Trigonometric Identities

Section 5.1

Precalculus PreAP/Dual, Revised ©2017

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Reciprocal & Quotient Identities

$\sin\theta = \dfrac{1}{\csc\theta}$	$\cos\theta = \dfrac{1}{\sec\theta}$	$\tan\theta = \dfrac{1}{\cot\theta}$
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$	$1 + \tan^2 x = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
Known as: $\boldsymbol{a^2 + b^2 = c^2}$	**"I TASTE SALT"**	**"I CUT CHEESECAKE"**

Co-Function Identities

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$	$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$	$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$
$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$	$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$	$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

Pythagorean Identities

$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = r^2$$

$$\cos\theta = \frac{x}{r}; \sin\theta = \frac{y}{r}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

"I TASTE SALT"

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

"I CUT CHEESECAKE"

PROVING IDENTITIES

A. **An identity is an equation that is true for all values of the variables for which every term of the equation is defined.**
B. **An odd function is a function that is symmetrical at the origin**
C. **An even function is a function that is symmetrical through the y-axis**

Steps in Proving Identities

A. Work at one side at a time
B. Identify any opportunities to factor an expression, such as adding fractions, squaring a binomial, etc…
C. Apply whatever sine and cosine functions are applicable
D. Simplify the equation by using all applicable theorems and identities
E. CANCEL, CANCEL, CANCEL
F. At its conclusion, Make sure that $A = A$
G. Remember, there is MORE THAN ONE WAY to solve an equation

Reciprocal Identities Theorem

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

Example 1

Use trigonometric identities to transform the left side of the equation into the right side $\left(0, \frac{\pi}{2}\right)$ for $\tan\theta\cos\theta = \sin\theta$

$$\tan\theta\cos\theta = \sin\theta$$

$$\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)\cancel{\cos\theta} = \sin\theta$$

$$\sin\theta = \sin\theta$$

EXAMPLE 2

Use trigonometric identities to transform the left side of the equation into the right side $\left(0, \frac{\pi}{2}\right)$ for $\cos\theta \sec\theta = 1$

$$1 = 1$$

YOUR TURN

Use trigonometric identities to transform the left side of the equation into the right side $\left(0, \frac{\pi}{2}\right)$ for $\csc\theta\tan\theta = \sec\theta$

$$\sec\theta = \sec\theta$$

Example 3

Use trigonometric identities to prove $\left(0, \frac{\pi}{2}\right)$ for $\cot^2\theta \cdot \sin^2\theta + \sin^2\theta = 1$

$$\cot^2 x \sin^2 x + \sin^2 x = 1$$

$$\sin^2 x\left(\cot^2 x + 1\right) = 1$$

$$\sin^2 x\left(\csc^2 x\right) = 1$$

$$\sin^2 x\left(\frac{1}{\sin^2 x}\right) = 1$$

$$1 = 1$$

Example 3 (Another Way)

Use trigonometric identities to prove $\left(0,\frac{\pi}{2}\right)$ for $\cot^2\theta \cdot \sin^2\theta + \sin^2\theta = 1$

$$\left(\frac{\cos^2 x}{\sin^2 x}\right)\sin^2 x + \sin^2 x = 1$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 = 1$$

EXAMPLE 4

Use trigonometric identities to prove $\left(0,\frac{\pi}{2}\right)$ for

$$(\csc\theta+\cot\theta)(\csc\theta-\cot\theta)=1$$

$$1=1$$

YOUR TURN

Use trigonometric identities to prove $\left(0, \frac{\pi}{2}\right)$ for

$$(\csc\theta + 1)(\csc\theta - 1) = \cot^2\theta$$

$$\cot^2\theta = \cot^2\theta$$

ASSIGNMENT

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