

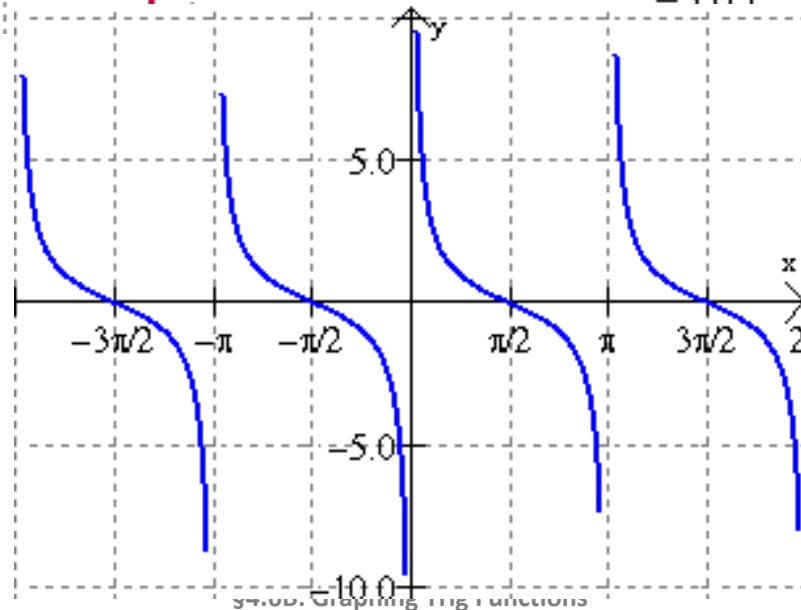
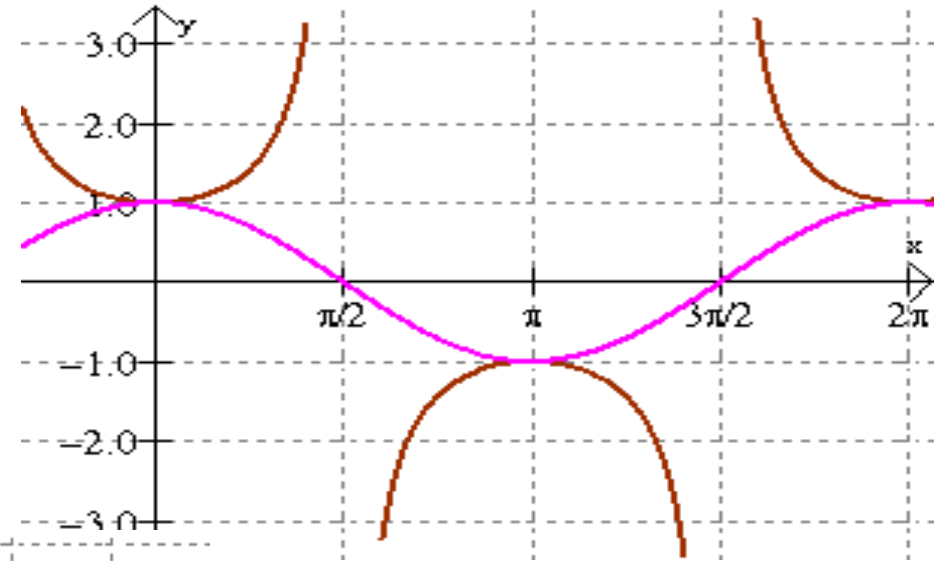
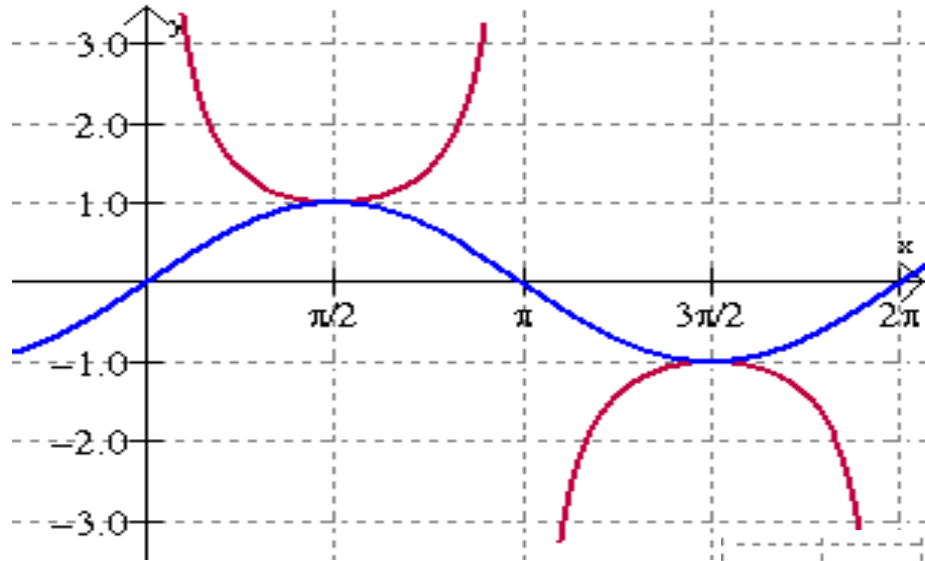
GRAPHING TRIGONOMETRIC FUNCTIONS

Section 4.6B

Precalculus PreAP/Dual, Revised ©2017

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REVIEW OF GRAPHS



TRANSFORMATIONS

A. Equation: $y = A \text{ trig function } B(x - C) + D$

B. A is the amplitude

1. a : vertically stretches by a factor of a ,
2. $\frac{1}{a}$: Vertically compresses by a factor of $1/a$

C. B is the period or frequency

1. *Period equation*: $\frac{2\pi}{B}$ for sine and cosine, $\frac{\pi}{B}$ for tangent
2. B : phase compresses by a factor of $\frac{\pi}{B}$
3. $\frac{1}{B}$: phase stretches by a factor of b

D. C is the phase shift

1. If there no GCF taken out, divide the coefficient

E. D is the vertical shift

F. Frequency is defined as the number of cycles per second

STEPS

- A. Identify A , B , C , and D from the equation, $y = A \text{ trig } B(x - C) + D$
- B. Identify the phase shift
 - 1. Period: $\frac{2\pi}{B}$ or $\frac{\pi}{B}$ (for Tan and Cot only)
- C. Use the period to identify the spacing
 - 1. Anchor Point Equation: $\frac{\textit{Period}}{4}$
- D. Start with the phase shift as the middle of the trig table (at the origin) and apply the spacing before and after

BASIC TABLE POINTS

$y = \sin(x)$	
x	y
C	0
	1
	0
	-1
	0

$y = \cos(x)$	
x	y
C	1
	0
	-1
	0
	1

$y = \tan(x)$	
x	y
	<i>Und</i>
	-1
C	0
	1
	<i>Und</i>

BASIC TABLE POINTS

$y = \csc(x)$	
x	y
C	Und
	1
	Und
	-1
	Und

$y = \sec(x)$	
x	y
	Und
C	1
	Und
	-1
	Und

$y = \cot(x)$	
x	y
C	Und
	1
	0
	-1
	Und

EXAMPLE 1

$$y = A \operatorname{trig} B(x - C) + D$$

$$A = 1, B = 1, C = 0, D = 1$$

Graph $y = \sin x + 1$ in one period and identify amplitude, period, vertical shift, phase shift, domain (entire graph), and range

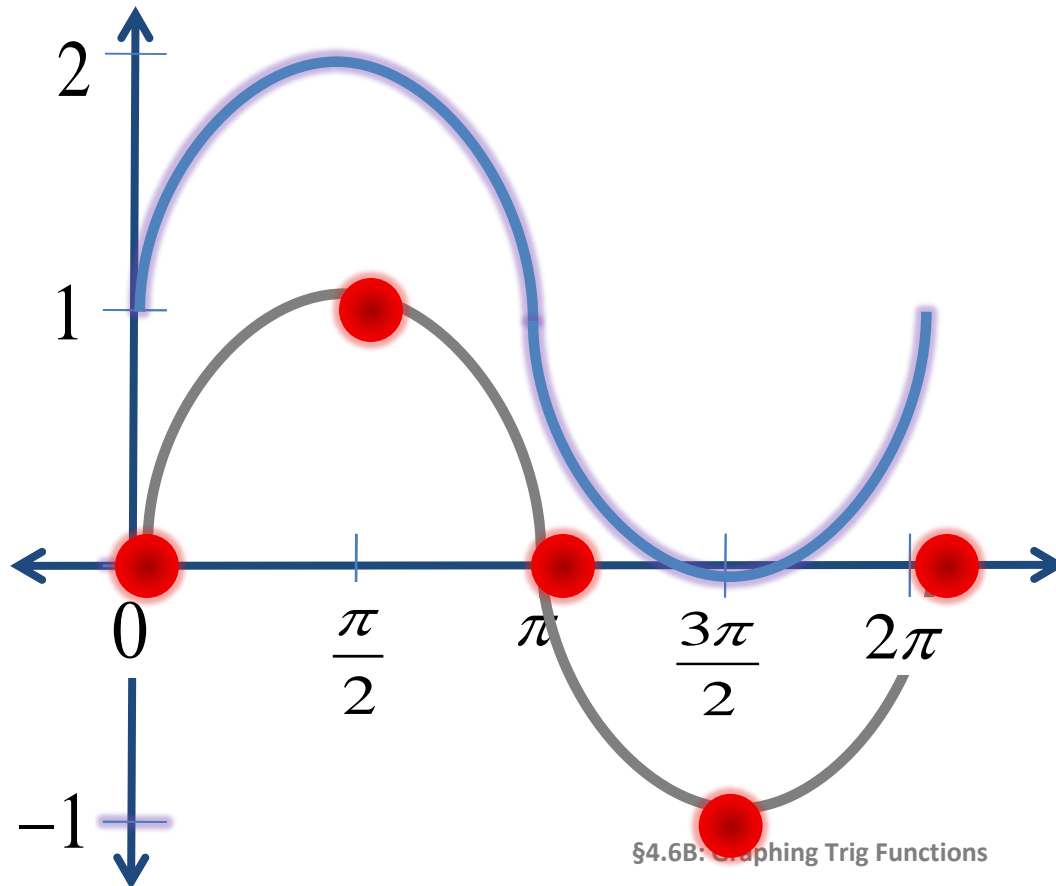
$y = \sin(x)$		$\sin(x) + 1$
0	0	1
$0 + \frac{\pi}{2} =$	1	2
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{2}$	-1	0
2π	0	1

Amplitude	1
Period	2π
Vertical Shift	Up 1
Phase Shift	None
Spacing (A.P.)	$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$
Domain	$(-\infty, \infty)$
Range	$[0, 2]$

EXAMPLE 1

Graph $y = \sin x + 1$ in one period and identify amplitude, period, vertical shift, phase shift, domain (entire graph), and range

$y = \sin(x) + 1$	
0	1
$\pi/2$	2
π	1
$3\pi/2$	0
2π	1



Amplitude	1
Period	2π
Vertical Shift	Up 1
Phase Shift	None
Spacing (A.P.)	$\frac{Period}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$
Domain	$(-\infty, \infty)$
Range	$[0, 2]$

EXAMPLE 2

$$y = A \operatorname{trig} B(x - C) + D$$

$$A = \frac{1}{2}, B = 1, C = 0, D = -1$$

Graph $y = \frac{1}{2} \cos \theta - 1$ in one period and identify amplitude, period, vertical shift, phase shift, domain (entire graph), and range

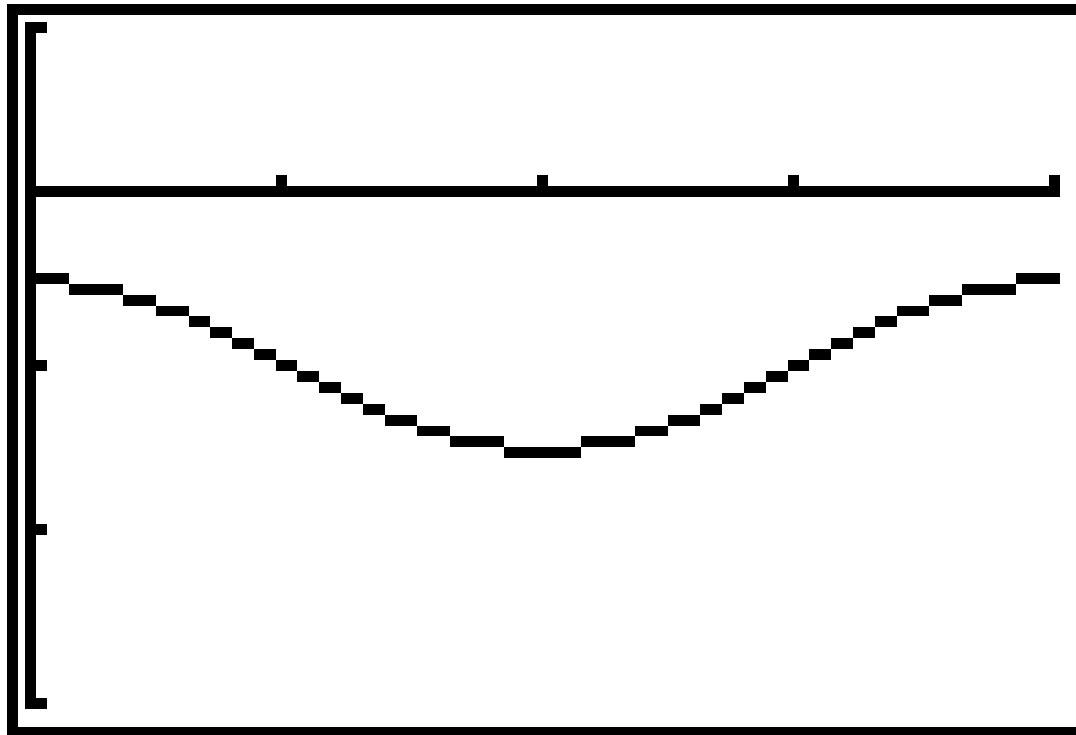
$y = \cos(\theta)$		$\frac{1}{2} \cos \theta$	$\frac{1}{2} \cos \theta - 1$
0	1	1/2	-1/2
$0 + \frac{\pi\cancel{c}}{2\cancel{2}} =$	0	0	-1
$\frac{\pi}{2} \frac{\cancel{\pi}}{2} =$	-1	-1/2	-3/2
$\frac{3\pi}{2}$	0	0	-1
2π	1	1/2	-1/2

Amplitude	$\frac{1}{2}$
Period	2π
Vertical Shift	Down 1
Phase Shift	None
Spacing (A.P.)	$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$
Domain	$(-\infty, \infty)$
Range	$\left[-\frac{3}{2}, -1\right]$

EXAMPLE 2

Graph $y = \frac{1}{2} \cos \theta - 1$ in one period and identify amplitude, period, vertical shift, phase shift, domain (entire graph), and range

$\frac{1}{2} \cos \theta - 1$	
0	-1/2
$\pi/2$	-1
π	-3/2
$3\pi/2$	-1
2π	-1/2

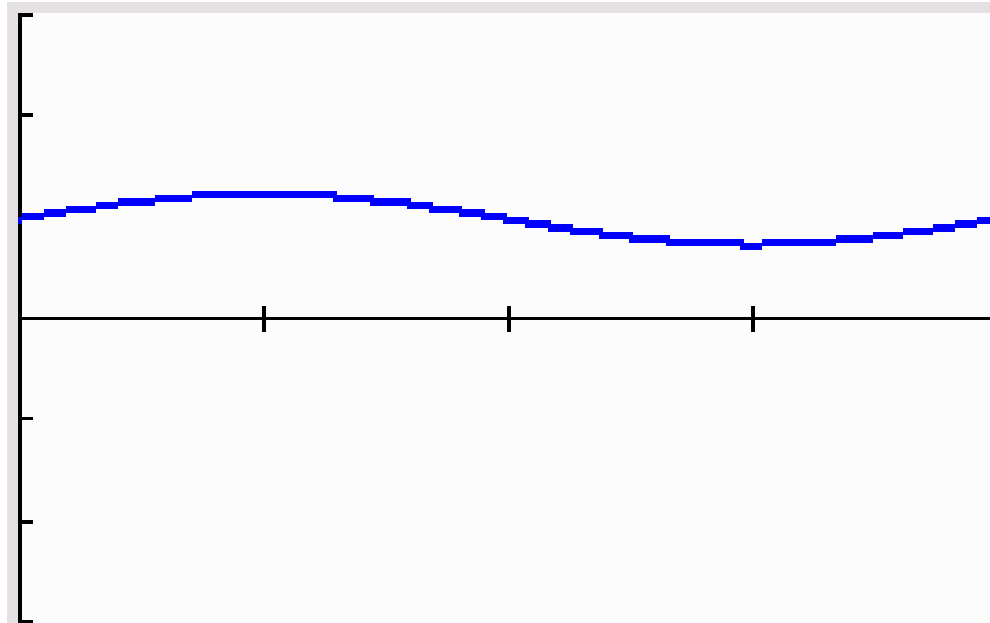


Amplitude	$\frac{1}{2}$
Period	2π
Vertical Shift	Down 1
Phase Shift	None
Spacing (A.P.)	$\frac{Period}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$
Domain	$(-\infty, \infty)$
Range	$[-\frac{3}{2}, -1]$

YOUR TURN

Graph $y = \frac{1}{4} \sin t + 1$ from $[-2\pi, 2\pi]$ and identify amplitude, period, vertical shift, phase shift, domain, and range

$y = \frac{1}{4} \sin t + 1$	
x	y
0	1
$\pi/2$	$5/4$
π	1
$3\pi/2$	$3/4$
2π	1



Amplitude	$\frac{1}{4}$
Period	2π
Vertical Shift	<i>Up 1</i>
Phase Shift	<i>None</i>
Spacing (A.P.)	$\frac{\pi}{2}$
Domain	$(-\infty, \infty)$
Range	$[\frac{3}{4}, \frac{5}{4}]$

EXAMPLE 3

$$y = A \operatorname{trig} B(x - C) + D$$

$$A = 2, B = 1, C = 0, D = 1$$

Given $y = 2 \tan x + 1$ from $[-\pi, \pi]$ and amplitude, period, vertical shift, phase shift, domain, and range

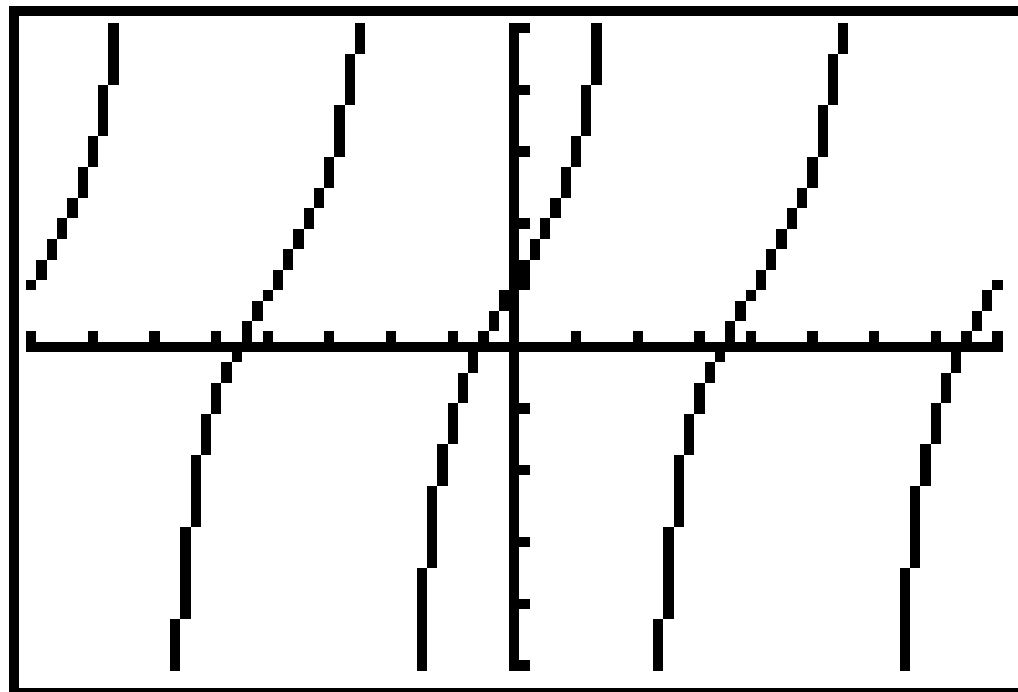
$y = \tan(x)$		$y = 2 \tan x$	$2 \tan x + 1$
$-\frac{\pi}{2}$	<i>Und</i>	<i>Und</i>	<i>Und</i>
$0 - \frac{\pi}{4} =$	-1	-2	-1
0	0	0	1
$0 + \frac{\pi}{4} =$	1	2	3
$\frac{\pi}{2}$	<i>Und</i>	<i>Und</i>	<i>Und</i>

Amplitude	<i>DNE</i>
Period	π
Vertical Shift	Up 1
Phase Shift	<i>None</i>
Spacing (A.P.)	$\frac{\text{Period}}{4} = \frac{\pi}{4} = \frac{\pi}{4}$
Domain	$(-\infty, \infty), x \neq \frac{\pi}{2}$
Range	$(-\infty, \infty)$

EXAMPLE 3

Given $y = 2 \tan x + 1$ from $[-\pi, \pi]$ and amplitude, period, vertical shift, and phase shift

x	$2 \tan x + 1$
$-\pi/2$	<i>Und</i>
$-\pi/4$	-1
0	1
$\pi/4$	3
$\pi/2$	<i>Und</i>



Amplitude	<i>DNE</i>
Period	π
Vertical Shift	Up 1
Phase Shift	<i>None</i>
Spacing (A.P.)	$\frac{\text{Period}}{4} = \frac{\pi}{4} = \frac{\pi}{4}$
Domain	$(-\infty, \infty)$
Range	$(-\infty, \infty)$

EXAMPLE 4

$$y = A \operatorname{trig} B(x - C) + D$$

$$A = \frac{1}{2}, B = 2, C = 0, D = 1$$

Given $y = \frac{1}{2} \csc 2x + 1$ from $[-2\pi, 2\pi]$ and amplitude, period, vertical shift, phase shift, domain, and range

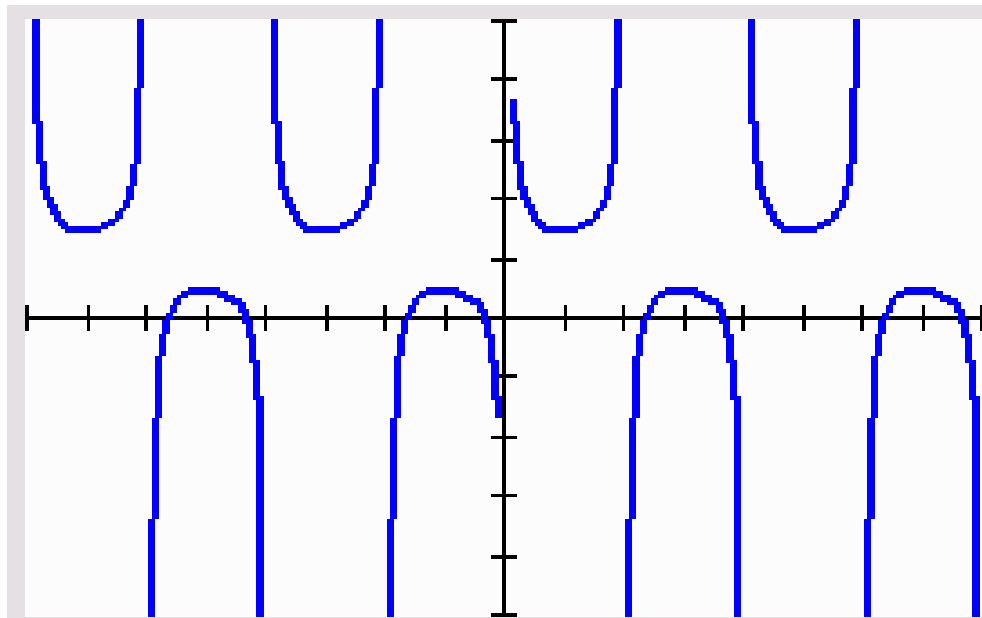
$y = \csc(x)$	$y = \frac{1}{2} \csc x$	$\frac{1}{2} \csc x + 1$
0	<i>Und</i>	<i>Und</i>
$\frac{\pi}{4}$	1	3/2
$\frac{\pi}{2}$	<i>Und</i>	<i>Und</i>
$\frac{3\pi}{4}$	-1	1/2
π	<i>Und</i>	<i>Und</i>

Amplitude	DNE
Period	π
Vertical Shift	Up 1
Phase Shift	None
Spacing (A.P.)	$\frac{\text{Period}}{4} = \frac{\pi}{4} = \frac{\pi}{4}$
Domain	$x \neq \frac{\pi}{2} + \pi n$
Range	$\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

EXAMPLE 4

Given $y = \frac{1}{2} \csc 2x + 1$ from $[-2\pi, 2\pi]$ and amplitude, period, vertical shift, phase shift, domain, and range

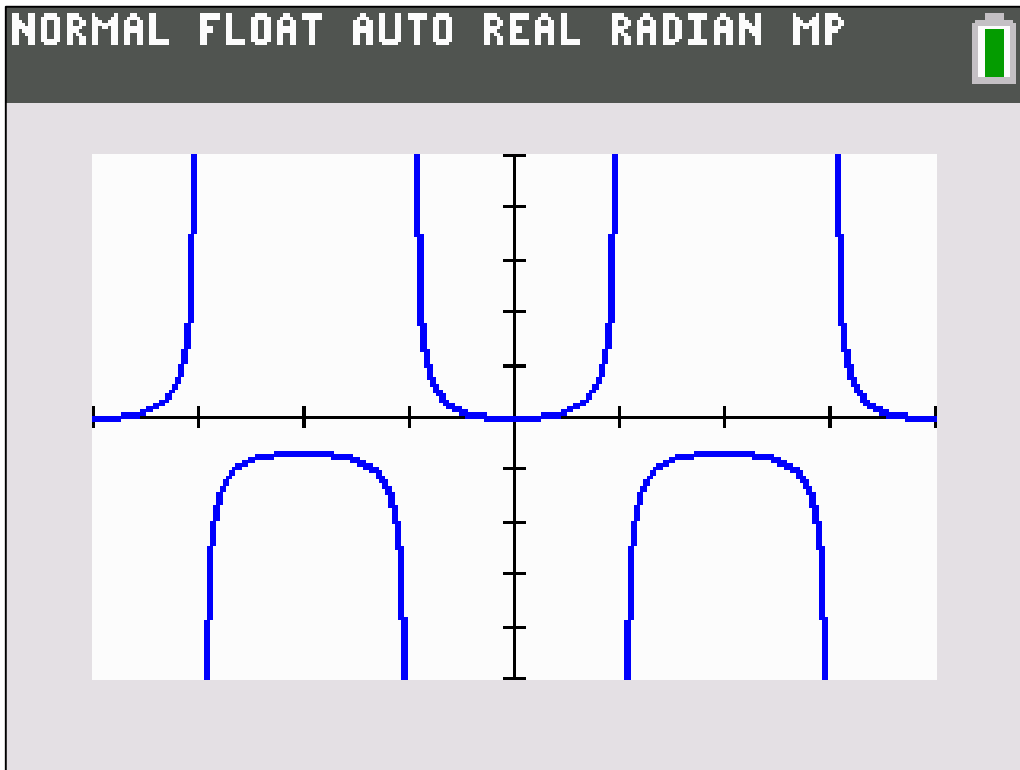
x	$\frac{1}{2} \csc x + 1$
0	<i>Und</i>
$\pi/4$	$3/2$
$\pi/2$	<i>Und</i>
$3\pi/4$	$1/2$
π	<i>Und</i>



Amplitude	<i>DNE</i>
Period	π
Vertical Shift	Up 1
Phase Shift	<i>None</i>
Spacing (A.P.)	$\frac{\text{Period}}{4} = \frac{\pi}{4} = \frac{\pi}{4}$
Domain	$x \neq \frac{\pi}{2} + \frac{\pi}{2}n$
Range	$\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

YOUR TURN

Given $y = \frac{1}{3} \sec x - 1$ from $[-2\pi, 2\pi]$ and identify period, vertical shift, phase shift, domain, and range



Amplitude	<i>DNE</i>
Period	2π
Vertical Shift	<i>Down 1</i>
Phase Shift	<i>None</i>
Spacing (A.P.)	$\frac{\pi}{2}$
Domain	$x \neq \pi + \pi n$
Range	$\left(-\infty, -\frac{2}{3}\right] \cup \left[-\frac{4}{3}, \infty\right)$

EXAMPLE 5 $y = A \operatorname{trig} B(x - C) + D$

Given $y = 5 \sin 2 \left(x - \frac{\pi}{6} \right) + 1$, identify amplitude, period, vertical shift, phase shift, and points to graph in one period

$$A = 5, B = 2, C = \frac{\pi}{6}, D = 1$$

$$y = 5 \sin 2 \left(x - \frac{\pi}{6} \right) + 1$$

Amplitude	5
Period	π
Vertical Shift	Up 1

EXAMPLE 5 $y = A \operatorname{trig} B(x - C) + D$
 $A = 5, B = 2, C = \frac{\pi}{6}, D = 1$

Given $y = 5 \sin 2 \left(x - \frac{\pi}{6} \right) + 1$, identify amplitude, period, vertical shift, phase shift, and points to graph in one period

Phase Shift

$$y = 5 \sin 2 \left(x - \frac{\pi}{6} \right) + 1$$

B is OUTSIDE of the parenthesis

Phase Shift

Right $\frac{\pi}{6}$

EXAMPLE 5 $y = A \operatorname{trig} B(x - C) + D$
 $A = 5, B = 2, C = \frac{\pi}{6}, D = 1$

Given $y = 5 \sin 2 \left(x - \frac{\pi}{6} \right) + 1$, identify amplitude, period, vertical shift, phase shift, and points to graph in one period

Anchor Points

$$\frac{\text{Period}}{4} = \frac{\pi}{4} = \frac{\pi}{4}$$

Phase Shift $\frac{\pi}{4}$

EXAMPLE 5 $y = A \operatorname{trig} B(x - C) + D$
 $A = 5, B = 2, C = \frac{\pi}{6}, D = 1$

Given $y = 5 \sin 2\left(x - \frac{\pi}{6}\right) + 1$, identify amplitude, period, vertical shift, phase shift, and points to graph in one period

x - Values

$y = \sin(x)$		$Y = 5\sin(x)$	$Y = 5\sin(x) + 1$
x	y	y	y
$\pi/6$	0	0	1
$5\pi/12$	1	5	6
$2\pi/3$	0	0	1
$11\pi/12$	-1	-5	-4
$7\pi/6$	0	0	1

$\frac{\pi}{4}$

EXAMPLE 6 $y = A \operatorname{trig} B(x - C) + D$

Given $y = 3 \cos 4 \left(x + \frac{\pi}{2} \right) - 1$, identify amplitude, period, vertical shift, phase shift, and points to graph in one period

$y = \cos(x)$		$Y = 3 \cos(x)$	$Y = 3 \cos(x) - 1$
x	y	y	y
$-\pi/2$	1	3	2
0	0	0	-1
$\pi/2$	-1	-3	-2
π	0	0	-1
$3\pi/2$	1	3	2

Amplitude	3
Period	$\frac{\pi}{2}$
Vertical Shift	Down 1
Phase Shift	Left $\frac{\pi}{2}$
Spacing (A.P.)	$\frac{\pi}{2}$
Domain	$(-\infty, \infty)$
Range	$(-\infty, -\frac{2}{3}] \cup [-\frac{4}{3}, \infty)$

EXAMPLE 7

Given $y = 4 \sin(\pi x + 2) - 5$, identify amplitude, period, vertical shift, and phase shift and points from $[-2\pi, 2\pi]$

$$y = 4 \sin(\pi x + 2) - 5$$

$$y = A \operatorname{trig} B(x - C) + D$$

$$\pi x + 2$$

$$\frac{\pi x}{\pi} + \frac{2}{\pi}$$

$$x + \frac{2}{\pi}$$

EXAMPLE 7

Given $y = 4 \sin(\pi x + 2) - 5$, identify amplitude, period, vertical shift, and phase shift and points from $[-2\pi, 2\pi]$

$$y = 4 \sin \pi \left(x + \frac{2}{\pi} \right) - 5$$

$$\text{Period: } \frac{2\pi}{\pi} = 2$$

$$\begin{aligned} \text{Phase Shift: } & (x - C) + D \\ & \left(x - \left(-\frac{2}{\pi} \right) \right) - 5 \end{aligned}$$

Amplitude: 4

Period: 2

Vertical Shift: Down 5

Phase Shift: Left $\frac{2}{\pi}$

EXAMPLE 7

Given $y = 4 \sin(\pi x + 2) - 5$, identify amplitude, period, vertical shift, and phase shift and points from $[-2\pi, 2\pi]$

Amplitude: 4

Period: 2

Vertical Shift: Down 5

Phase Shift: Left $\frac{2}{\pi}$

x	Period: π	Phase Shift: Left $2/\pi$	$y = \sin x$	$y = 4 \cos(\pi x + 2)$	$y = 4 \cos(\pi x + 2) - 5$
0	0	$0 - 2/\pi$	0	0	-5
$\pi/2$	$\pi/4$	$\pi/4 - 2/\pi$	1	4	-1
π	$\pi/2$	$\pi/2 - 2/\pi$	0	0	-5
$3\pi/2$	$3\pi/4$	$3\pi/4 - 2/\pi$	-1	-4	-9
2π	π	$\pi - 2/\pi$	0	0	-5

YOUR TURN

Given $y = 2 \sin 3 \left(x + \frac{\pi}{3} \right) - 5$, identify amplitude, period, vertical shift, phase shift, and points to graph in one period

$y = \sin(x)$		$Y = 2\sin(x)$	$Y = 2\sin(x) - 5$
x	y	y	y
$-\pi/3$	0	0	-5
$\pi/3$	1	2	-3
π	0	0	-5
$5\pi/3$	-1	-2	-7
$8\pi/3$	0	0	-5

Amplitude	2
Period	$\frac{\pi}{3}$
Vertical Shift	Down 5
Phase Shift	Left $\frac{\pi}{3}$
Spacing (A.P.)	$\frac{2\pi}{3}$
Domain	$(-\infty, \infty)$
Range	$(-\infty, -\frac{2}{3}] \cup [-\frac{4}{3}, \infty)$

EXAMPLE 8

Graph $y = -2\cos 3\left(x + \frac{\pi}{3}\right)$ in one period and identify amplitude, period, vertical shift, phase shift, domain, and range

$$y = -2\cos 3\left(x + \frac{\pi}{3}\right) + 0$$

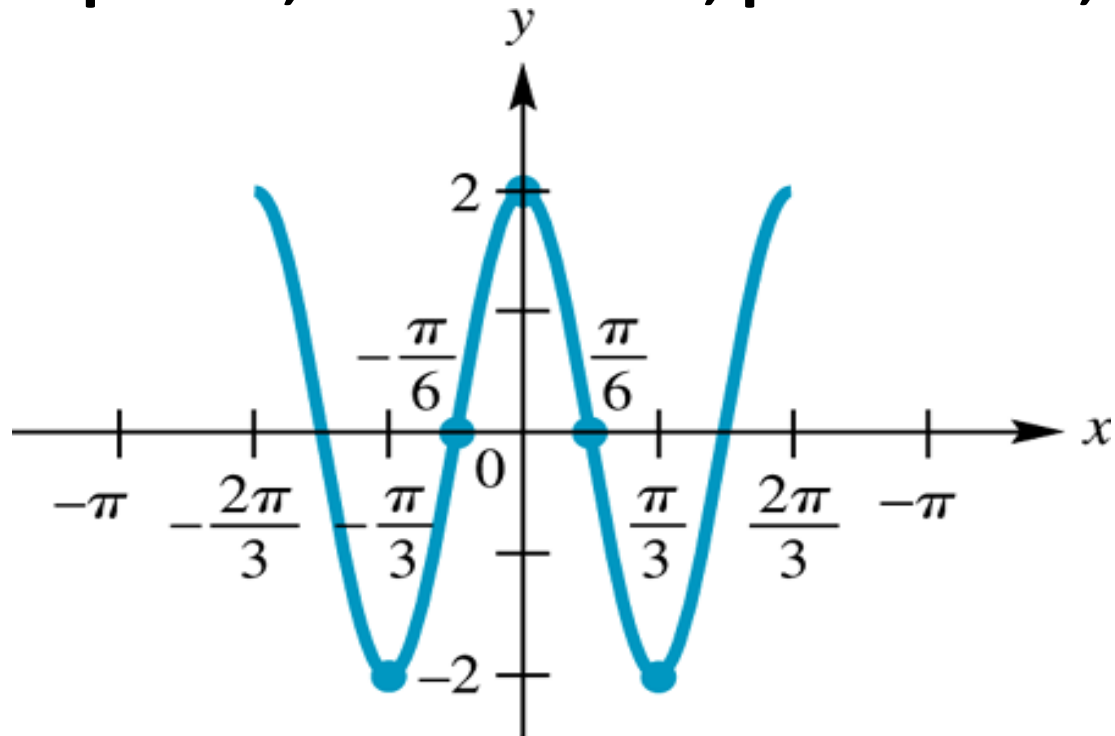
$$\text{Period} : \frac{2\pi}{3} \quad \text{Phase Shift} : \frac{\pi}{3} = \text{Left } \frac{\pi}{3}$$

$$\text{Spacing} : \frac{\text{Period}}{4} = \frac{2\pi/3}{4} = \frac{\pi}{6}$$

$y = \cos(x)$		$-2\cos 3(x)$
x	y	y
$-\pi/3$	1	-2
$-\pi/6$	0	0
0	-1	2
$\pi/6$	0	0
$\pi/3$	1	-2

EXAMPLE 8

Graph $y = -2\cos 3\left(x + \frac{\pi}{3}\right)$ in one period and identify amplitude, period, vertical shift, phase shift, domain, and range



Amplitude	2; Reflected
Period	$\frac{2\pi}{3}$
Vertical Shift	<i>None</i>
Phase Shift	<i>Left $\frac{\pi}{3}$</i>
Domain	$(-\infty, \infty)$
Range	$[-2, 2]$

EXAMPLE 9

Graph $y = 2 \sin(4x + \pi) - 1$ from $[0, 2\pi]$ and identify amplitude, period, vertical shift, phase shift, domain, and range

$$y = A \sin B(x - C) + D$$

$$y = 2 \sin(4x + \pi) - 1$$

$$\frac{4x}{4} + \frac{\pi}{4}$$

$$\text{Period} : \frac{\pi}{2}$$

$$\text{Phase Shift} : \frac{\pi}{4} = \text{Left } \frac{\pi}{4}$$

$$4 \left(x + \frac{\pi}{4} \right)$$

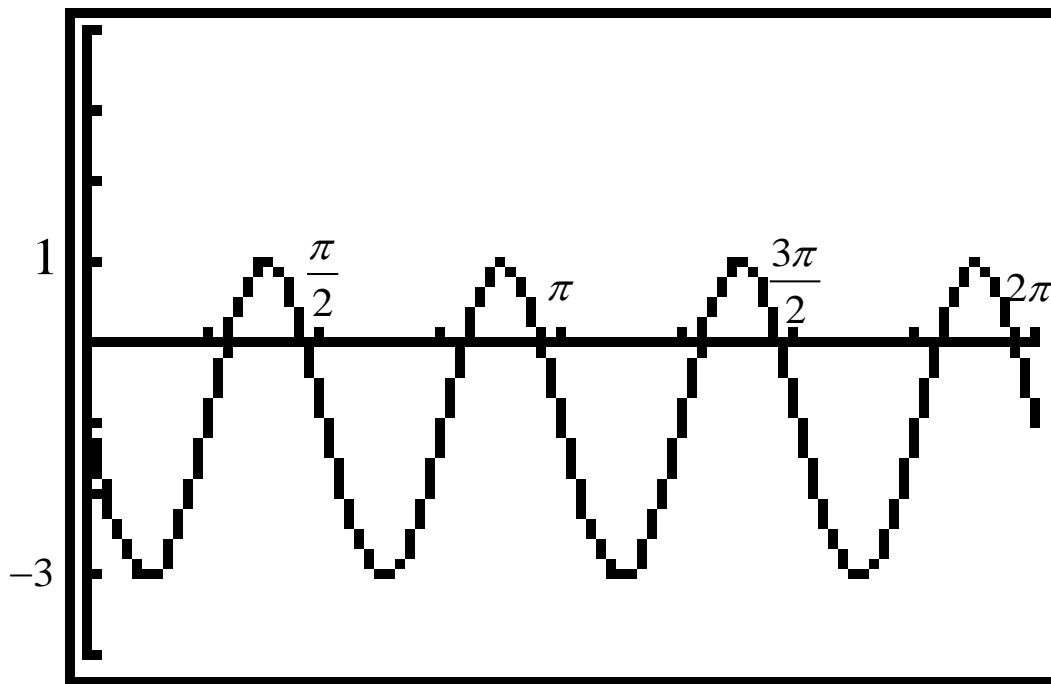
$$\text{Spacing} : \frac{\text{Period}}{4} = \frac{\pi/2}{4} = \frac{\pi}{8}$$

$y = 2 \sin 4 \left(x + \frac{\pi}{4} \right) - 1$	
x	y
$-\pi/4$	-1
$-\pi/8$	1
0	-1
$\pi/8$	-3
$\pi/4$	-1

EXAMPLE 9

Graph $y = 2 \sin(4x + \pi) - 1$ from $[0, 2\pi]$ and identify amplitude, period, vertical shift, phase shift, domain, and range

$y = 2\sin 4\left(x + \frac{\pi}{4}\right) - 1$	
x	y
$-\pi/4$	-1
$-\pi/8$	1
0	-1
$\pi/8$	-3
$\pi/4$	-1



Amplitude	2
Period	$\frac{\pi}{2}$
Vertical Shift	Down 1
Phase Shift	Left $\frac{\pi}{4}$
Anchor Points	$\frac{\pi}{8}$
Domain	$(-\infty, \infty)$
Range	$[-3, 1]$

EXAMPLE 10

Given $y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$ from $[-\pi, \pi]$ and amplitude, period, vertical shift, phase shift, domain, and range

$$y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$$

$$\text{Period} : \frac{\pi}{1} = \pi$$

$$\text{Phase Shift} : \text{Left } \frac{\pi}{2}$$

$$\text{Spacing} : \frac{\text{Period}}{4} = \frac{\pi}{4}$$

$y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$	
x	y
$-\pi$	<i>Und</i>
$-3\pi/4$	1
$-\pi/2$	-1
$-\pi/4$	-3
0	<i>Und</i>

EXAMPLE 10

Given $y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$ from $[-\pi, \pi]$ and identify period, vertical shift, phase shift, domain, and range

$y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$	
x	y
$-\pi$	<i>Und</i>
$-3\pi/4$	1
$-\pi/2$	-1
$-\pi/4$	-3
0	<i>Und</i>

Amplitude	<i>DNE</i>
Period	π
Vertical Shift	<i>Down 1</i>
Phase Shift	<i>Left</i> $\frac{\pi}{2}$
Anchor Points	$\frac{\pi}{4}$
Domain	$(-\infty, \infty); x \neq \pi n$
Range	$(-\infty, -2] \cup [2, \infty)$

EXAMPLE 10

Given $y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$ from $[-\pi, \pi]$ and amplitude, period, vertical shift, phase shift, domain, and range

$$y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$$

$$\text{Period} : \frac{\pi}{1} = \pi$$

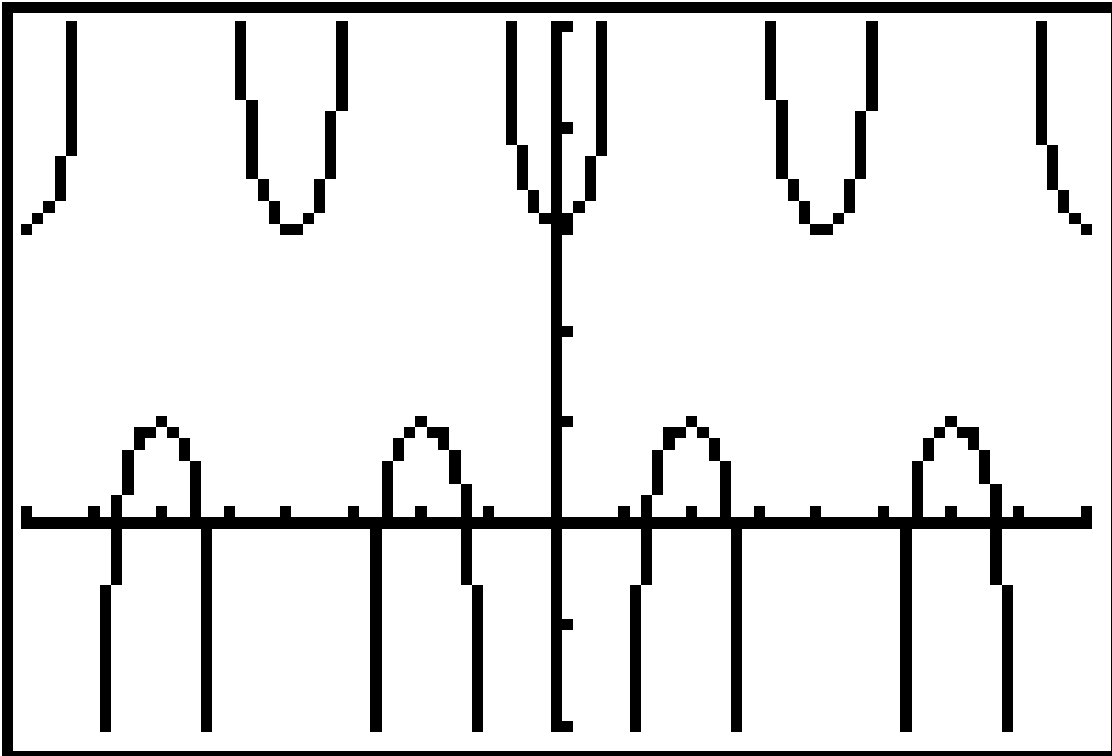
$$\text{Phase Shift} : \text{Left } \frac{\pi}{2}$$

$$\text{Spacing} : \frac{\text{Period}}{4} = \frac{\pi}{4}$$

$y = -2 \tan \left(x + \frac{\pi}{2} \right) - 1$	
x	y
$-\pi$	<i>Und</i>
$-3\pi/4$	1
$-\pi/2$	-1
$-\pi/4$	3
0	<i>Und</i>

YOUR TURN

Given $y = \sec 2(x + \pi) + 2$ from $[-2\pi, 2\pi]$ and identify period, vertical shift, phase shift, domain, and range



Amplitude	<i>DNE</i>
Period	π
Vertical Shift	<i>Up 2</i>
Phase Shift	<i>Left π</i>
Anchor Points	$\frac{\pi}{4}$
Domain	$(-\infty, \infty); x \neq 2\pi n$
Range	$(-\infty, 1] \cup [3, \infty)$

ASSIGNMENT

Worksheet