

# TRIG FUNCTIONS OF ANY ANGLE

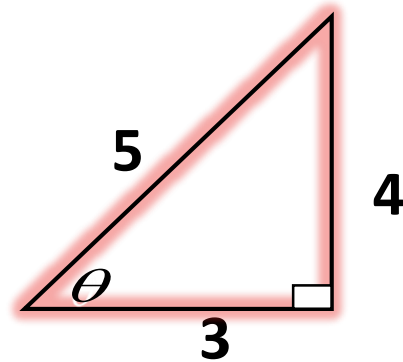
## Section 4.4

Precalculus PreAP/Dual, Revised ©2017

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# REVIEW

Determine the Trigonometric Functions for  $\theta$



$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

# EQUATION IN STANDARD FORM

A. For  $\theta$  be an angle in standard position with any point  $(x, y)$

1.  $\sin \theta = \underline{y/r}$

2.  $\cos \theta = \underline{x/r}$

3.  $\tan \theta = \underline{y/x}$

4.  $\csc \theta = \underline{r/y}$

5.  $\sec \theta = \underline{r/x}$

6.  $\cot \theta = \underline{x/y}$

B. To establish the radius, the equation is  $r = \sqrt{x^2 + y^2}$

C. Think of “ASTC: All Students Take Calculus”

1. **A:** All points are always positive in Quadrant I

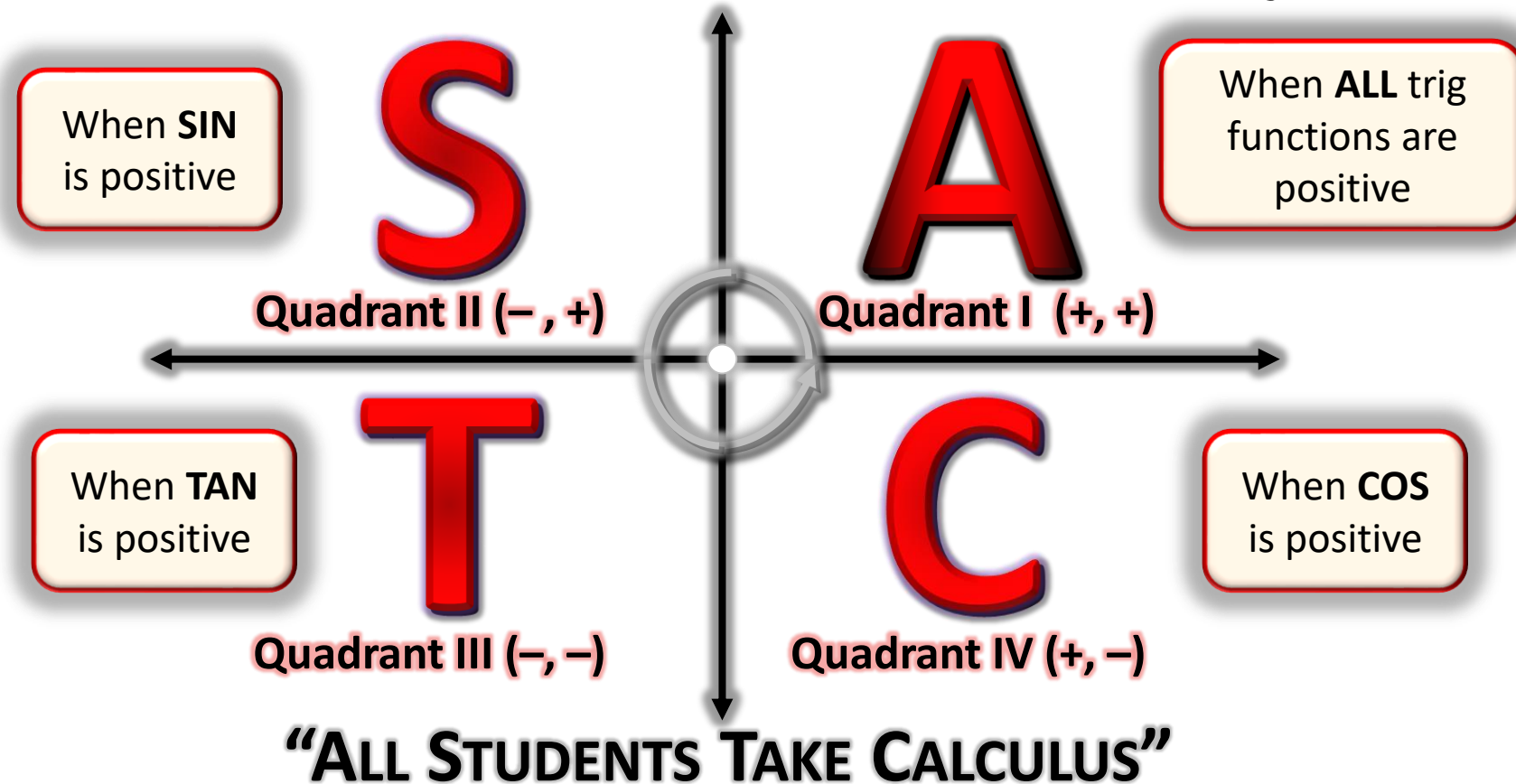
2. **S:** Sine points are positive in Quadrant II

3. **T:** Tan points are positive in Quadrant III

4. **C:** Cosine points are positive in Quadrant IV

# EQUATION IN STANDARD FORM

For  $\theta$  be an angle in standard position with any point  $(x, y)$ :

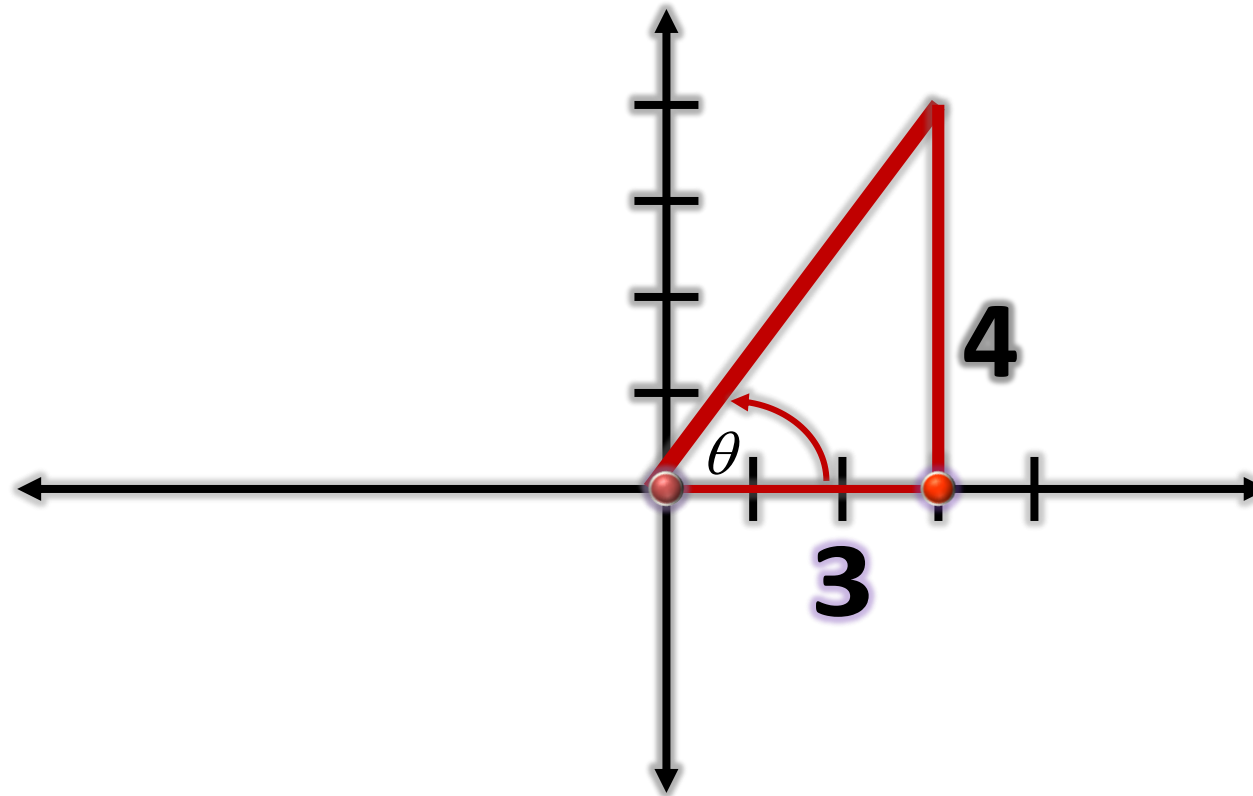


# STEPS IN EVALUATING FUNCTIONS GIVEN A POINT

- A. Draw a picture from a coordinate plane**
- B. Identify and plot the point onto the coordinate plane**
- C. Determine the missing side using the radius equation**
- D. Use Trigonometric Functions to solve**

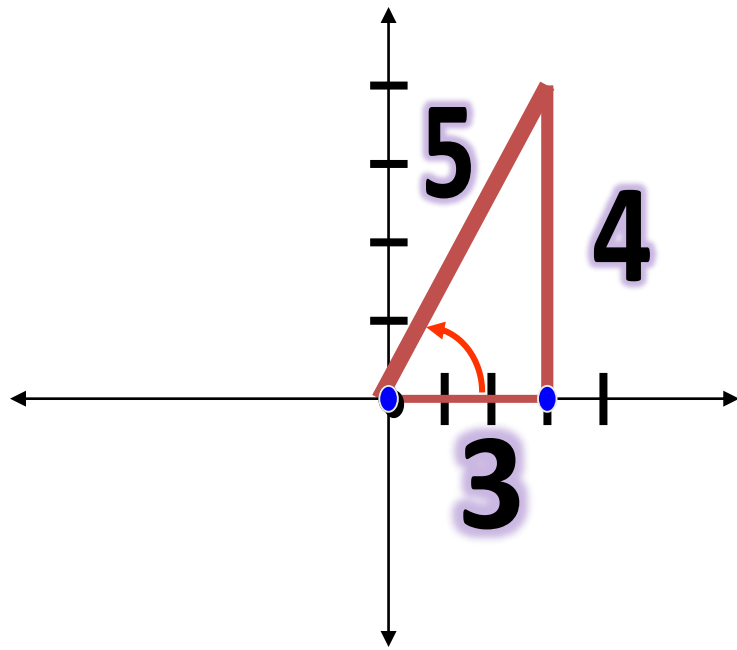
# EXAMPLE 1

Let  $(3, 4)$  be a point on the terminal side of  $\theta$ . Determine the value of the six trigonometric functions for  $\theta$ .



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Let  $(3, 4)$  be a point on the terminal side of  $\theta$ . Determine the value of the six trigonometric functions for  $\theta$ .



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(3)^2 + (4)^2}$$

$$r = \sqrt{25}$$

$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

## EXAMPLE 2

Let  $\left(\frac{\sqrt{10}}{10}, -\frac{3\sqrt{10}}{10}\right)$  be a point on the terminal side of  $\theta$ . Determine the value of the six trigonometric functions for  $\theta$ .

$$\begin{aligned}\sin \theta &= -\frac{3\sqrt{10}}{10} & \cos \theta &= \frac{\sqrt{10}}{10} & \tan \theta &= -3 \\ \csc \theta &= -\frac{\sqrt{10}}{3} & \sec \theta &= \sqrt{10} & \cot \theta &= -\frac{1}{3}\end{aligned}$$



# YOUR TURN

Let  $(1, -1)$  be a point on the terminal side of  $\theta$ . Determine the value of the six trigonometric functions for  $\theta$ .

$$\sin \theta = -\frac{\sqrt{2}}{2} \quad \cos \theta = \frac{\sqrt{2}}{2} \quad \tan \theta = -1$$

$$\csc \theta = -\sqrt{2} \quad \sec \theta = \sqrt{2} \quad \cot \theta = -1$$

# EXAMPLE 3

Let  $\theta$  be in Quadrant II. Given  $\sin \theta = \frac{1}{3}$ , determine the value of the six trigonometric functions for  $\theta$ .

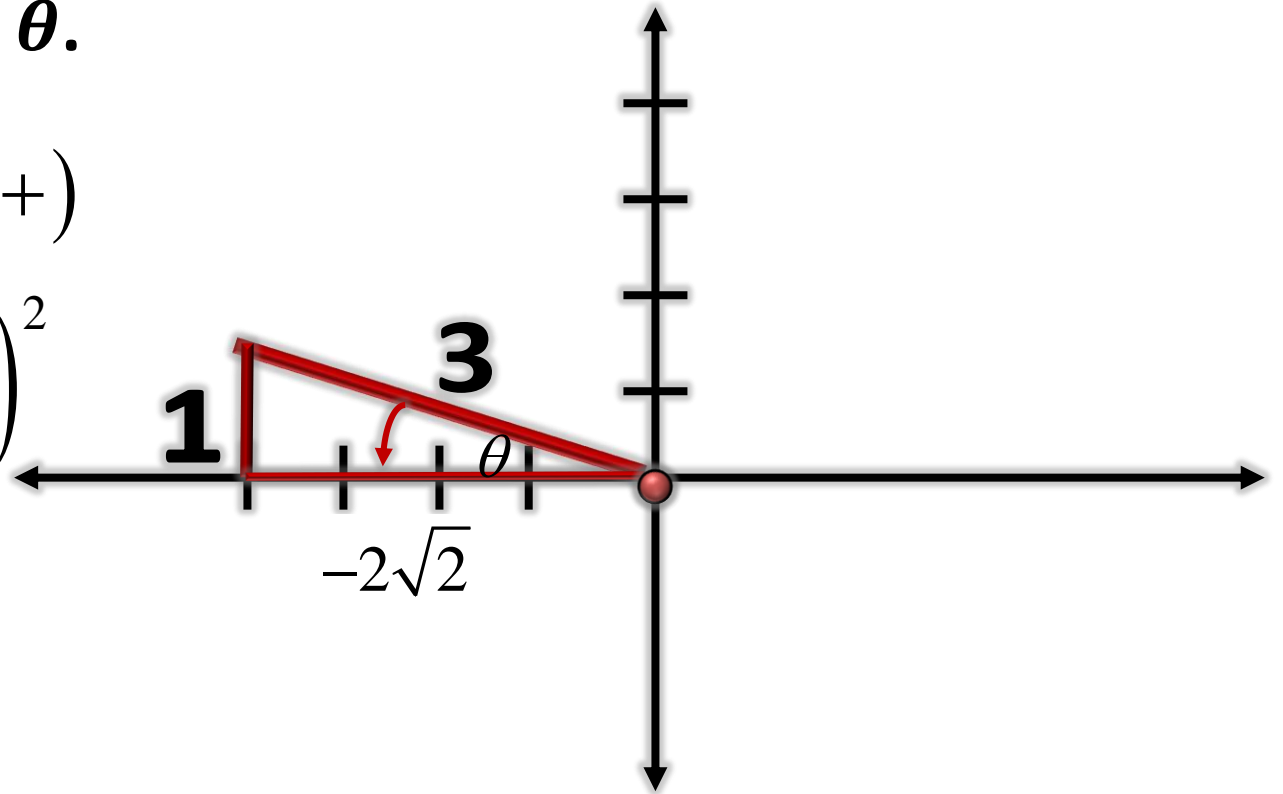
$$\sin \theta = \frac{y}{r} \quad \text{Quadrant II} = (-, +)$$

$$r = \sqrt{x^2 + y^2} \quad (3)^2 = (\sqrt{x^2 + 1})^2$$

$$3 = \sqrt{x^2 + (1)^2} \quad 9 = x^2 + 1$$

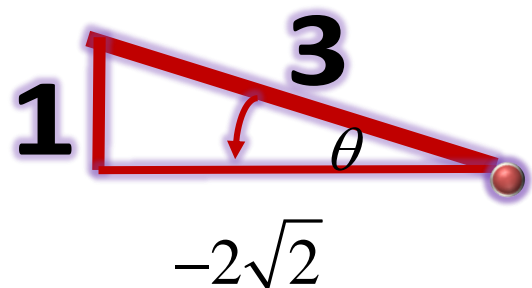
$$3 = \sqrt{x^2 + 1} \quad x^2 = 8$$

$$x = -2\sqrt{2}$$



## EXAMPLE 3

Let  $\theta$  be in Quadrant II. Given  $\sin \theta = \frac{1}{3}$ , determine the value of the six trigonometric functions for  $\theta$ .



$$\sin \theta = \frac{1}{3}$$

$$\csc \theta = 3$$

$$\cos \theta = -\frac{2\sqrt{2}}{3} \quad \tan \theta = -\frac{\sqrt{2}}{4}$$

$$\sec \theta = -\frac{3\sqrt{2}}{4} \quad \cot \theta = -2\sqrt{2}$$

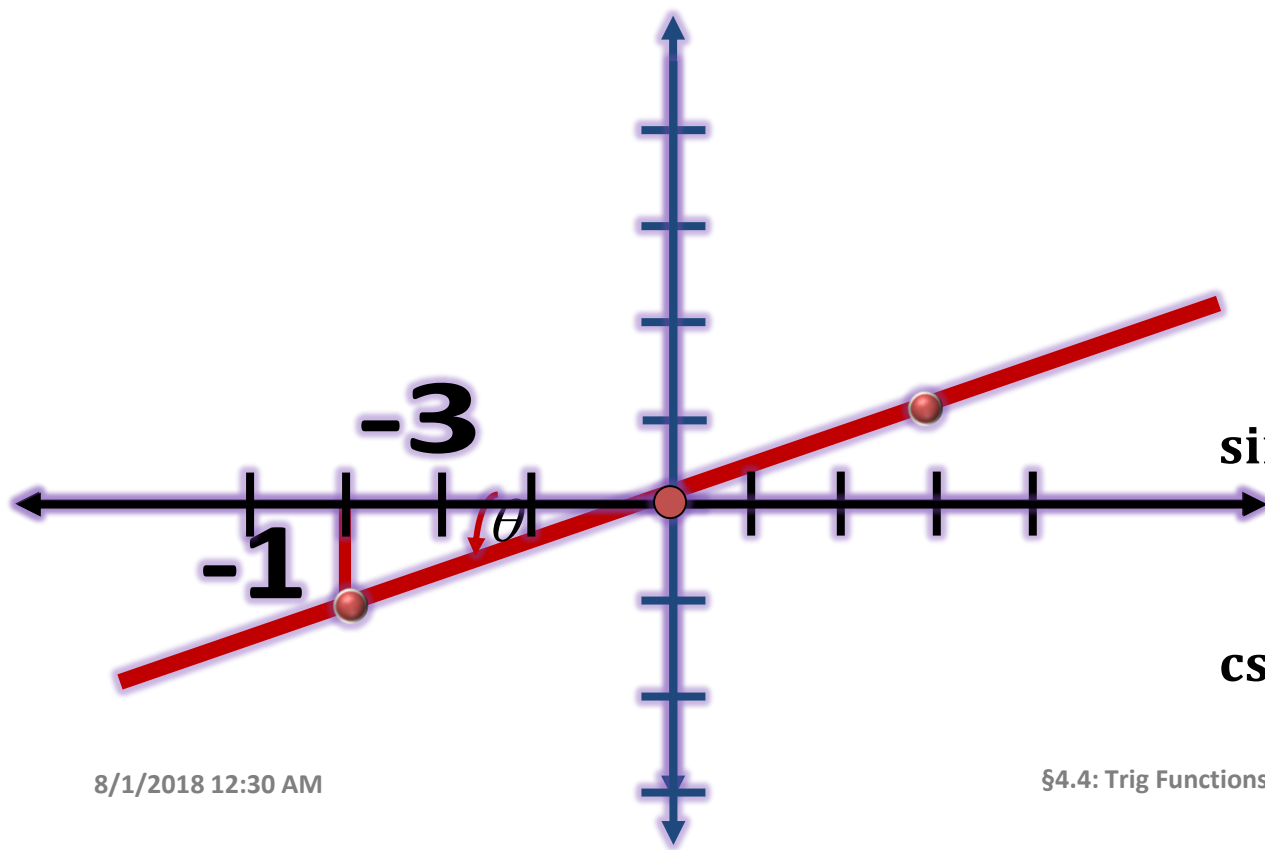
## EXAMPLE 4

Let  $\cot \theta < 0$ . Given  $\csc \theta = 4$ , determine the value of the six trigonometric functions for  $\theta$ .

$$\begin{aligned}\sin \theta &= \frac{1}{4} & \cos \theta &= -\frac{\sqrt{15}}{4} & \tan \theta &= -\frac{\sqrt{15}}{15} \\ \csc \theta &= 4 & \sec \theta &= -\frac{4\sqrt{15}}{15} & \cot \theta &= -\sqrt{15}\end{aligned}$$

# EXAMPLE 5

Given  $y = \frac{1}{3}x$  and  $\theta$  is in Quadrant III, determine the value of the six trigonometric functions for  $\theta$ .



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-3)^2 + (-1)^2}$$

$$r = \sqrt{9+1}$$

$$r = \sqrt{10}$$

$$\sin \theta = \frac{\sqrt{10}}{10} \quad \cos \theta = \frac{\sqrt{2}}{2} \quad \tan \theta = \frac{1}{3}$$

$$\csc \theta = -\sqrt{10} \quad \sec \theta = -\sqrt{2} \quad \cot \theta = 3$$

# YOUR TURN

Let  $\tan \theta > 0$ . Given  $\cos \theta = -\frac{2}{3}$ , determine the value of the six trigonometric functions for  $\theta$ .

$$\begin{aligned}\sin \theta &= -\frac{\sqrt{5}}{3} & \cos \theta &= -\frac{2}{3} & \tan \theta &= \frac{\sqrt{5}}{2} \\ \csc \theta &= -\frac{3\sqrt{5}}{5} & \sec \theta &= -\frac{3}{2} & \cot \theta &= \frac{2\sqrt{5}}{5}\end{aligned}$$

# ASSIGNMENT

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**11, 15-23 odd, 27-35 odd (omit 29)**