u}, u > 0$

Derivative of Natural Log

http://www.math.uri.edu/~bkaskosz/flashmo/derplot/

WHY?

$$y = \ln x$$

$$e^y = x$$

$$\frac{d}{dx}\left(e^y\right) = \frac{d}{dx}\left(x\right)$$

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln u = \frac{1}{x}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\ln u\right] = \frac{u'}{u}$$

Steps

A. If necessary, use the log properties FIRST by EXPANDING
B. Apply the derivative
C. Simplify

EXAMPLE 1

Solve $\frac{dy}{dx}$ if $y = [\ln(2x)]$

$$\ln(2x) = \ln 2 + \ln x$$

$$\frac{d}{dx}\left[\ln(2x)\right] = \frac{d}{dx}(\ln 2) + \frac{d}{dx}(\ln x)$$

$$\frac{d}{dx}\left[\ln(2x)\right] = \frac{(2)'}{2} + \frac{(x)'}{x}$$

$$\frac{d}{dx}\left[\ln(2x)\right] = \frac{0}{2} + \frac{1}{x}$$

$$\frac{1}{x}$$

EXAMPLE 1 (EASIER WAY)

Solve $\frac{dy}{dx}$ if $y = [\ln(2x)]$

$$\frac{d}{dx}\ln(u) = \frac{u'}{u}$$

$$\frac{d}{dx}\left[\ln(2x)\right] = \frac{2}{2x}$$

$$\boxed{\frac{1}{x}}$$

EXAMPLE 2

Solve for the derivative for $f(x) = \left[\ln\left(x^2 + 1\right)\right]$

$$\frac{2x}{x^2+1}$$

EXAMPLE 3

Solve for the derivative $f(x) = (\ln x)^5$

Hint: cannot use u'/u **on this problem because of the exponent**

$$\frac{5}{x}(\ln x)^4$$

EXAMPLE 4

Solve for the derivative : $f(x) = \ln(x^4)$

$$\frac{4}{x}$$

YOUR TURN

Solve for the derivative : $f(x) = x \ln(x)$

$$1 + \ln x$$

EXAMPLE 5

Solve for the derivative: $\boldsymbol{f(x) = \left[\ln \sqrt{(x^2 - 3)}\right]}$

$$\ln\left(x^2 - 3\right)^{1/2}$$

$$\left(\frac{1}{2}\right)\frac{u'}{u}$$

$$\left(\frac{1}{2}\right)\frac{2x}{x^2 - 3}$$

$$\boxed{\frac{x}{x^2 - 3}}$$

EXAMPLE 6

Solve $\frac{dy}{dx}$ if $y = [\sqrt{x} \cdot \ln(x)]$

$$f(x)g'(x) + g(x)f'(x)$$

$$\left[\left(\sqrt{x}\right)\left(\frac{1}{x}\right)\right] + \left[\left(\ln x\right)\left(\frac{1}{2}x^{-1/2}\right)\right]$$

$$\left(\frac{\sqrt{x}}{x}\right) + \left(\frac{\ln x}{2x^{1/2}}\right)$$

EXAMPLE 7

Solve $\frac{dy}{dx}$ if $y = \ln\left(\sqrt{\frac{x+1}{x-1}}\right)$

$$y = \ln\left(\frac{x+1}{x-1}\right)^{1/2}$$

$$y = \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$$

$$y' = \frac{1}{2}\left[\frac{d}{dx}\ln(x+1) - \frac{d}{dx}\ln(x-1)\right]$$

$$y' = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right]$$

EXAMPLE 7

Solve $\frac{dy}{dx}$ if $y = \ln\left(\sqrt{\frac{x+1}{x-1}}\right)$

$$y' = \frac{1}{2}\left[\frac{1(x-1)}{(x+1)(x-1)} - \frac{1(x+1)}{(x-1)(x+1)}\right]$$

$$y' = \frac{1}{2}\left[\frac{x-1-(x+1)}{(x+1)(x-1)}\right]$$

$$y' = \frac{1}{2}\left[\frac{-2}{(x+1)(x-1)}\right]$$

$$y' = -\frac{1}{(x+1)(x-1)}$$

YOUR TURN

Solve $\frac{dy}{dx}$ **if** $y = \ln \frac{x}{\sqrt{x^2+2}}$

$$\frac{1}{x} - \frac{x}{x^2+2}$$

EXAMPLE 8

Solve $\frac{dy}{dx}$ if $y = \ln|\cos x|$

$$y' = \frac{u'}{u}$$

$$y' = \frac{(\cos x)'}{\cos x}$$

$$y' = \frac{-\sin x}{\cos x}$$

$$\boxed{-\tan x}$$

YOUR TURN

Solve $\frac{dy}{dx}$ if $y = \ln|\csc x|$

$-\cot x$

AP Multiple Choice Practice Question 1 (non-calculator)

Solve $\frac{dy}{dx}$ for $y = \ln\sqrt{x^2+4}$

(A) $\frac{1}{x}$

(B) $\frac{x}{\sqrt{x^2+4}}$

(C) $\frac{x}{x^2+4}$

(D) $\frac{2x}{\sqrt{x^2+4}}$

AP Multiple Choice Practice Question 1 (non-calculator)

Solve $\frac{dy}{dx}$ for $y = \ln\sqrt{x^2+4}$

Vocabulary	Connections and Process	Answer and Justifications
Natural Log	$\frac{u'}{u} = \left(\frac{1}{2}\right)\frac{2x}{x^2+4}$ $\frac{u'}{u} = \frac{x}{x^2+4}$	C

ASSIGNMENT

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5-8 all, 41-63 odd (omit 55), 64, 65A, 73, 75