

Write an integral to find the area under the graph of f for the given function and interval. Then, write the expression as a limit and solve the integral.

1) $f(x) = x^3$, $[0, 3]$

2) $f(x) = \sqrt[3]{x}$, $1 \leq x \leq 27$

Integral and Answer: _____

Integral and Answer: _____

Limit: _____

Limit: _____

3) $f(x) = 1 + x^5$, $1 \leq x \leq 2$

4) $f(x) = x^3 + x - 1$, $2 \leq x \leq 5$

Integral and Answer: _____

Integral and Answer: _____

Limit: _____

Limit: _____

5) $f(x) = \frac{1}{x+1}$, $0 \leq x \leq 1$

6) $f(x) = \cos x$, $[\pi, 2\pi]$

Integral (DO NOT Answer): _____

Integral and Answer: _____

Limit: _____

Limit: _____

Write the limit using interval notation. Do NOT evaluate the integral.

$$7) \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{2k}{n} \right)^2 \left(\frac{2}{n} \right)$$

$$8) \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{3}{n} \sqrt[3]{\frac{3k}{n}}$$

Integral: _____

Integral: _____

$$9) \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt[3]{2 + \frac{3}{n}i} \left(\frac{3}{n} \right)$$

$$10) \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4}{n} \left(2 + \sqrt{3 + \frac{4k}{n}} \right)$$

Integral: _____

Integral: _____

$$11) \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\left(1 + \frac{5k}{n} \right)^3 + 3 \left(1 + \frac{5k}{n} \right) + \left(1 + \frac{5k}{n} \right) + 2 \right] \left(\frac{2}{n} \right)$$

$$12) \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\ln \left(1 + \left(\frac{e-1}{n} \right) k \right) \right] \left(\frac{e-1}{n} \right)$$

Integral: _____

Integral: _____

12) Use the limit $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\left(\left(\frac{k}{n} \right)^2 + \frac{k}{n} + 1 \right) \left(\frac{1}{n} \right) \right]$ to complete the following problems.

(a) Write the first four terms of the sum.

(b) Evaluate the sum if $n = 5$. Then, write the limit for a left Riemann's Sum.

(c) Write the limit as a definite integral and evaluate the definite integral using the Fundamental Theorem.