Simple Harmonic Motion

Section 4.8

Precalculus PreAP/Dual, Revised ©2018

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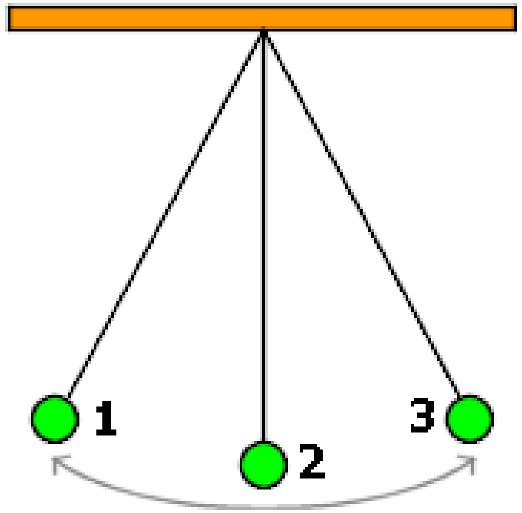
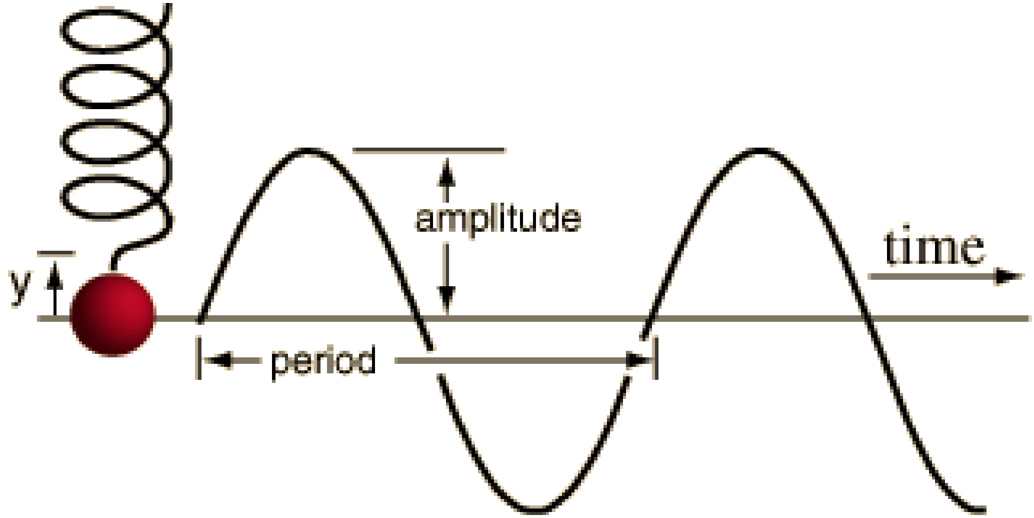
Simple Harmonic Motion

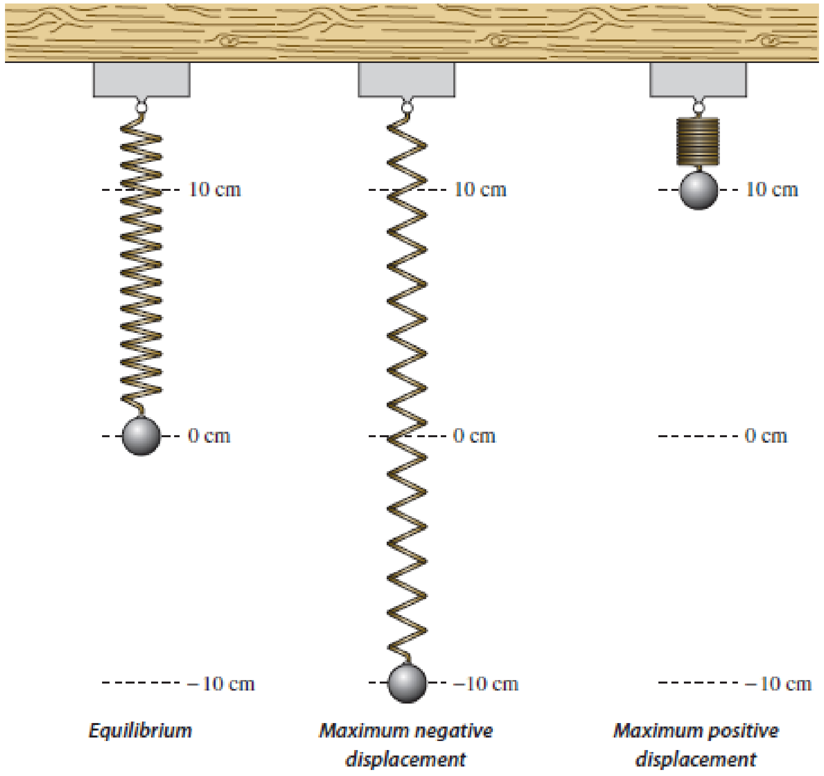
A. **<u>Simple harmonic motion</u> is a special kind of vibrational motion in which the acceleration, a of the object is directly proportional to the negative its displacement, d from its rest position.**
B. **The periodic nature of the trigonometric functions is useful for describing motion of a point on an object that vibrates, oscillates, rotates or is moved by wave motion.**
C. **<u>Displacement</u> is shortest distance from the initial to the final position**



EQUATIONS

A. $d = A\,\text{Trig}\,B(t - C) + D$
   1. $|A|$ is amplitude or maximum displacement
   2. Period = $\frac{2\pi}{|B|}$
   3. Frequency = $\frac{|B|}{2\pi}$





VIDEO

GSP File

YouTube Clip: https://www.youtube.com/watch?v=XjyOYluqnDM

EXAMPLE 1

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(A) Find the maximum displacement

(B) Determine the frequency

(C) Find the value of d when $t = 4$

(D) Find the least positive value of t for which $d = 0$

EXAMPLE 1A

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(A) Find the maximum displacement

$$d = 6\cos\frac{3\pi}{4}t$$

$$Amplitude = 6$$

EXAMPLE 1B

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(B) Determine the frequency

$$d = 6\cos\frac{3\pi}{4}t$$

$$Frequency = \frac{B}{2\pi}$$

$$Frequency = \frac{\frac{3\pi}{4}}{2\pi}$$

$$Frequency = \frac{3}{8}cycle$$

EXAMPLE 1C

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(C) the value of d when $t = 4$

$$d = 6\cos\frac{3\pi}{4}t$$

$$6\cos\frac{3\pi}{4}(4) = 0$$

$$6\cos 3\pi = 0$$

$$6(-1)$$

$$-6$$

Example 1D

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(D) Find the least positive value of t for which $d = 0$

$$d = 6\cos\frac{3\pi}{4}t$$

$$\frac{6\cos\frac{3\pi}{4}t}{6} = \frac{0}{6}$$

$$\cos\frac{3\pi}{4}t = 0$$

$$\frac{3\pi}{4}t = \cos^{-1}(0)$$

Example 1D

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(D) Find the least positive value of t for which $d = 0$

$$\frac{3\pi}{4}t = \cos^{-1}(0)$$

$$\left(\frac{4}{3\pi}\right)\frac{3\pi}{4}t = \frac{\pi}{2}\left(\frac{4}{3\pi}\right)$$

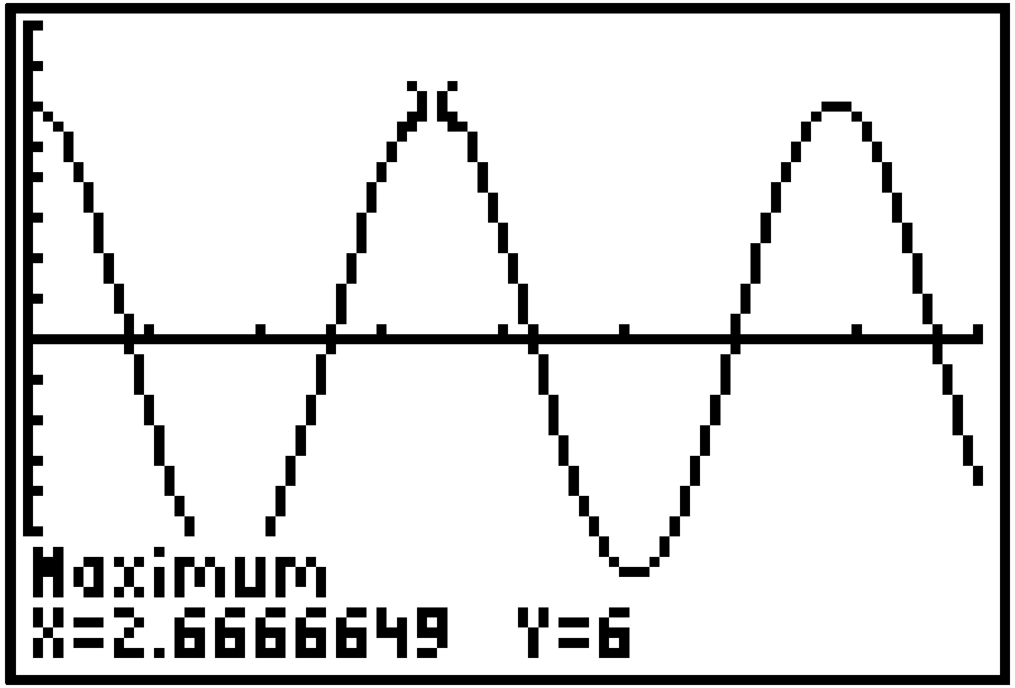
$$t = \frac{2}{3}$$

GRAPHING CALCULATOR

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(A) Find the maximum displacement

$Amplitude = 6$

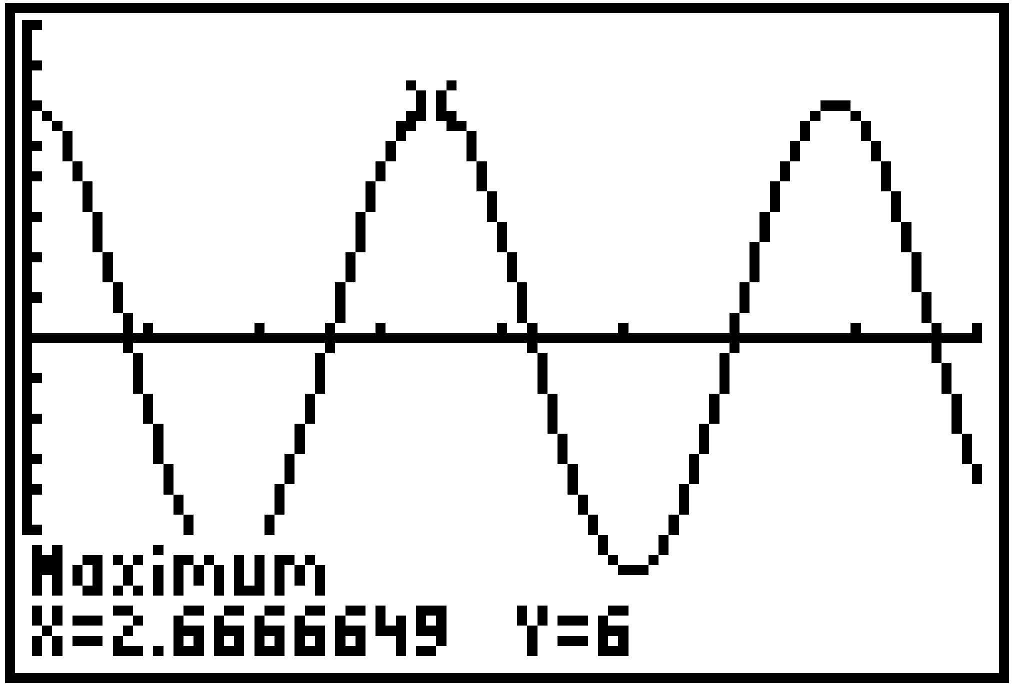


GRAPHING CALCULATOR

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$**.**

(B) Determine the frequency

$$Frequency = \frac{3}{8}cycle$$



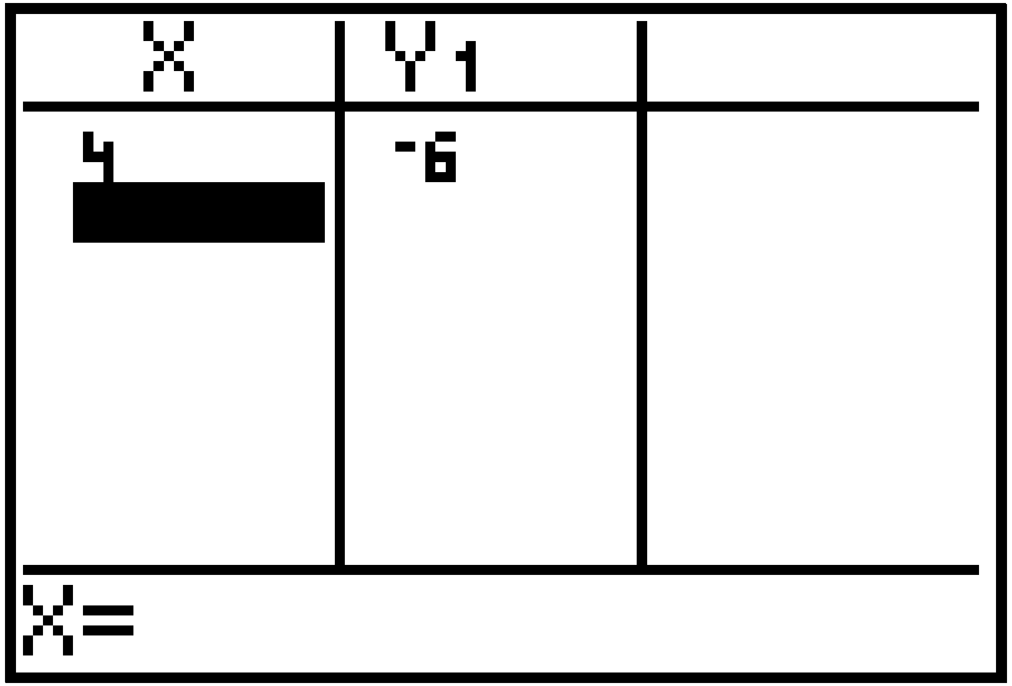
$$F = \frac{2.66667}{2\pi}$$

GRAPHING CALCULATOR

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(C) the value of d when $t = 4$

-6

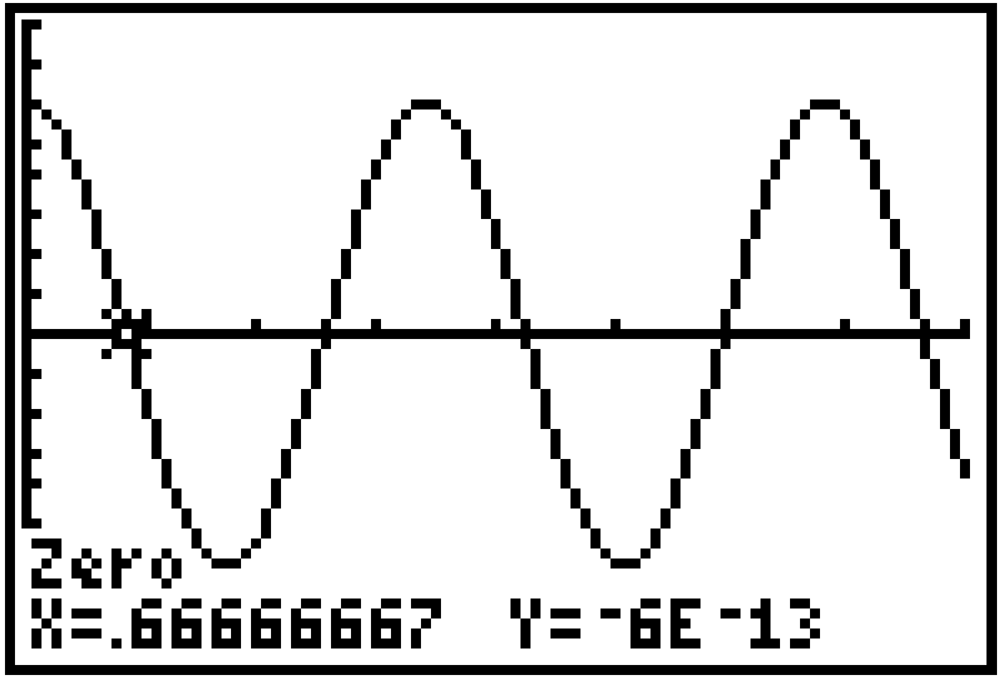


GRAPHING CALCULATOR

Given the equation for the simple harmonic motion, $d = 6\cos\frac{3\pi}{4}t$.

(D) Find the least positive value of t for which $d = 0$

$t = \frac{2}{3}$



YOUR TURN

Given the equation for the simple harmonic motion, $d = 5\sin\frac{\pi}{4}t$.

(A) Find the maximum displacement

(B) Determine the frequency

(C) Find the value of d when $t = 4$

(D) Find the least positive value of t for which $d = 0$

$A)5$

$B)\frac{1}{8}$

$C)0$

$D)8$

Simple Harmonic Motion

A. An important situation in which simple harmonic motion occurs is in the production of sound.
B. Sound is produced by a regular variation in air pressure from the normal pressure. If the pressure varies in simple harmonic motion, then a pure sound is produced.
C. The tone of the sound depends on the frequency, and the loudness depends on the amplitude.

EXAMPLE 2

A tuba player plays the note E and sustains the sound for some time. For a pure E the variation in pressure from normal air pressure is given by $V(t) = 0.2 \sin 80\pi t$ where V is measured in pounds per square inch and t is measured in seconds.

(A) Find the amplitude, period, and frequency of V.

(B) Sketch a graph of V.

(C) If the tuba player increases the loudness of the note, how does the equation for V change?

(D) If the player is playing the note incorrectly and it is a little flat, how does the equation for V change?

EXAMPLE 2A

A tuba player plays the note E and sustains the sound for some time. For a pure E the variation in pressure from normal air pressure is given by $V(t) = 0.2 \sin 80\pi t$ where V is measured in pounds per square inch and t is measured in seconds.

(A) Find the amplitude, period, and frequency of V.

$$Amplitude = |0.2| = 0.2$$

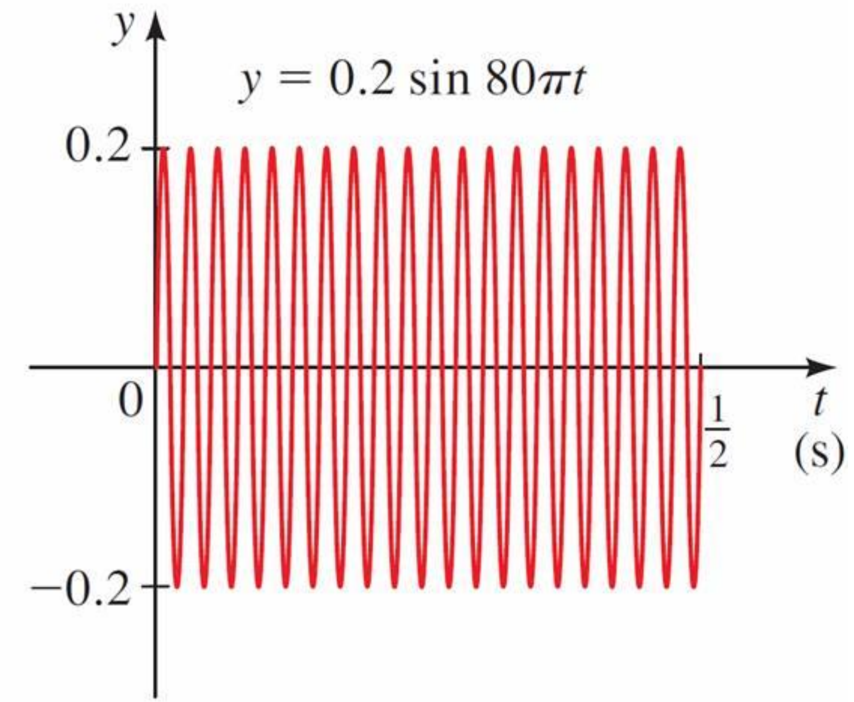
$$Period = \frac{2\pi}{80\pi} = \frac{1}{40}$$

$$Frequency = \frac{80\pi}{2\pi} = 40$$

EXAMPLE 2B

A tuba player plays the note E and sustains the sound for some time. For a pure E the variation in pressure from normal air pressure is given by $V(t) = 0.2\sin 80\pi t$ where V is measured in pounds per square inch and t is measured in seconds.

(B) Sketch a graph of V.



Example 2c

A tuba player plays the note E and sustains the sound for some time. For a pure E the variation in pressure from normal air pressure is given by $V(t) = 0.2 \sin 80\pi t$ where V is measured in pounds per square inch and t is measured in seconds.

(C) If the tuba player increases the loudness of the note, how does the equation for V change?

If the player increases the loudness the amplitude increases.
So the number 0.2 is replaced by a larger number.

EXAMPLE 2D

A tuba player plays the note E and sustains the sound for some time. For a pure E the variation in pressure from normal air pressure is given by $V(t) = 0.2 \sin 80\pi t$ where V is measured in pounds per square inch and t is measured in seconds.

(D) If the player is playing the note incorrectly and it is a little flat, how does the equation for V change?

If the note is flat, then the frequency is decreased.Thus, the coefficient oftis less than 80π.

Steps

A. Establish the D first by drawing a horizontal line cutting the graph in half

B. Find A where the amplitude is the distance above or below the line to the top or bottom of the curve

C. Find B by finding the length of curve to receive the period, $b = \frac{2\pi}{period}$

D. Find C for sine and cosine graphs

1. Sine starts on the line and goes up
2. Cosine starts on the top of the curve

RECALL

A = Amplitude: $\frac{1}{2}(Max - Min)$

D = Baseline = $\frac{1}{2}(Max + Min)$

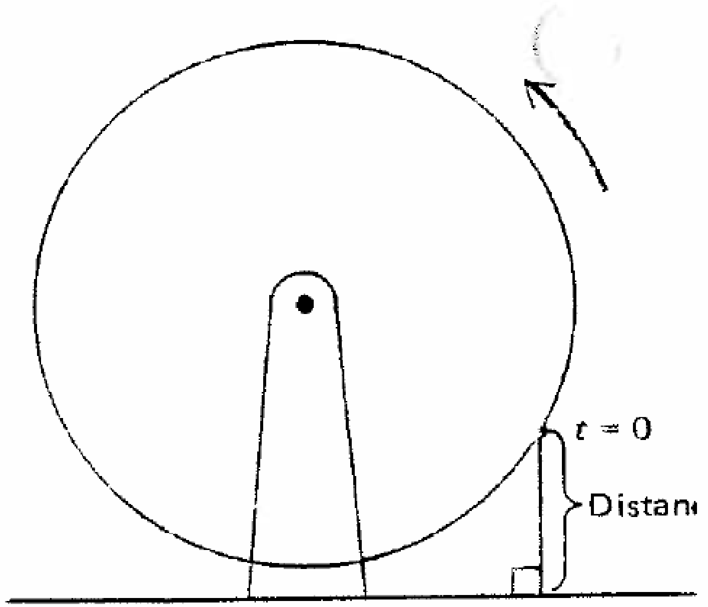
$$B = \frac{2\pi}{period}$$

Equation: $y = A\,\text{trig}\ \frac{2\pi}{B}(x - C) + D$

Example 3

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled,, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

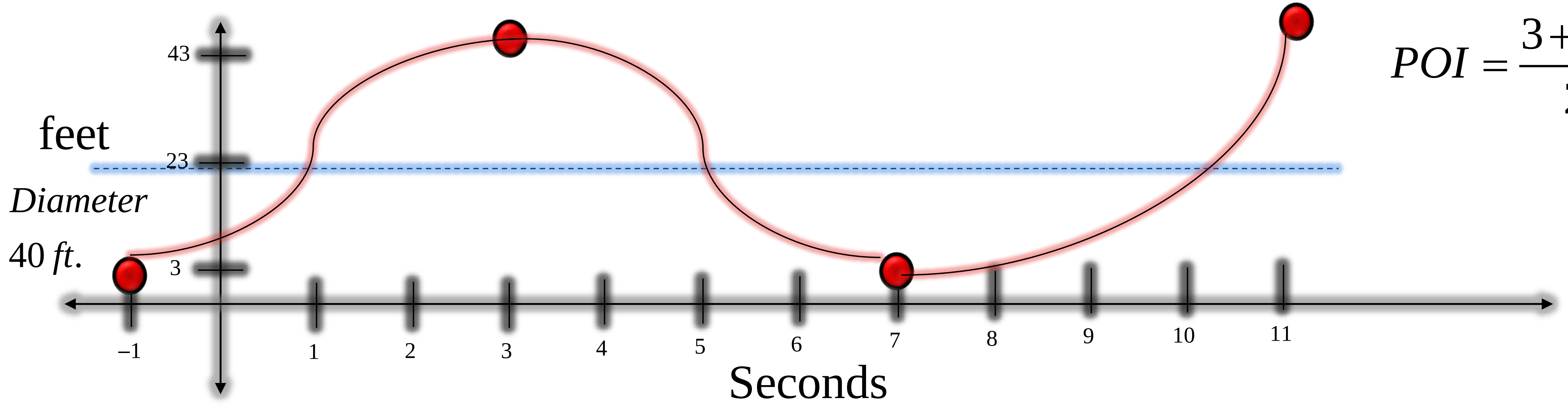
(A) Sketch a graph of the sinusoid.
(B) What is the lowest you can go as the Ferris wheel turns, and why is this number greater than zero?
(C) Write an equation of this sinusoidal.
(D) Predict your height above the ground when $t = 6$, $t = 9$, and $t = 0$
(E) What is the value of t the second time you are 18 feet above the ground?



EXAMPLE 3A

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

(A) Sketch a graph of the sinusoid.

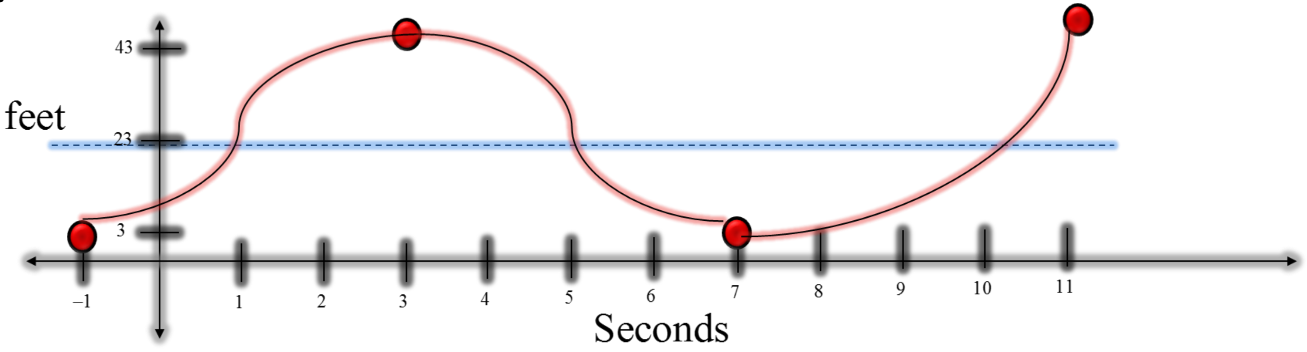


$$POI = \frac{3+43}{2} = 23$$

Example 3B

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

(B) What is the lowest you can go as the Ferris wheel turns, and why is this number greater than zero?



The lowest point is 3 feet because the Ferris wheel never hits the ground, which is 0 feet.

EXAMPLE 3C

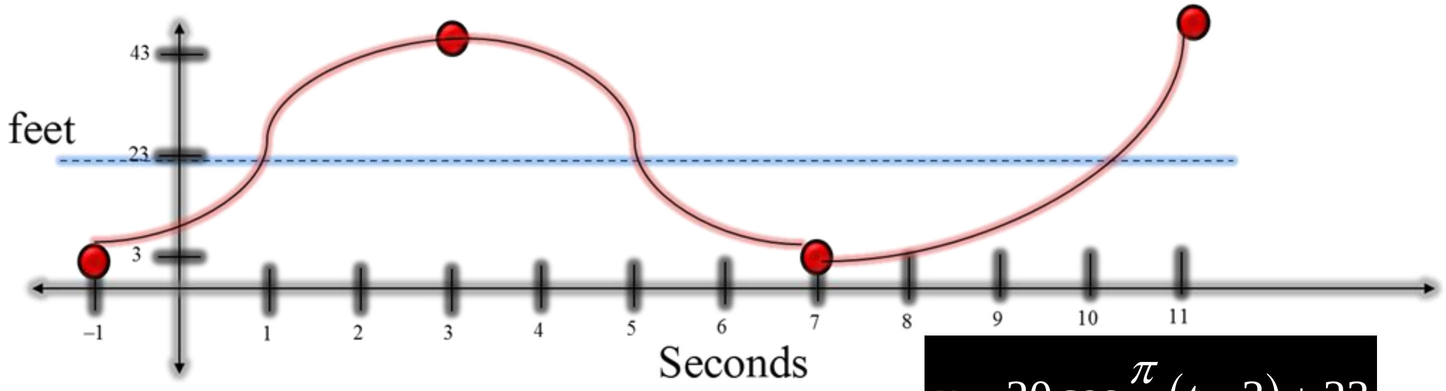
As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

(C) Write an equation of this sinusoidal.

$$Amplitude = |20| = 20$$

$$Period = \frac{2\pi}{7-(-1)} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$A = 20, B = 8, D = 23$$



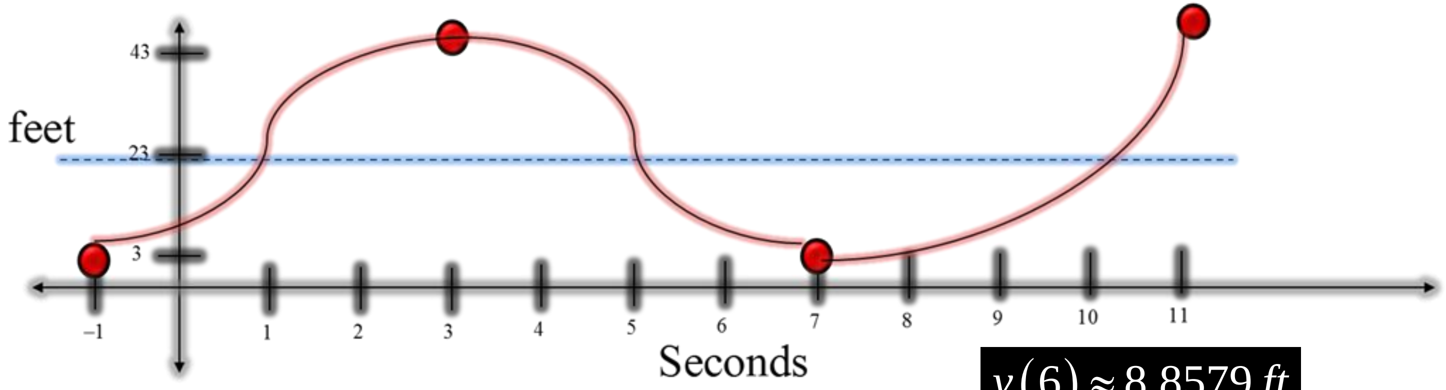
$$y = 20\cos\frac{\pi}{4}(t-3)+23$$

$$y = 20\sin\frac{\pi}{4}(t-1)+23$$

Example 3D

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

(D) Predict your height above the ground when $t = 6$, $t = 9$, and $t = 0$



$$y = 20\cos\frac{\pi}{4}(t-3)+23$$

$$y = 20\cos\frac{\pi}{4}(6-3)+23$$

$$y = 20\cos\frac{\pi}{4}(9-3)+23$$

$$y = 20\cos\frac{\pi}{4}(0-3)+23$$

$y(6) \approx 8.8579\ ft$

$y(9) \approx 23\ ft$

$y(0) \approx 8.8579\ ft$

EXAMPLE 3E

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

(E) What is the value of t the second time you are 18 feet above the ground? (calc)

$$y = 20\cos\frac{\pi}{4}(t-3) + 23$$

$$18 = 20\cos\frac{\pi}{4}(t-3) + 23$$

$$-5 = 20\cos\frac{\pi}{4}(t-3)$$

$$-\frac{1}{4} = \cos\frac{\pi}{4}(t-3)$$

EXAMPLE 3E

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

(E) What is the value of t the second time you are 18 feet above the ground?

$$-\frac{1}{4} = \cos\frac{\pi}{4}(t-3)$$

$$\cos^{-1}\left(-\frac{1}{4}\right) = \frac{\pi}{4}t - \frac{3\pi}{4}$$

$$1.8234 \approx \frac{\pi}{4}t - \frac{3\pi}{4}$$

$$1.8234 + \frac{3\pi}{4} \approx \frac{\pi}{4}t$$

$$t \approx 5.32172 \text{ sec.}$$

Example 3E

As you ride the Ferris wheel, your distance from the ground caries sinusoidally with time. When the last seat is filled, and the Ferris wheel starts, your seat is at the position shown below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes revolution once every 8 seconds. The diameter of the wheel is 40 feet.

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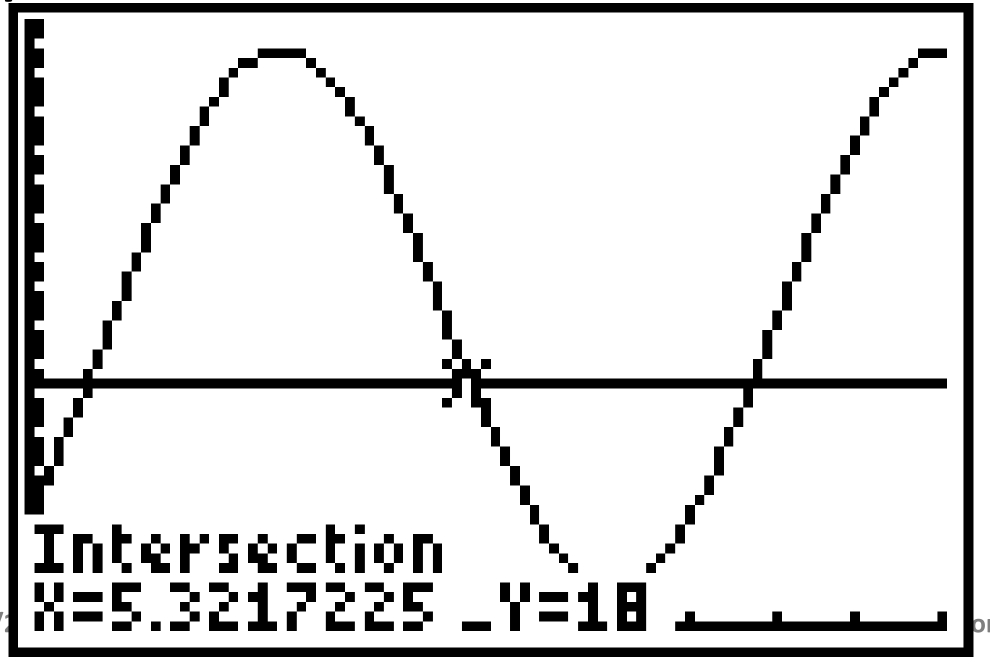


$$t \approx 5.32172 \text{ sec.}$$

Example 4

Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the point was its highest, 16 feet above the water's surface. The wheel's diameter was 18 feet, and its completed a revolution every 10 seconds.

(a) Sketch a graph of the sinusoidal.

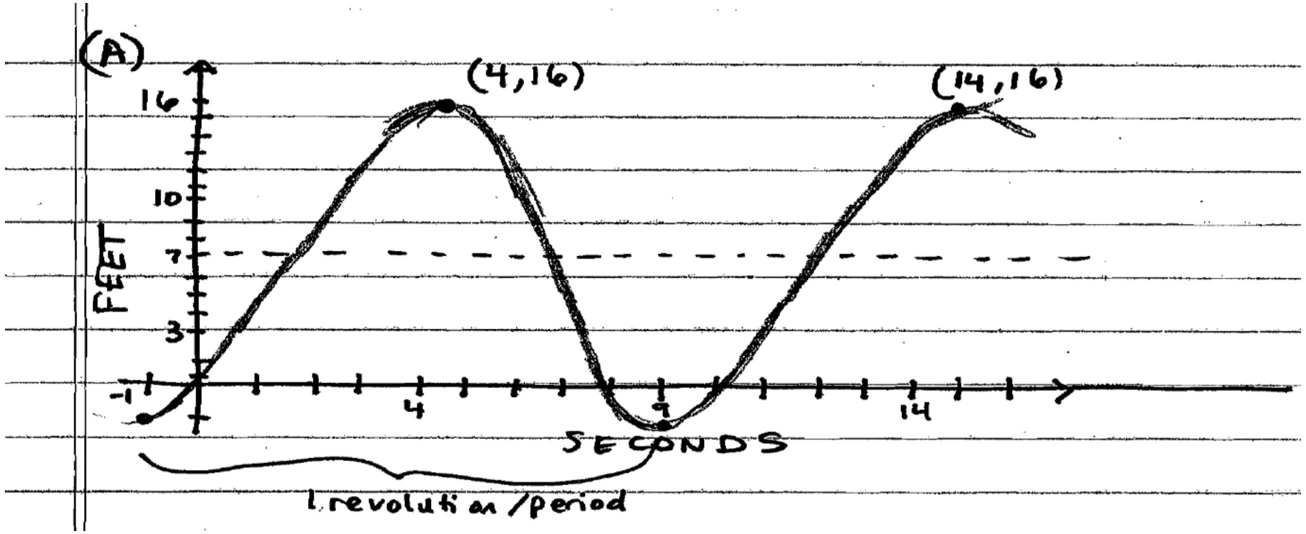
(b) Write the equation of the sinusoid.

(c) How far above the surface was the point when Mark's stopwatch read i) 5 seconds, and ii) 17 seconds?

Example 4a

Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the point was its highest, 16 feet above the water's surface. The wheel's diameter was 18 feet, and its completed a revolution every 10 seconds.

(A) Sketch a graph of the sinusoid.



Example 4b

Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the point was its highest, 16 feet above the water's surface. The wheel's diameter was 18 feet, and its completed a revolution every 10 seconds.

(B) Write a cosine equation of the sinusoid.

$$y = 9\cos\frac{\pi}{5}\left(t-4\right)+7$$

Example 4c

Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the point was its highest, 16 feet above the water's surface. The wheel's diameter was 18 feet, and its completed a revolution every 10 seconds.

(C) How far above the surface was the point when Mark's stopwatch read i) 5 seconds, and ii) 17 seconds?

$$y(5) = 14.2812\ ft,\ y(17) = 4.2188\ ft$$

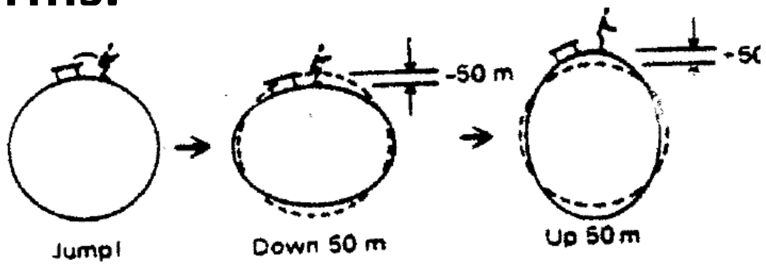
Example 5

Suppose that one day all 400 million people in the US climb up on tables. At time, $t = 0$, we all jump off. The resulting shock as we hit the earth's surface will start the entire earth vibrating in such a way that its surface first move down from its normal position and then moves up on equal distance above its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assuming that the amplitude is 50 meters.

(a) At what time will the first maximum (i.e. the greatest distance above the normal position to occur?)

(b) Write the equation expressing the displacement in terms.

(c) Predict the displacement when $t = 21$.



EXAMPLE 5A

Suppose that one day all 400 million people in the US climb up on tables. At time, $t = 0$, we all jump off. The resulting shock as we hit the earth's surface will start the entire earth vibrating in such a way that its surface first move down from its normal position and then moves up on equal distance above its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assuming that the amplitude is 50 meters.

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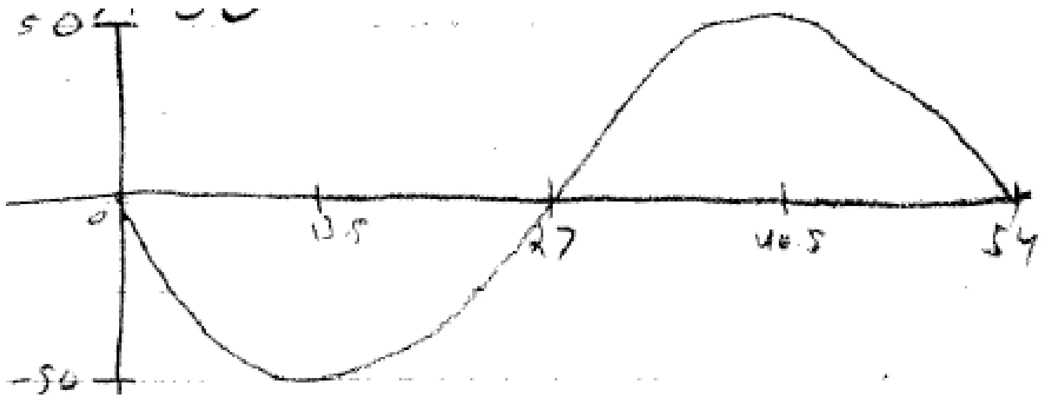


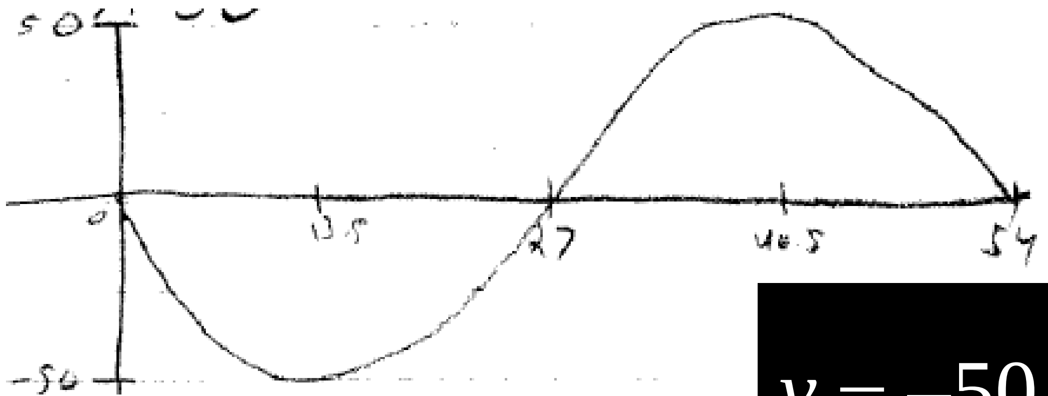
$$Max = \frac{27+54}{2} = 40.5\,\text{min}.$$

40.5 minutes

Example 5b

Suppose that one day all 400 million people in the US climb up on tables. At time, $t = 0$, we all jump off. The resulting shock as we hit the earth's surface will start the entire earth vibrating in such a way that its surface first move down from its normal position and then moves up on equal distance above its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assuming that the amplitude is 50 meters.

(b) Write a sine equation expressing the displacement in terms.



$$y = -50\sin\frac{\pi}{27}(t-0)$$

Example 5c

Suppose that one day all 400 million people in the US climb up on tables. At time, $t = 0$, we all jump off. The resulting shock as we hit the earth's surface will start the entire earth vibrating in such a way that its surface first move down from its normal position and then moves up on equal distance above its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assuming that the amplitude is 50 meters.

(c) Predict the displacement when $t = 21$.

$\approx -32.1394m$

YOUR TURN

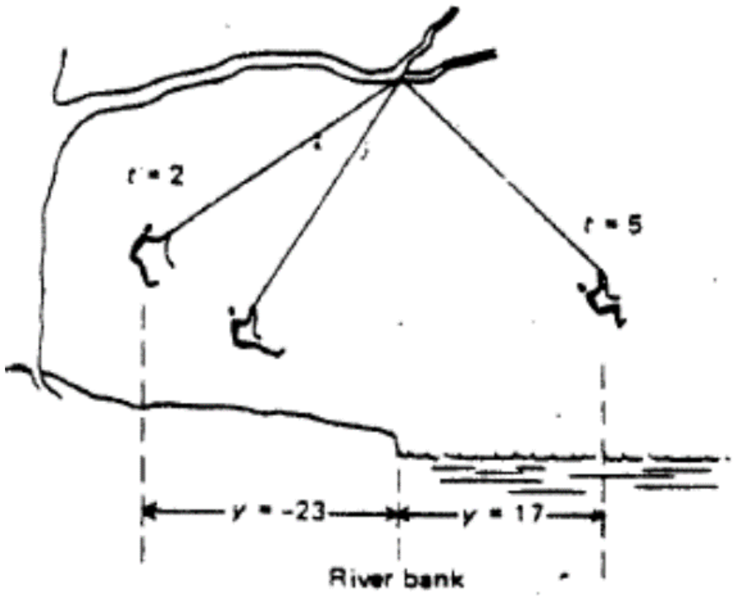
Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the river bank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and y be the number of meters Tarzan is from the river bank. Assume y varies sinusoidally with t, and that y is positive when Tarzan is over water and negative when he is over land.

Jane finds that when $t = 2$, Tarzan is at one end of his swing, when $y = -23$. She finds that when $t = 5$ he reaches the other end of his swing and $y = 17$.

(a) Sketch a graph of this sinusoidal function

(b) Write an equation from the river bank in terms of t.

(c) Predict y when $t = 2.8$ and $t = 15$



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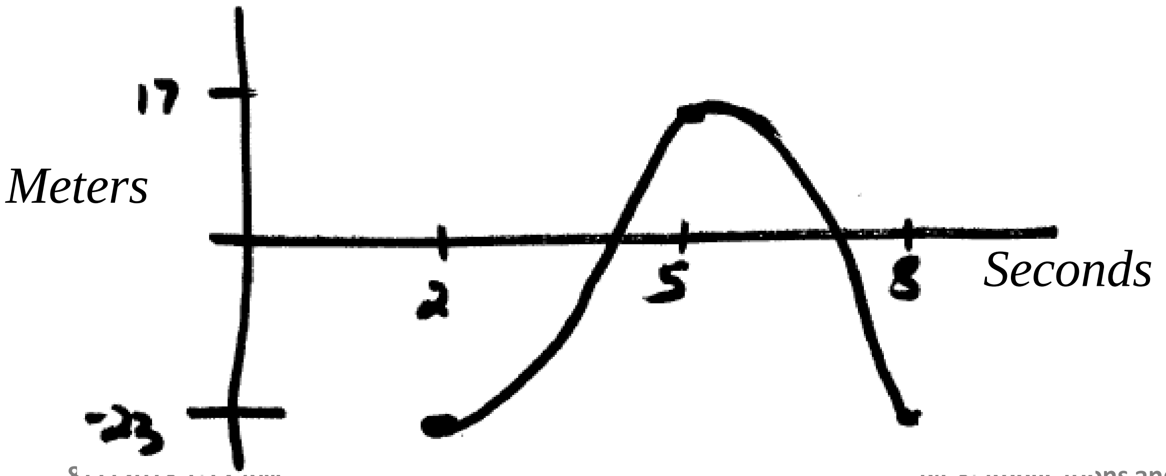

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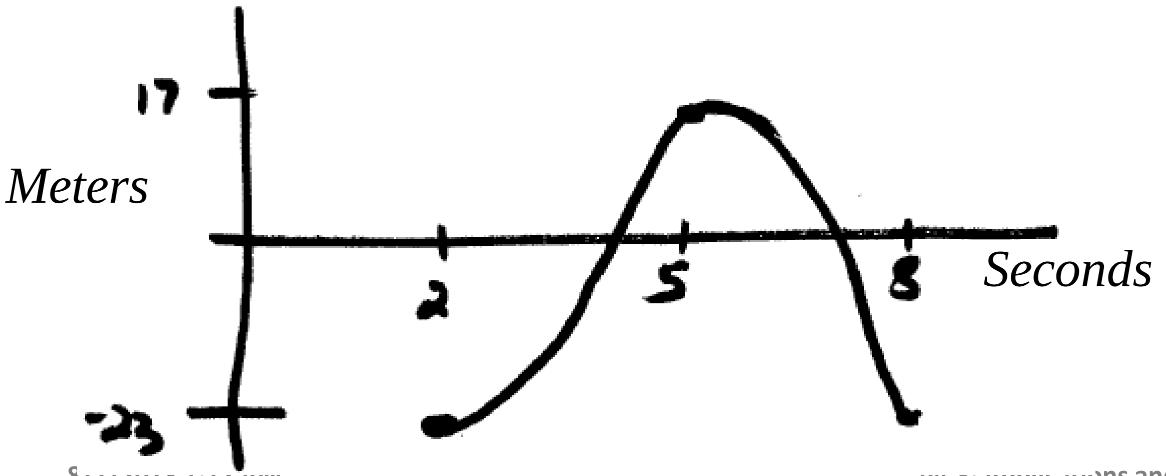
(b) Write an equation from the river bank in terms of t.



$$y = 20\cos\frac{\pi}{3}\left(x-5\right)-3$$

$$y = -20\cos\frac{\pi}{3}\left(x-2\right)-3$$

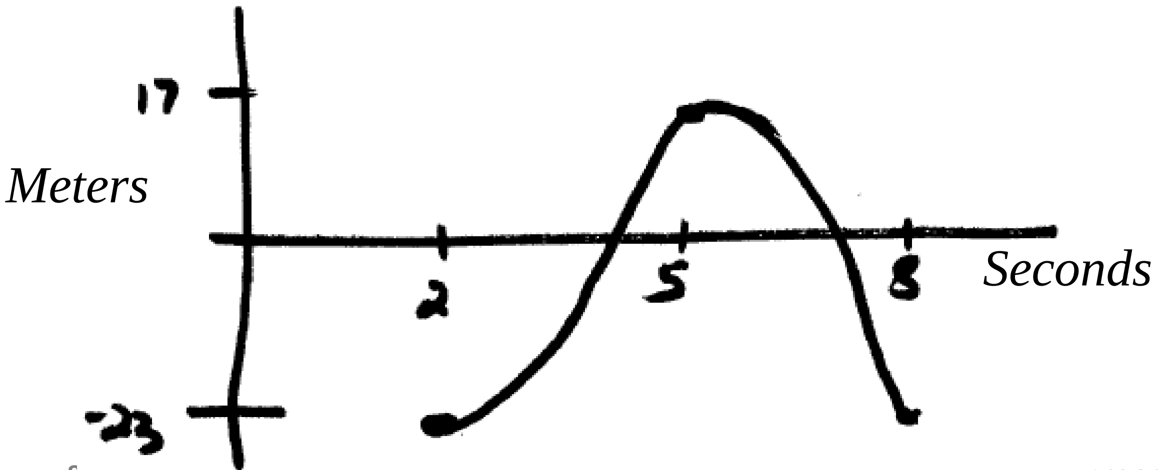
$$y = 20\sin\frac{\pi}{3}\left(x-3.5\right)-3$$

Your Turn

Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the river bank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and y be the number of meters Tarzan is from the river bank. Assume y varies sinusoidally with t, and that y is positive when Tarzan is over water and negative when he is over land.

Jane finds that when $t = 2$, Tarzan is at one end of his swing, when $y = -23$. She finds that when $t = 5$ he reaches the other end of his swing and $y = 17$.

(c) Predict y when $t = 2.8$ and $t = 15$



$$y(2.8) \approx -16.3826m$$
$$y(15) \approx -13m.$$

ASSIGNMENT

Worksheet