

RIGHT TRIANGLE TRIGONOMETRY

Section 4.3

Precalculus PreAP/Dual, Revised ©2017

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BACKGROUND INFORMATION

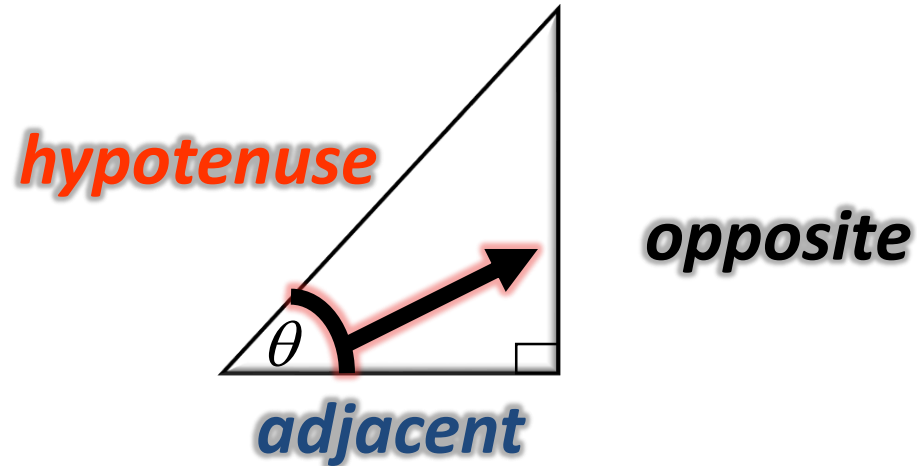
A. Trigonometry

1. Comes from Greek word – Trigonon, which means 3 angles
2. “Metry” means measure in Greek

B. Trigonometry Ratios

1. Main: Sine, Cosine, Tangent
2. Reciprocal: Secant, Cosecant, Cotangent

RIGHT TRIANGLES



- A. Consider a right triangle, one of whose acute angles is θ
- B. The three sides of a triangle are hypotenuse, opposite, and adjacent side of θ
- C. To determine what is the opposite side, look at θ and extend the line to determine the *opposite*

STEPS IN SOLVING FOR TRIANGLES

- A. Solve for x , using Pythagorean Theorem
- B. Determine the missing sides by identifying θ
- C. Use Trigonometry Functions to find what's needed

TRIGONOMETRIC RATIOS

$$\text{Sine } \theta = \frac{\textit{Opposite}}{\textit{Hypotenuse}}$$

SIN

$$\text{Cosecant } \theta = \frac{\textit{Hypotenuse}}{\textit{Opposite}}$$

CSC

$$\text{Cosine } \theta = \frac{\textit{Adjacent}}{\textit{Hypotenuse}}$$

COS

$$\text{Secant } \theta = \frac{\textit{Hypotenuse}}{\textit{Adjacent}}$$

SEC

$$\text{Tangent } \theta = \frac{\textit{Opposite}}{\textit{Adjacent}}$$

TAN

$$\text{Cotangent } \theta = \frac{\textit{Adjacent}}{\textit{Opposite}}$$

COT

CO-FUNCTIONS

$$\text{Sine } \theta = \frac{\textit{Opposite}}{\textit{Hypotenuse}}$$

SIN



$$\text{Cosine } \theta = \frac{\textit{Adjacent}}{\textit{Hypotenuse}}$$

COS

$$\text{Cosecant } \theta = \frac{\textit{Hypotenuse}}{\textit{Opposite}}$$

COS



$$\text{Secant } \theta = \frac{\textit{Hypotenuse}}{\textit{Adjacent}}$$

SEC

$$\text{Tangent } \theta = \frac{\textit{Opposite}}{\textit{Adjacent}}$$

TAN

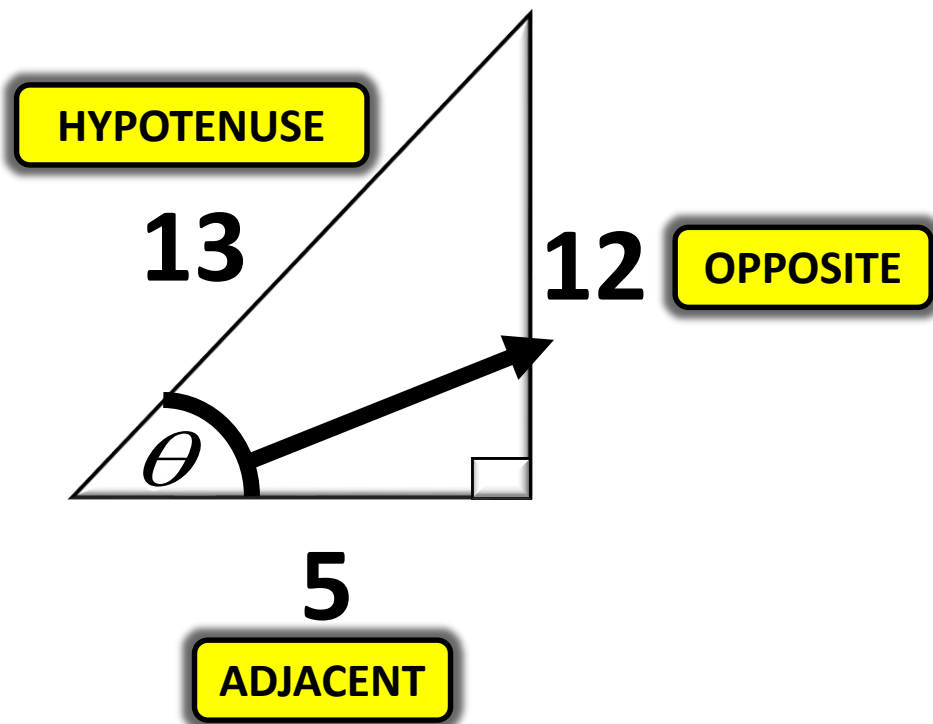


$$\text{Cotangent } \theta = \frac{\textit{Adjacent}}{\textit{Opposite}}$$

COT

EXAMPLE 1

Solve for x and determine all trig functions of θ



$$x^2 + y^2 = r^2$$

$$x^2 + (12)^2 = (13)^2$$

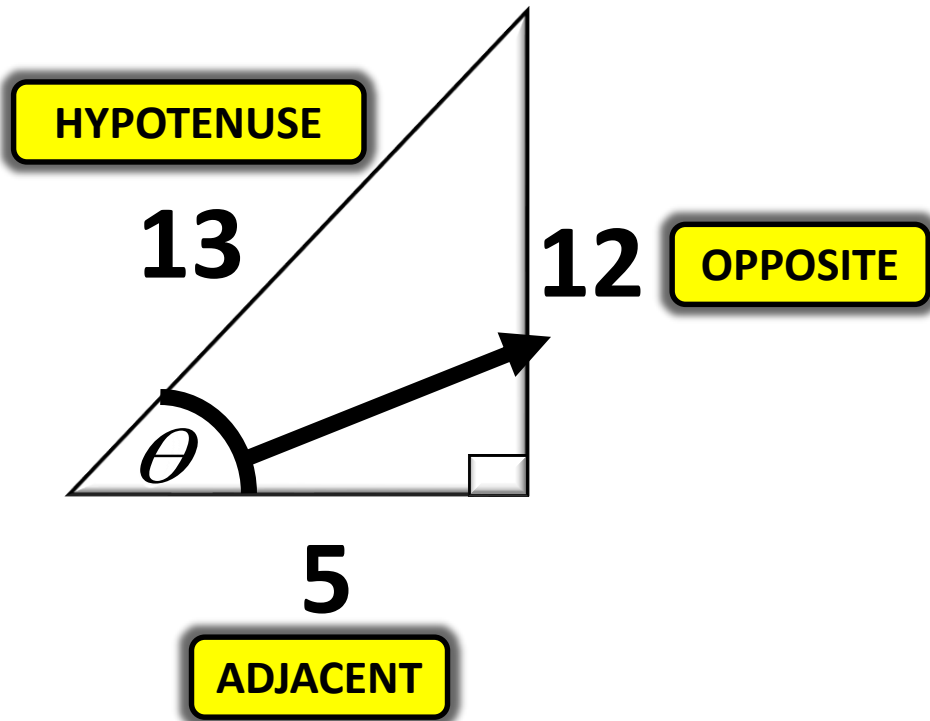
$$x^2 + 144 = 169$$

$$x^2 = 25$$

$$x = 5$$

EXAMPLE 1

Solve for x and determine all trig functions of θ



$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

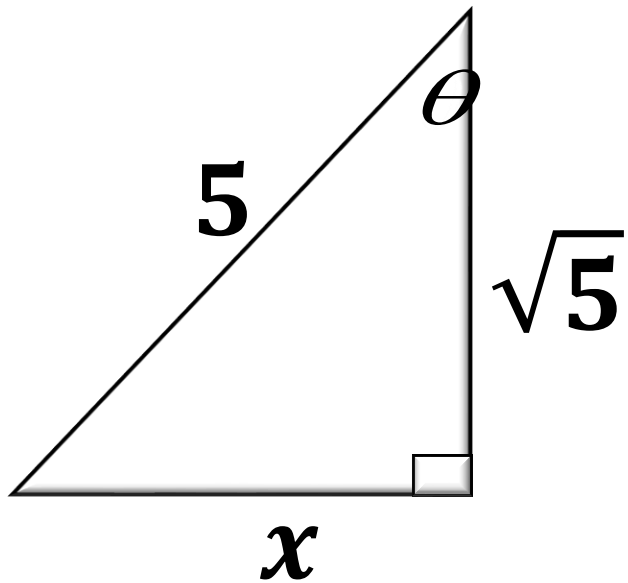
$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = \frac{5}{12}$$

EXAMPLE 2

Solve for x and determine all trig functions of θ



$$\sin \theta = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{\sqrt{5}}{5}$$

$$\tan \theta = 2$$

$$\csc \theta = \frac{\sqrt{5}}{2}$$

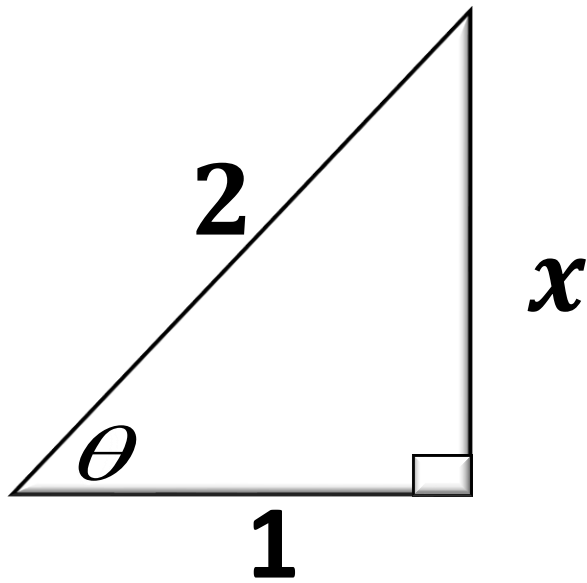
$$\sec \theta = \sqrt{5}$$

$$\cot \theta = \frac{1}{2}$$

$$x = 2\sqrt{5}$$

YOUR TURN

Solve for x and determine all trig functions of θ



$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\csc \theta = \frac{2\sqrt{3}}{3}$$

$$\sec \theta = 2$$

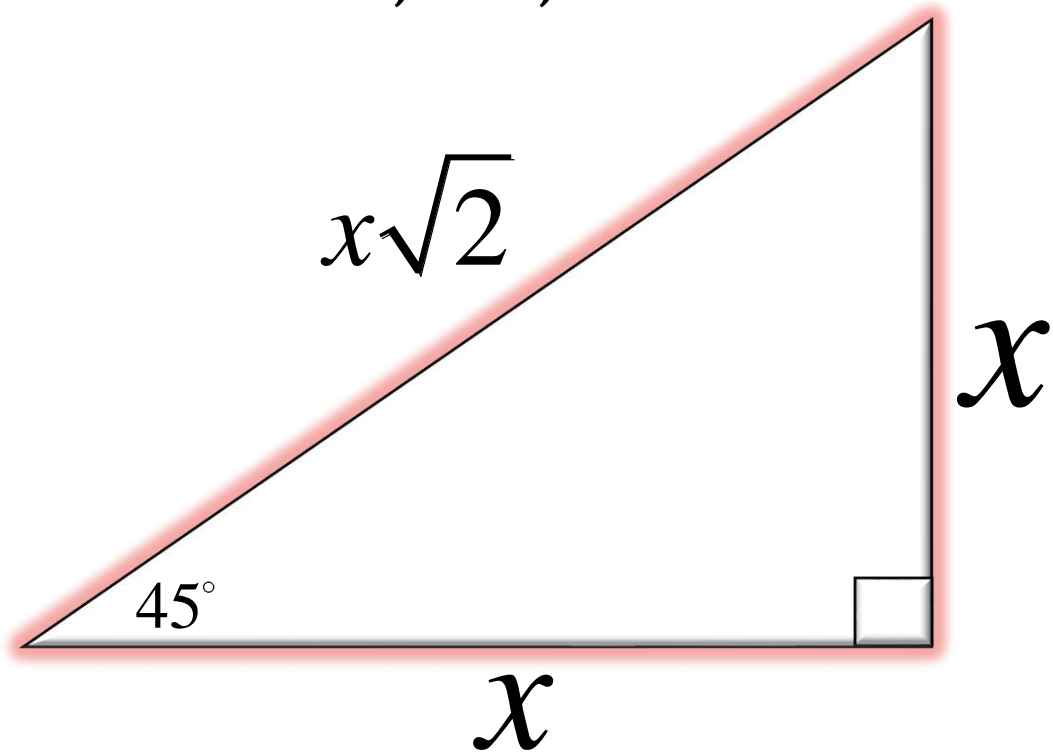
$$\cot \theta = \frac{\sqrt{3}}{3}$$

$$x = \sqrt{3}$$

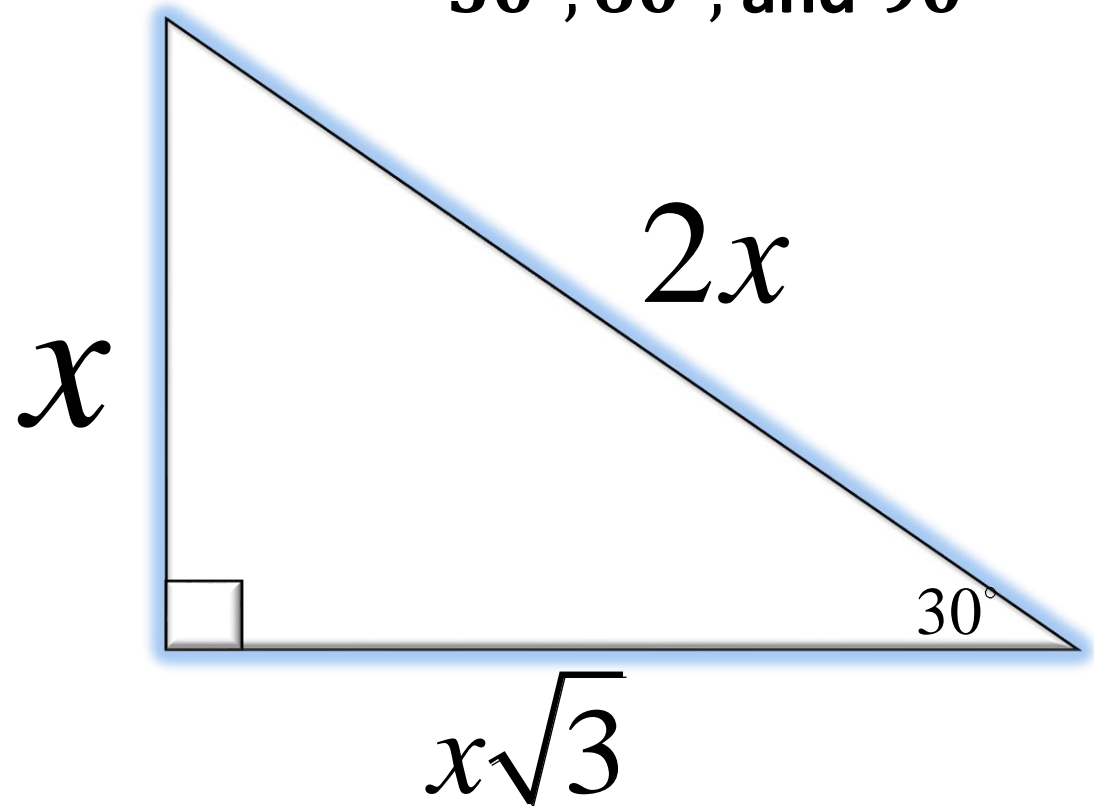
SPECIAL RIGHT TRIANGLES

Drawing is not to scale

$45^\circ, 45^\circ,$ and 90°

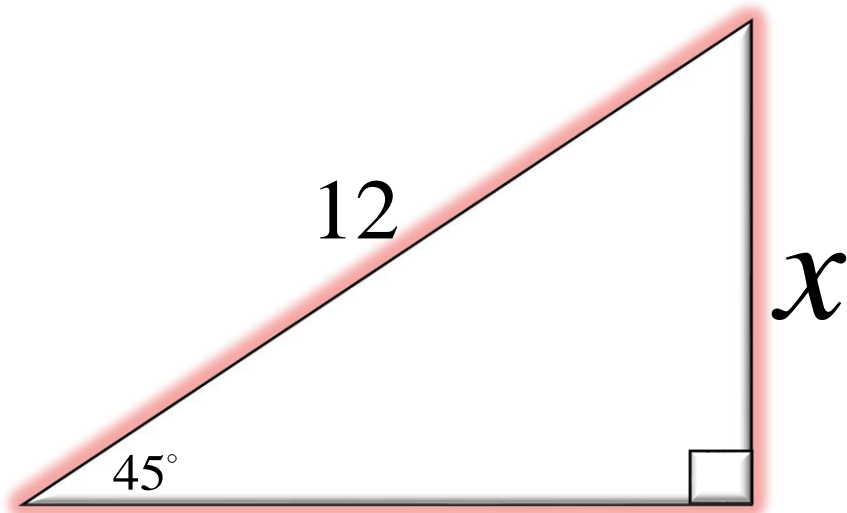


$30^\circ, 60^\circ,$ and 90°



EXAMPLE 3

Find the exact value of x without using a calculator



$$x\sqrt{2} = 12$$

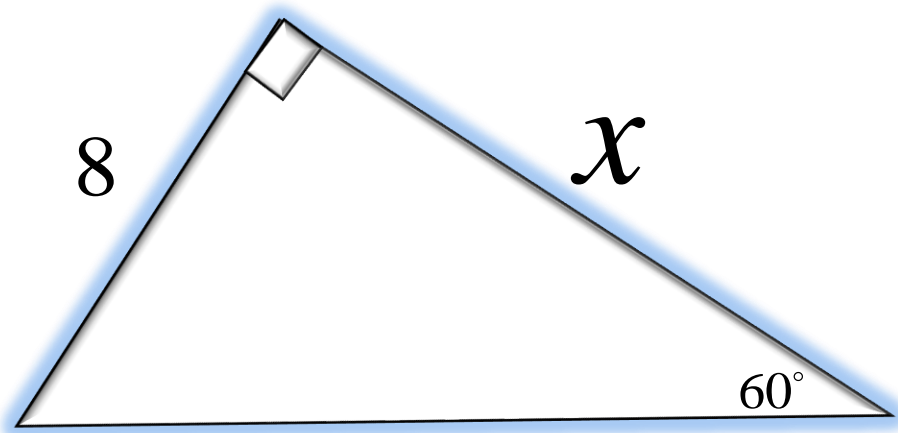
$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{12}{\sqrt{2}}$$

$$x = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2}$$

$$x = 6\sqrt{2}$$

EXAMPLE 4

Find the exact value of x without using a calculator

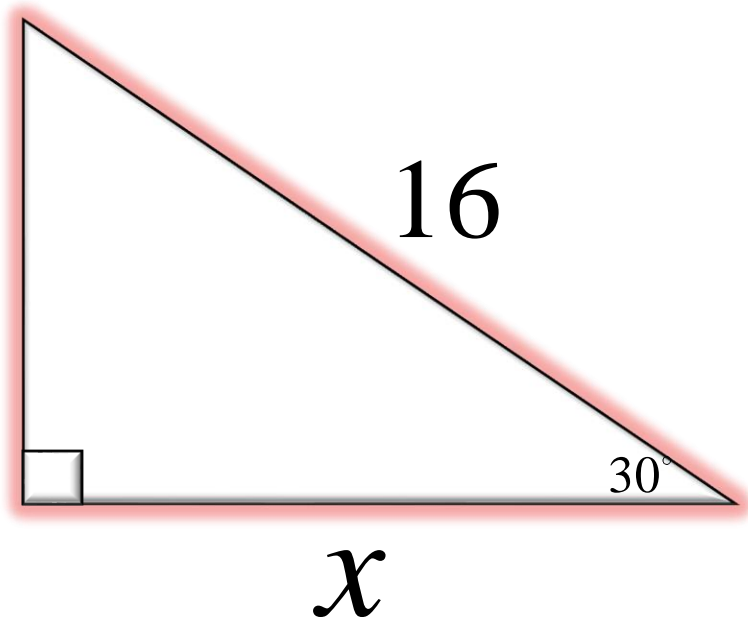


$$\begin{aligned}x\sqrt{3} &= 8 \\ \frac{x\sqrt{3}}{\sqrt{3}} &= \frac{8}{\sqrt{3}} \\ x &= \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}\end{aligned}$$

$$x = \frac{8\sqrt{3}}{3}$$

YOUR TURN

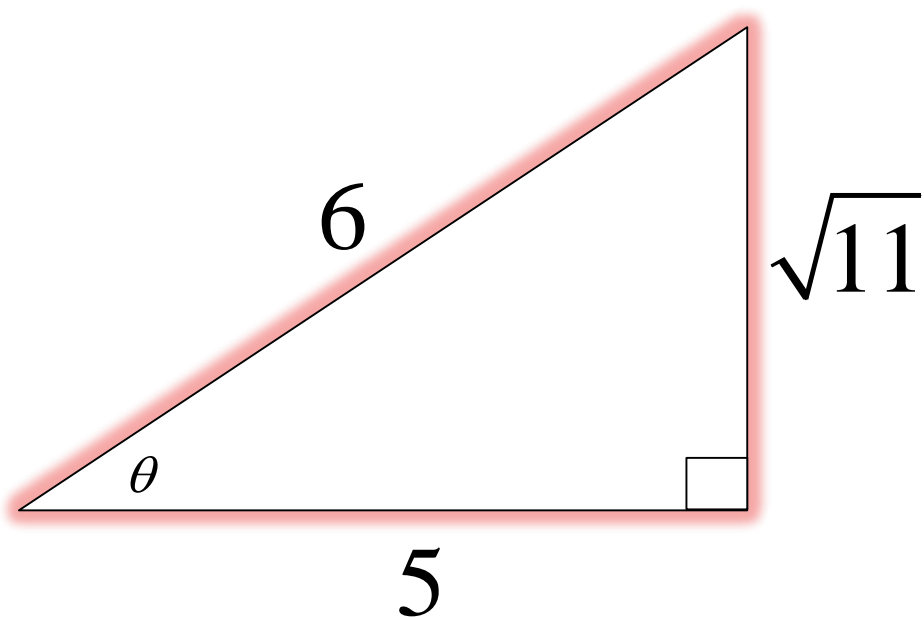
Find the exact value of x without using a calculator



$$x = 8\sqrt{3}$$

EXAMPLE 5

Given $\cos \theta = \frac{5}{6}$, identify the third side and solve for $\sec \theta$ and $\cot \theta$



$$x^2 + y^2 = r^2$$

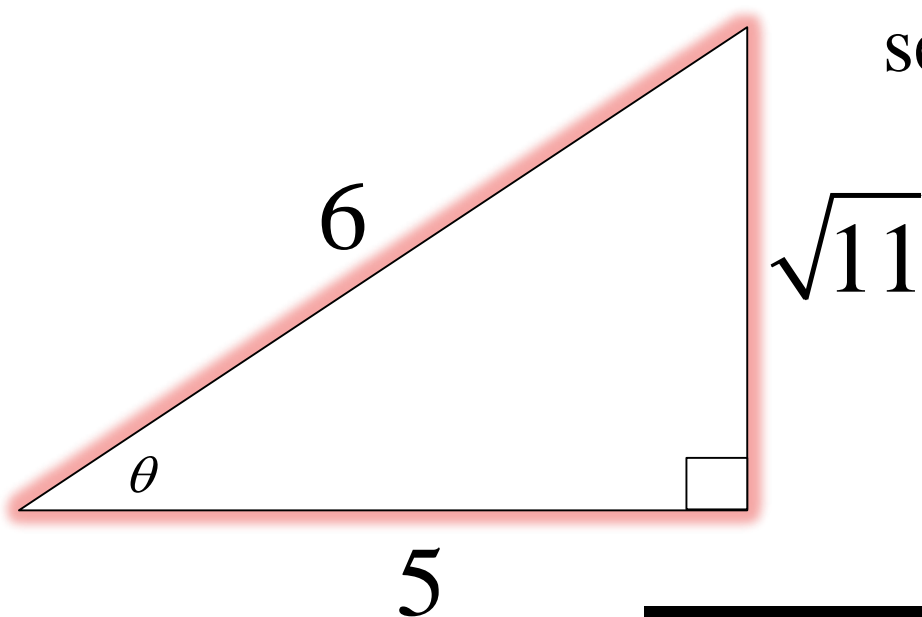
$$(5)^2 + y^2 = (6)^2$$

$$y^2 = 11$$

$$y = \sqrt{11}$$

EXAMPLE 5

Given $\cos \theta = \frac{5}{6}$, identify the third side and solve for $\sec \theta$ and $\cot \theta$



$$\sec \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

$$\sec \theta = \frac{6}{5}$$

$$\cot \theta = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\text{third side} = \sqrt{11}; \sec \theta = \frac{6}{5}, \cot \theta = \frac{5\sqrt{11}}{11}$$

EXAMPLE 6

Given $\cot \theta = 5$, identify the third side and solve for $\tan \theta$ and $\cot (90^\circ - \theta)$

$$\text{third side} = \sqrt{26}; \tan \theta = \frac{1}{5}, \cot (\theta - 90^\circ) = \frac{1}{5}$$

YOUR TURN

Given $\csc \theta = 9$, identify the third side and solve for $\cos \theta$ and $\cot \theta$

$$\text{third side} = 4\sqrt{5}; \cos \theta = \frac{4\sqrt{5}}{9}, \cot \theta = 4\sqrt{5}$$

ASSIGNMENT

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5, 8, 13-19 odd, 41-45 odd, 63-66 all