Finding Area

Section 4.3

Calculus AP/Dual, Revised ©2017

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When I Say Derivative, You Think…

SLOPE

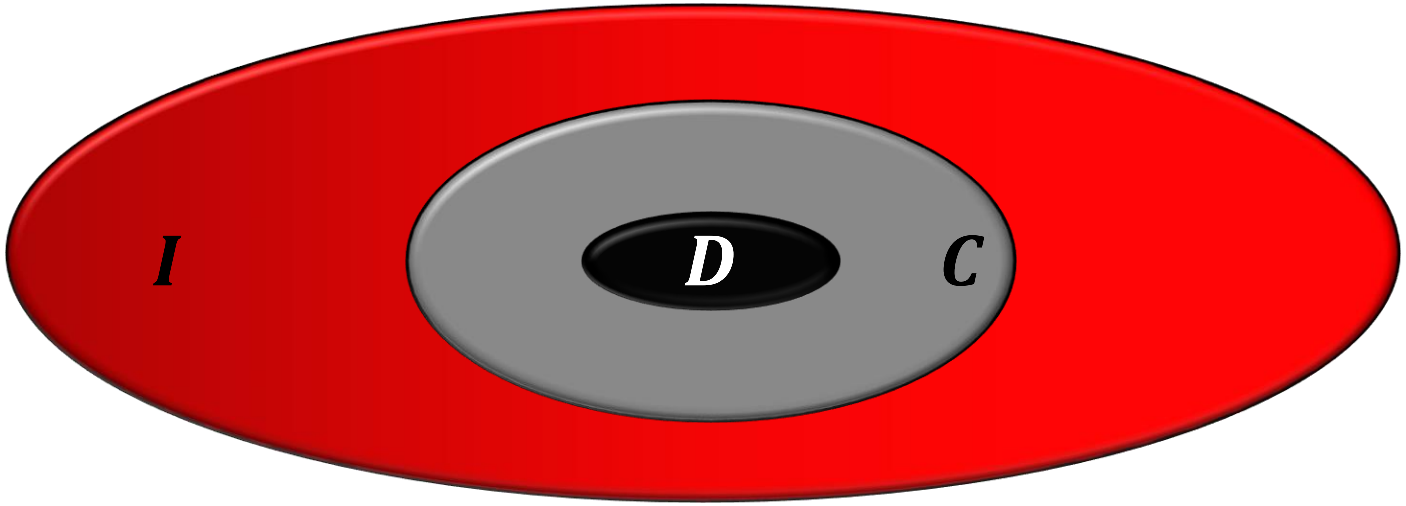
When I Say Integral, You Think…

AREA

Theorem

A. **If a function f is continuous on the closed interval, $[a, b]$, then f is integral on $[a, b]$**

Relationship



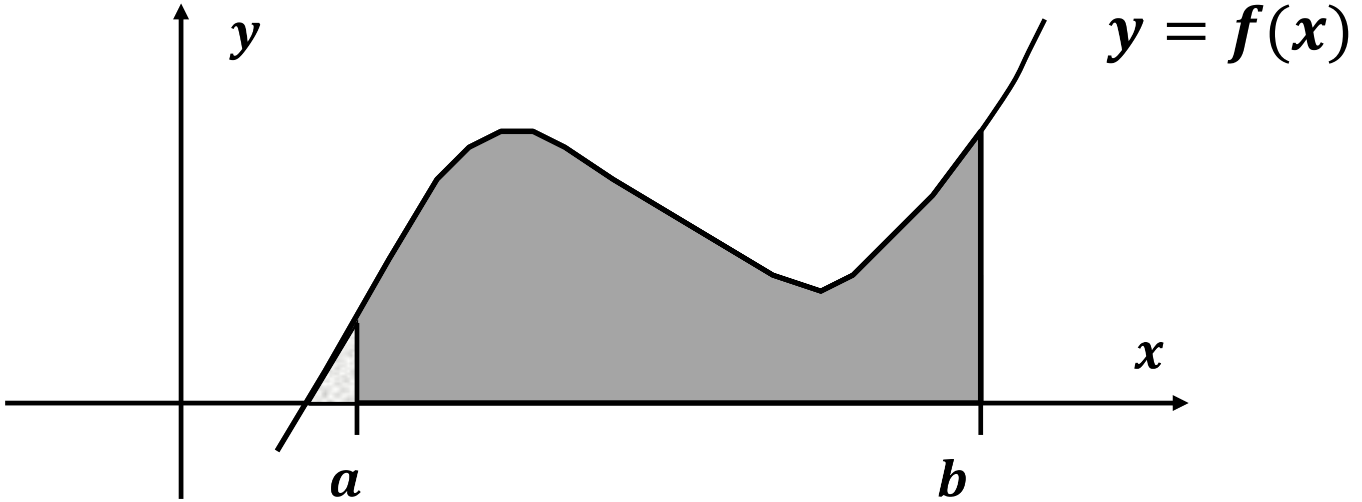
D – differentiable functions, strongest condition … all Differentiable functions are continuous and integrable.

C – continuous functions, all continuous functions are integrable, but not all are diffentiable.

I – integrable functions, weakest condition … it is possible they are not continuous and not differentiable

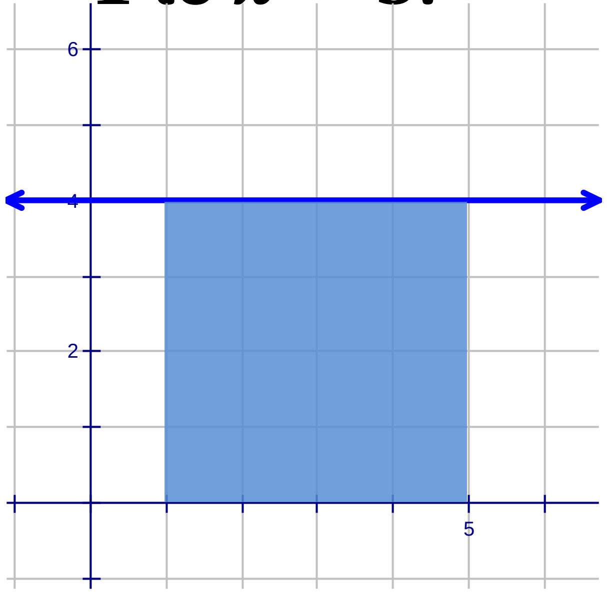
Evaluating Definite Integrals

A. Given $\int_a^b f(x)dx$: The Definite Integral represents the <u>Area of the Region</u> under the curve, $y = f(x)$, bounded by the x-axis, and the vertical lines $x = a$, and $x = b$



EXAMPLE 1

Set up a Definite Integral for finding the area of the shaded region. Then use integration and geometry to find the area for $f(x) = 4$ from $x = 1$ to $x = 5$.



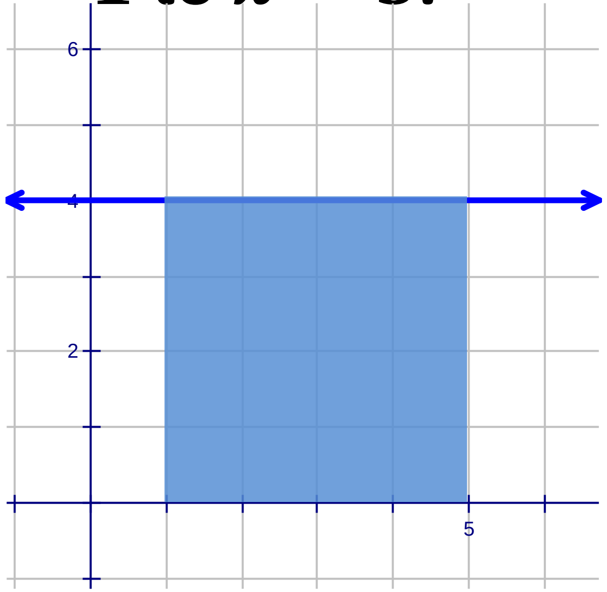
$$A = lw$$

$$A = (4)(4)$$

$$16\ units^2$$

EXAMPLE 1

Set up a Definite Integral for finding the area of the shaded region. Then use integration and geometry to find the area for $f(x) = 4$ from $x = 1$ to $x = 5$.

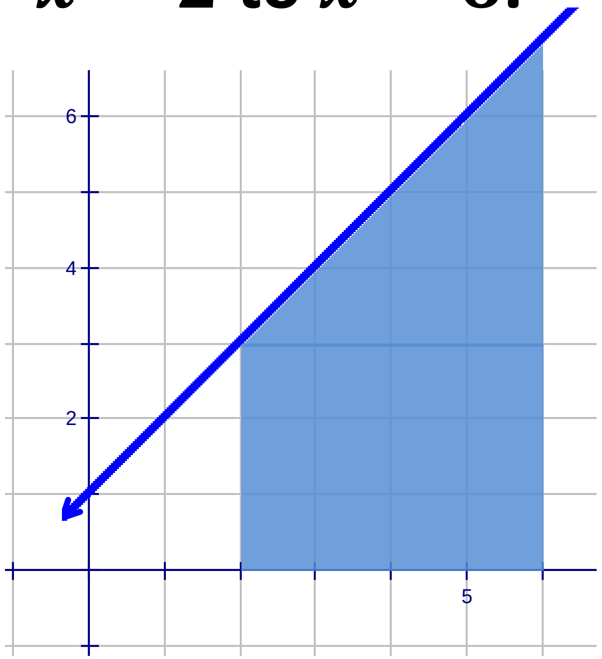


$$\int_1^5 4dx$$

$16\ units^2$

EXAMPLE 2

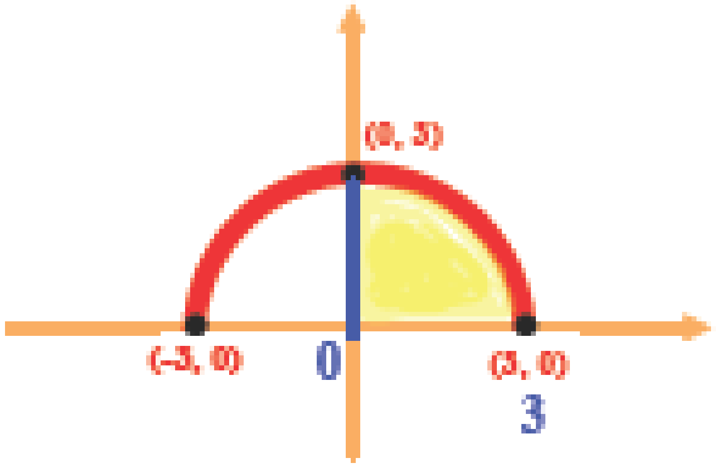
Set up a Definite Integral for finding the area of the shaded region. Then use integration and geometry to find the area for $f(x) = x + 1$ from $x = 2$ to $x = 6$.



$20\ units^2$

EXAMPLE 3

Set up a Definite Integral for finding the area of the shaded region. Then use geometry to find the area for $f(x) = \sqrt{9 - x^2}$ from $x = 0$ to $x = 3$.



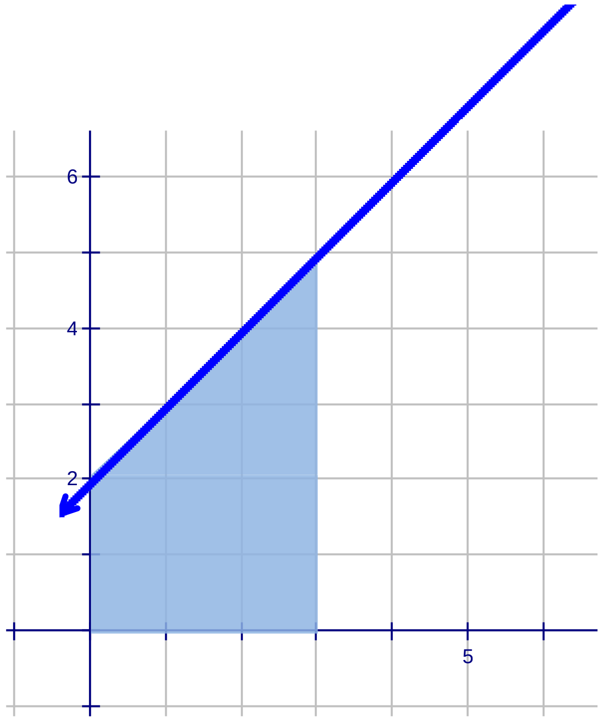
$$\int_0^3 \sqrt{9 - x^2}\,dx$$

$$A = \frac{\pi r^2}{4} = \frac{\pi (3)^2}{4}$$

$$\frac{9}{4}\pi \ units^2$$

Your Turn

Set up a Definite Integral for finding the area of the shaded region. Then use integration and geometry to find the area for $f(x) = x + 2$ from $x = 0$ to $x = 3$.



$10.5\ units^2$

Special Definite Integrals

A. $\int_a^a f(x)\,dx = 0$, where f is defined as $x = a$

B. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$, where f is integrable on $[a, b]$

C. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where f is integrable on the three closed intervals determined by a, b, and c.

D. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$, where k is constant and f and g are integrable

E. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^c f(x)\,dx \pm \int_c^b g(x)\,dx$, where f and g are integrable

EXAMPLE 4

Solve $\int_{2}^{2}(x+5)\,dx$

$$\int_{a}^{a} f\left(x\right)dx$$

$$\int_{2}^{2}\left(x+5\right)dx$$

$$0$$

EXAMPLE 5

Given $\int_0^3 (x+2)\,dx = \frac{21}{2}$**, solve** $\int_3^0 (x+2)\,dx$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_0^3 (x+2)\,dx = -\int_3^0 (x+2)\,dx$$

$$\frac{21}{2} = -\frac{21}{2}$$

$$-\frac{21}{2}$$

EXAMPLE 6

Given $\int_2^5 x^4\,dx = 40$, $\int_2^5 x\,dx = 7$, **and** $\int_2^5 dx = 3$ **evaluate**

$\int_2^5 (10 - 3x - 2x^4)\,dx$ $\quad \int_a^b kf(x)\,dx = k\int_b^a f(x)\,dx$

$$\int_2^5 10dx - \int_2^5 3x\,dx - \int_2^5 2x^4\,dx$$

$$10\int_2^5 dx - 3\int_2^5 x\,dx - 2\int_2^5 x^4\,dx$$

$$10(3) - 3(7) - 2(40)$$

$$30 - 21 - 80$$

$$-71$$

Your Turn

Given $\int_1^2 x^2\,dx = \frac{7}{3}$ **and** $\int_1^2 x\,dx = \frac{3}{2}$**, evaluate** $\int_1^2 3(x^2 + x)\,dx$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_1^2 3\left(x^2 + x\right)dx = 3\int_1^2 x^2\ dx\ \ + 3\int_1^2 x\ dx$$

$$3\left(\frac{7}{3}\right) + 3\left(\frac{3}{2}\right)$$

$$\frac{23}{2}$$

AP Multiple Choice Practice Question 1 (non-calculator)

If $\int_{1}^{10} f(x)\, dx = 4$ and $\int_{10}^{3} f(x)\, dx = 7$, then $\int_{1}^{3} f(x)\, dx =$

(A) -3

(B) 0

(C) 3

(D) 11

AP Multiple Choice Practice Question 1 (non-calculator)

If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^{3} f(x)\,dx =$

Vocabulary	Process and Connections	Answer
Integral Area	Lower Bound: 1, Upper Bound: 3 $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ $a=1, b=3, c=10$ $\int_1^3 f(x)dx = \int_1^{10} f(x)dx + \int_{10}^3 f(x)dx$ $\int_1^3 f(x)dx = 4+7=11$	D

Assignment

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13-43 odd, 47 A-E