

# TRIGONOMETRIC IDENTITIES

## Section 5.1

Precalculus PreAP/Dual, Revised ©2017

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# RECIPROCAL & QUOTIENT IDENTITIES

$\sin \theta = \frac{1}{\csc \theta}$	$\cos \theta = \frac{1}{\sec \theta}$	$\tan \theta = \frac{1}{\cot \theta}$
$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

# PYTHAGOREAN IDENTITIES

$\sin^2 x + \cos^2 x = 1$	$1 + \tan^2 x = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
Known as: $a^2 + b^2 = c^2$	<b>“I TASTE SALT”</b>	<b>“I CUT CHEESECAKE”</b>

# CO-FUNCTION IDENTITIES

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$	$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$	$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$
$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$	$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$	$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$

# PYTHAGOREAN IDENTITIES

$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = r^2$$

$$\cos \theta = \frac{x}{r}; \sin \theta = \frac{y}{r}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

# PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

"I TASTE SALT"

# PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

**"I CUT CHEESECAKE"**

# PROVING IDENTITIES

- A. An identity is an equation that is true for all values of the variables for which every term of the equation is defined.
- B. An odd function is a function that is symmetrical at the origin
- C. An even function is a function that is symmetrical through the y-axis



# STEPS IN PROVING IDENTITIES

- A. Work at one side at a time**
- B. Identify any opportunities to factor an expression, such as adding fractions, squaring a binomial, etc...**
- C. Apply whatever sine and cosine functions are applicable**
- D. Simplify the equation by using all applicable theorems and identities**
- E. CANCEL, CANCEL, CANCEL**
- F. At its conclusion, Make sure that  $A = A$**
- G. Remember, there is MORE THAN ONE WAY to solve an equation**

# RECIPROCAL IDENTITIES THEOREM

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

# QUOTIENT IDENTITIES

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$$

# EXAMPLE 1

Use trigonometric identities to transform the left side of the equation into the right side  $\left(0, \frac{\pi}{2}\right)$  for  $\tan \theta \cos \theta = \sin \theta$

$$\tan \theta \cos \theta = \sin \theta$$

$$\left(\frac{\sin \theta}{\cos \theta}\right) \cos \theta = \sin \theta$$

$$\sin \theta = \sin \theta$$

## EXAMPLE 2

Use trigonometric identities to transform the left side of the equation into the right side  $\left(0, \frac{\pi}{2}\right)$  for  $\cos \theta \sec \theta = 1$

$$1 = 1$$

# YOUR TURN

Use trigonometric identities to transform the left side of the equation into the right side  $\left(0, \frac{\pi}{2}\right)$  for  $\csc \theta \tan \theta = \sec \theta$

$$\sec \theta = \sec \theta$$

## EXAMPLE 3

Use trigonometric identities to prove  $\left(0, \frac{\pi}{2}\right)$  for  $\cot^2 \theta \cdot \sin^2 \theta + \sin^2 \theta = 1$

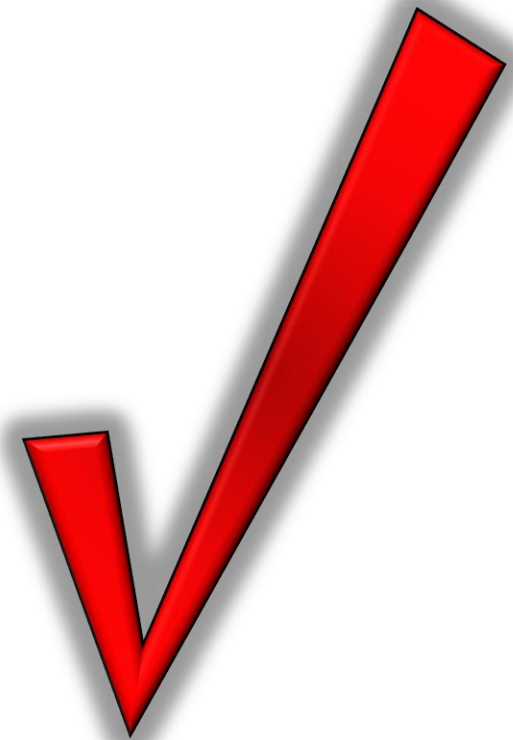
$$\cot^2 x \sin^2 x + \sin^2 x = 1$$

$$\sin^2 x (\cot^2 x + 1) = 1$$

$$\sin^2 x (\csc^2 x) = 1$$

~~$$\sin^2 x \left( \frac{1}{\sin^2 x} \right) = 1$$~~

$$\mathbf{1 = 1}$$



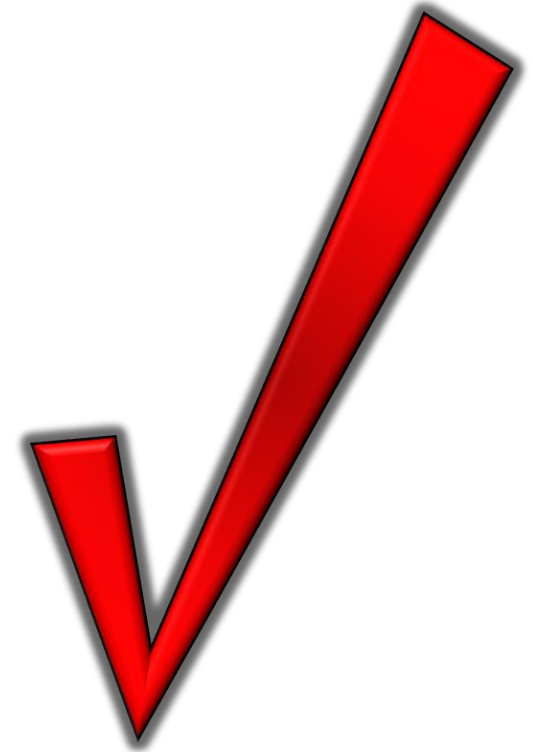
## EXAMPLE 3 (ANOTHER WAY)

Use trigonometric identities to prove  $\left(0, \frac{\pi}{2}\right)$  for  $\cot^2 \theta \cdot \sin^2 \theta + \sin^2 \theta = 1$

$$\left( \frac{\cos^2 x}{\sin^2 x} \right) \sin^2 x + \sin^2 x = 1$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 = 1$$





## EXAMPLE 4

Use trigonometric identities to prove  $\left(0, \frac{\pi}{2}\right)$  for  
 $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$

$$1 = 1$$

# YOUR TURN

Use trigonometric identities to prove  $\left(0, \frac{\pi}{2}\right)$  for  
 $(\csc \theta + 1)(\csc \theta - 1) = \cot^2 \theta$

$$\cot^2 \theta = \cot^2 \theta$$

# ASSIGNMENT

**Page 287**

**47-56 all**