§4.2B: Limits of Riemann's Sum
"I WILL …
…approximate and use the limit process using Riemann's Sum."

I. Limit Equation of Riemann's Sum

A. $\lim_{n\to\infty}\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$; $k =$ ______________________, $n =$ ______________________________

B. It is more traditional to use _______ for left or midpoint sums, and _____ for right sums.

C. $\sum_{k=1}^{n} 1 =$ __________________

D. $\sum_{k=1}^{n} k =$ __________________

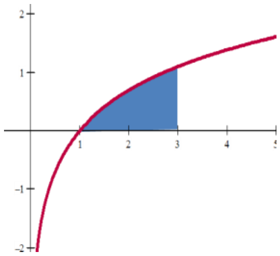
E. $\sum_{k=1}^{n} k^2 =$ __________________

F. $\sum_{k=1}^{n} k^3 =$ __________________

G. $\sum_{k=0}^{n} r^k =$ __________________, $r \neq 1$

Ex 1: Write the limit which is equal to $\int_0^5 x\,dx$ and then solve for the integral.
Ex 2: Write the limit which is equal to $\int_0^2 x^2\,dx$ and then solve for the integral.

Ex 3: Use the graph to write the limit for the area of the shaded region given the graph is $f(x) = \ln x$.



Your Turn: Write the limit which is equal to $\int_2^5 (3x^2 + 1)\, dx$.

Ex 4: Write the limit using Integral Notation, $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{3}{n}\right) \sqrt[3]{\frac{3k}{n}}$.

Ex 5: Write the limit using Integral Notation, $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n}\right) \left[\left(3 + \frac{2k}{n}\right)^3 + \left(3 + \frac{2k}{n}\right)\right]$.

Ex 6: Write the limit using Integral Notation, $\lim_{n\to\infty} \sum_{k=1}^{n} \left[1 + \left(1 + \frac{7k}{n}\right)^5\right]\left(\frac{7}{n}\right)$.

Your Turn: Write the limit using Integral Notation, $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{3}{n}\right)\left[\left(2 + \frac{3k}{n}\right)^4 + 4\right]$.

II. Types of Endpoints:

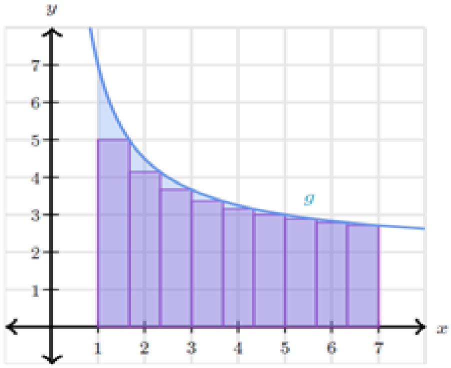
A. Left-Side Endpoint equation: ____

B. Right-Side Endpoint equation: ____

Ex 7: Write a left-endpoint and right-endpoint Riemann Sum R_4 that approximates $\int_0^1 \sqrt{x}\ dx$. Do not solve.

Your Turn: Write a left-endpoint and right-endpoint Riemann Sum R_{50} that approximates $\int_0^1 \sqrt{x}$. Do not solve.

AP 1) Which of the following approximates the area between $g(x) = \frac{5}{x} + 2$ and the x-axis on the interval [1,7] using a Right Riemann sum with 9 equal subdivisions?



(A) $\sum_{k=1}^{9} \frac{15}{3+2k} + 2$ (B) $\sum_{k=0}^{8} \frac{15}{3+2k} + 2$ (C) $\sum_{k=1}^{9} \left(\frac{15}{3+2k} + 2\right) \cdot \frac{2}{3}$ (D) $\sum_{k=0}^{8} \left(\frac{15}{3+2k} + 2\right) \cdot \frac{2}{3}$

Vocabulary	Process and Connections	Answer and Justifications

Assignment: Worksheet