Unit Circle Applications

Section 4.2A

Precalculus PreAP/Dual, Revised ©2017

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Reciprocal Identities Theorem

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

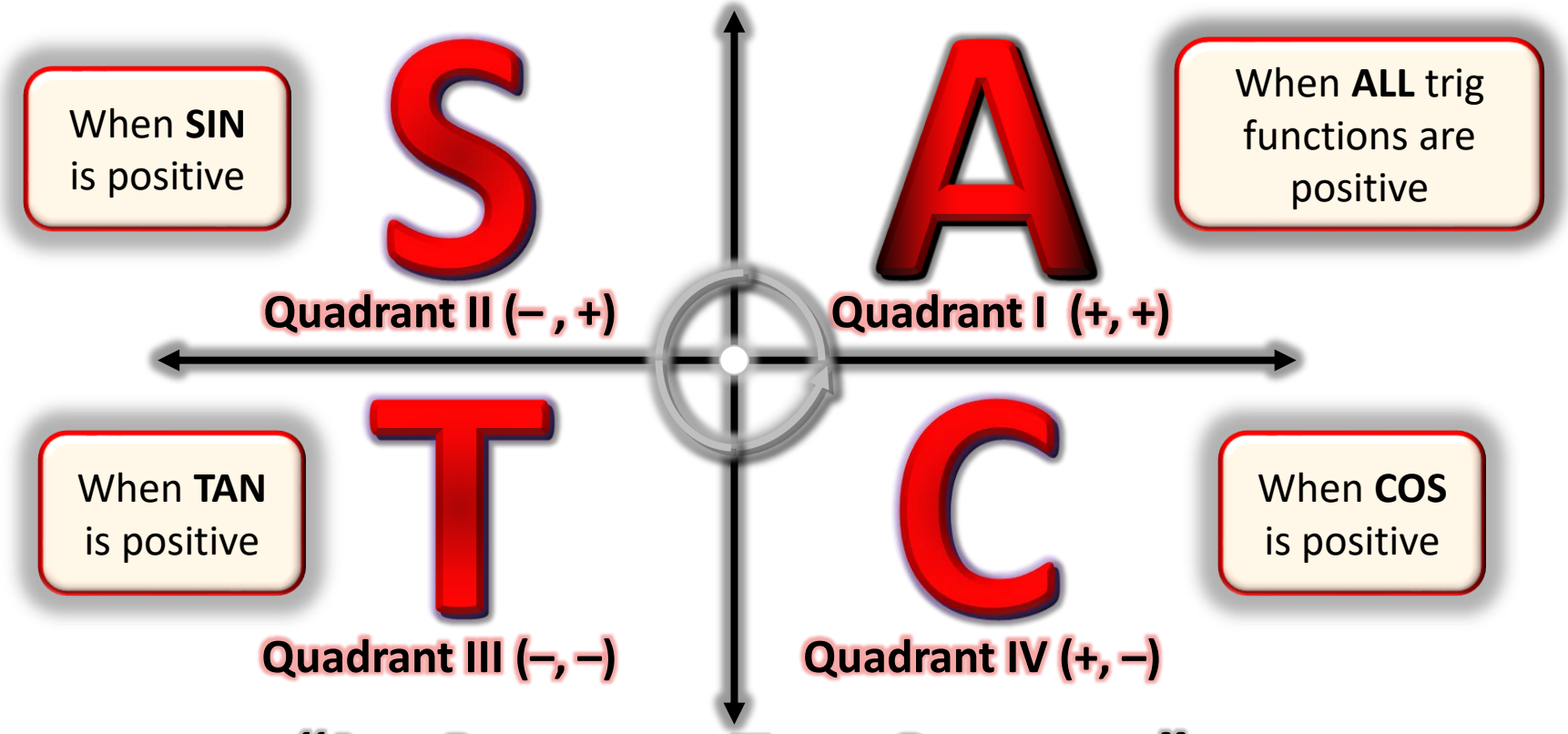
REMINDER

$$\cos\theta = \frac{A}{H} \qquad \sin\theta = \frac{O}{H}$$

$$(\cos\theta, \sin\theta)$$

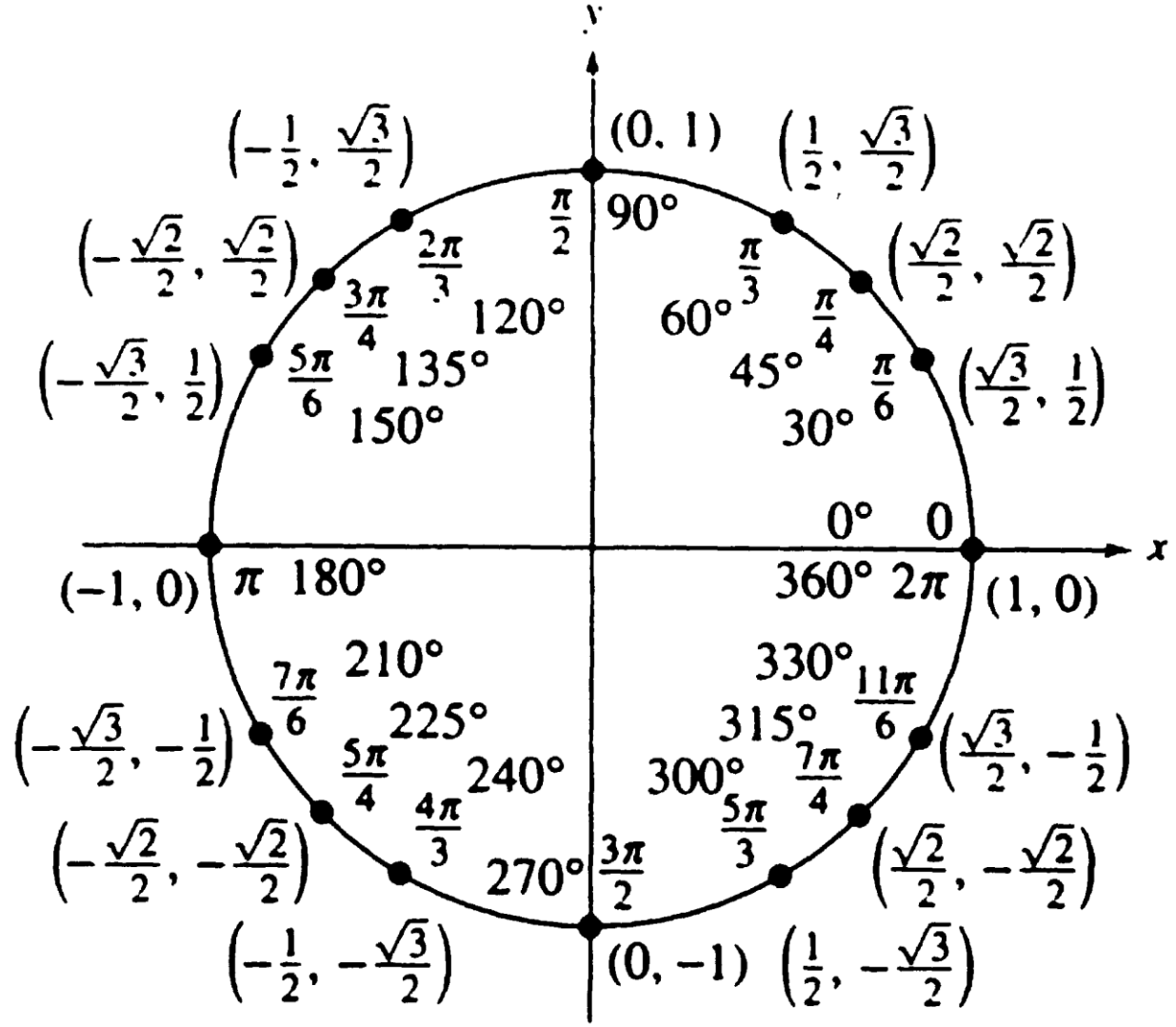
Equation in Standard Form

For θ be an angle in standard position with any point (x, y):



"All Students Take Calculus"

THE UNIT CIRCLE

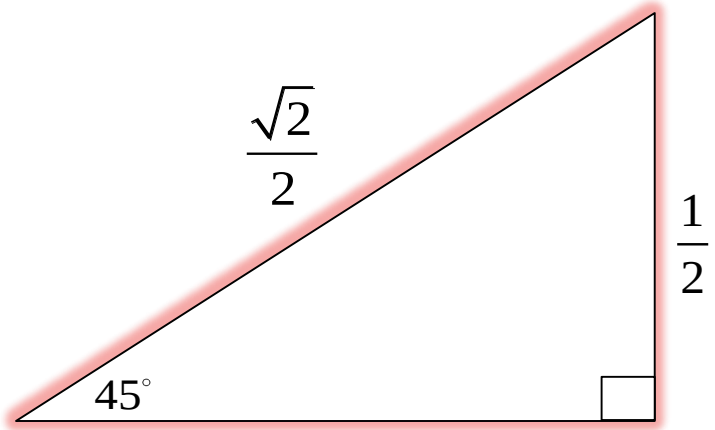


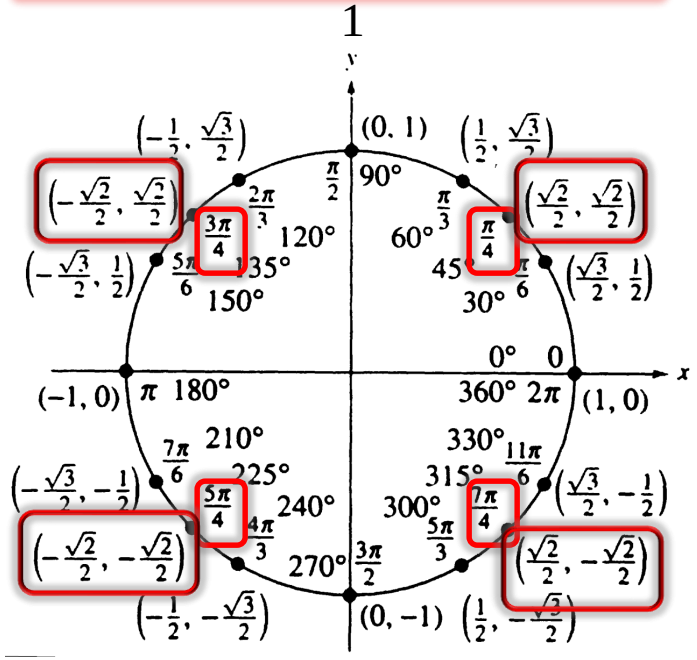
The Unit Circle

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{O}{H} = \frac{1}{H/O} = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{A}{H} = \frac{1}{H/A} = \frac{1}{\sec\theta}$$

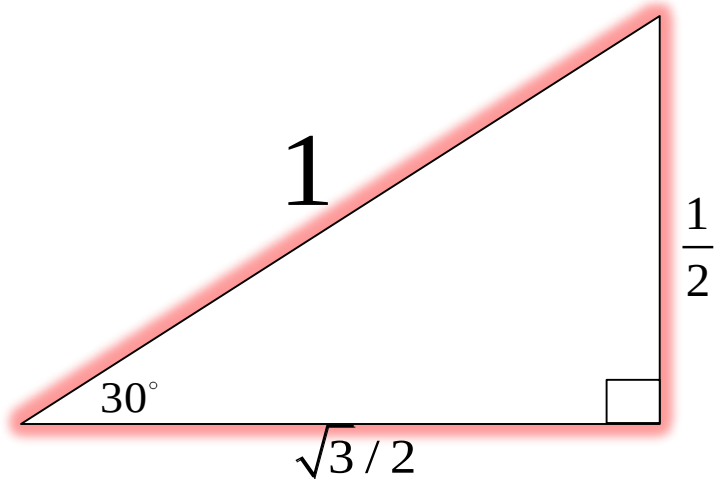
The Unit Circle: $\pi/4$ Family

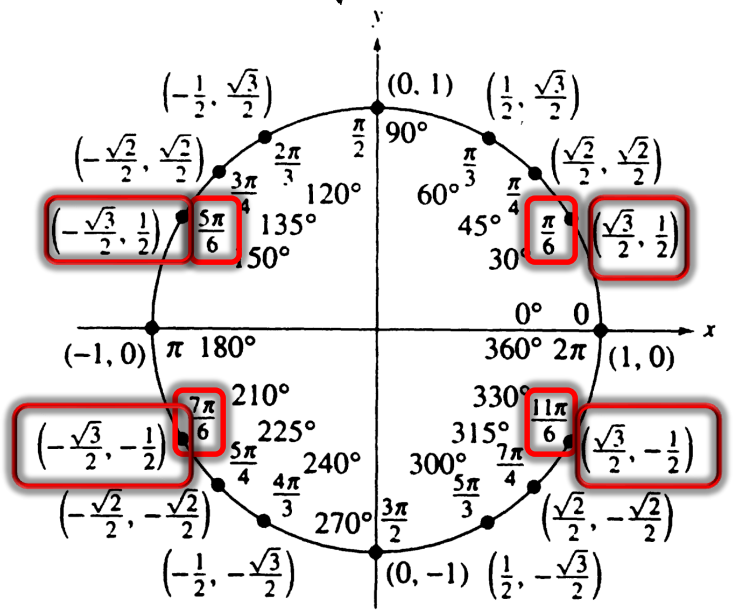




	$\cos\theta$	$\sin\theta$
Reference Angle: 45°		
$\pi/4$ 45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$3\pi/4$ 135°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$5\pi/4$ 225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$7\pi/4$ 315°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$

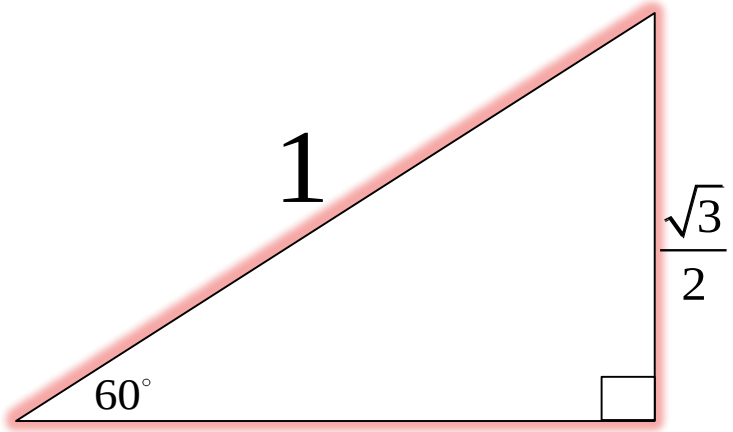
The Unit Circle: $\pi/6$ Family

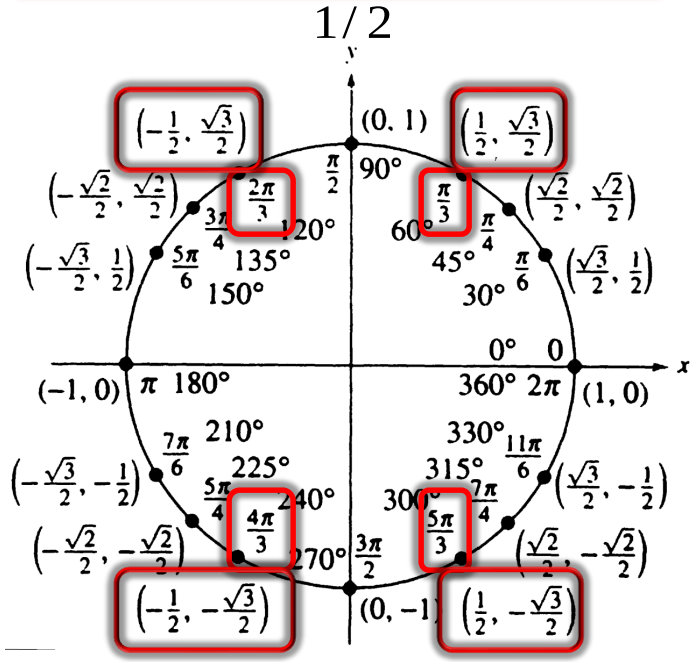




	$\cos\theta$	$\sin\theta$
Reference Angle: 30°		
$\pi/6$ 30°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$5\pi/6$ 150°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$7\pi/6$ 210°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$11\pi/6$ 330°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

The Unit Circle: $\pi/3$ Family

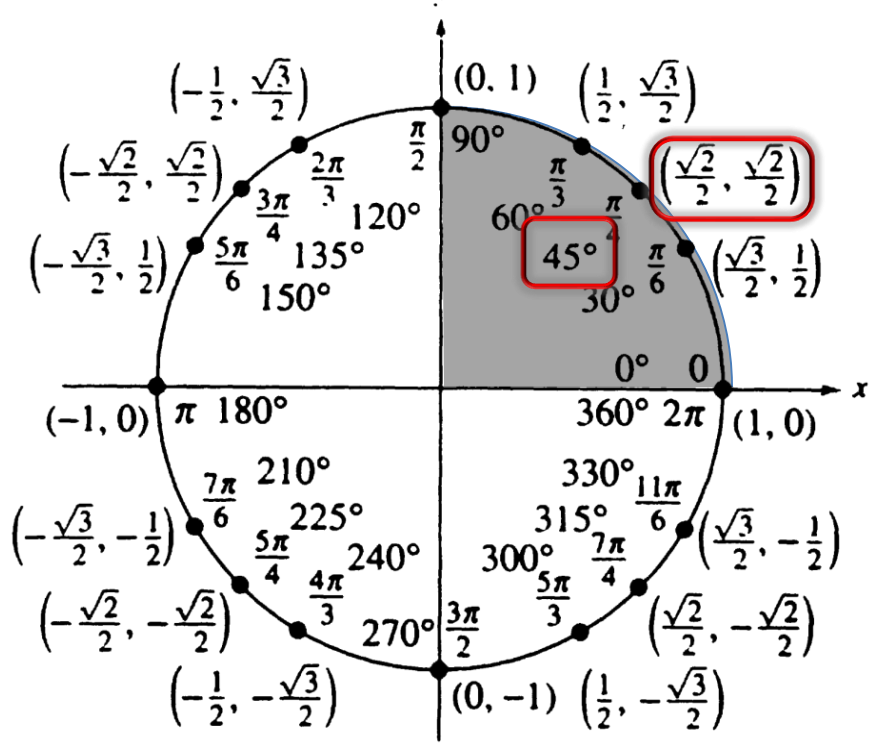




	$\cos\theta$	$\sin\theta$
Reference Angle: 60°		
$\pi/3$ 60°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$2\pi/3$ 120°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$4\pi/3$ 240°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$5\pi/3$ 300°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$

EXAMPLE 1

Solve $\cos 45°$ without a calculator



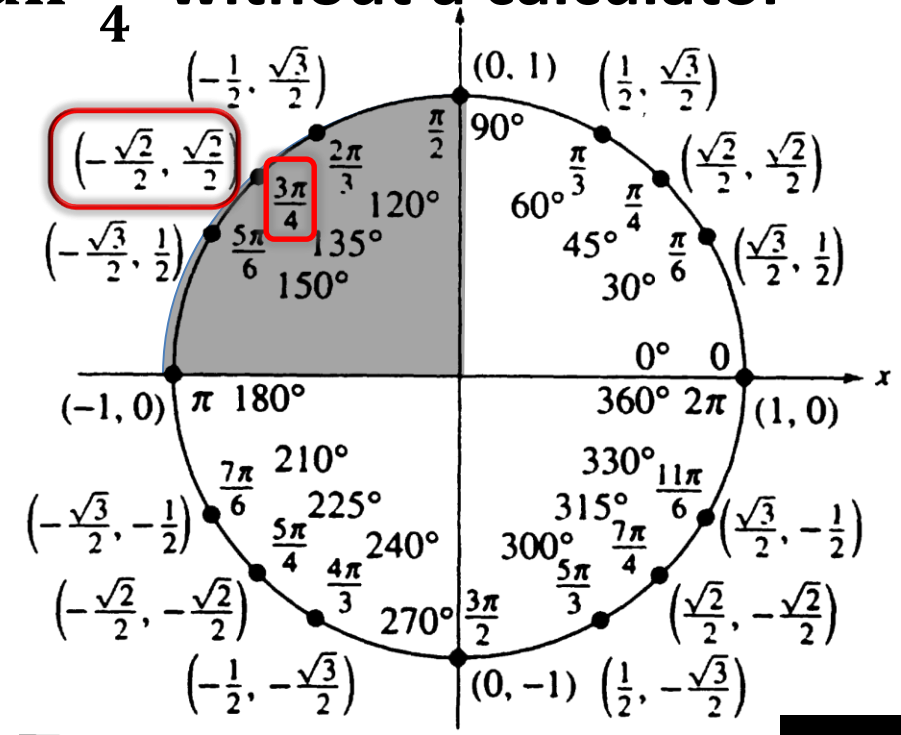
$$\cos 45°$$

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{2}$$

EXAMPLE 2

Solve $\tan\frac{3\pi}{4}$ without a calculator



$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} \qquad \frac{\sqrt{2}}{2} \div -\frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} \cdot -\frac{2}{\sqrt{2}}$$

$$-1$$

EXAMPLE 3

Solve $\tan \frac{11\pi}{6}$ without a calculator

$$-\frac{\sqrt{3}}{3}$$

YOUR TURN

Solve $\tan\frac{\pi}{3}$ without a calculator

$\sqrt{3}$

EXAMPLE 4

Solve $\tan\left(-\frac{\pi}{3}\right)$ without a calculator

$$-\sqrt{3}$$

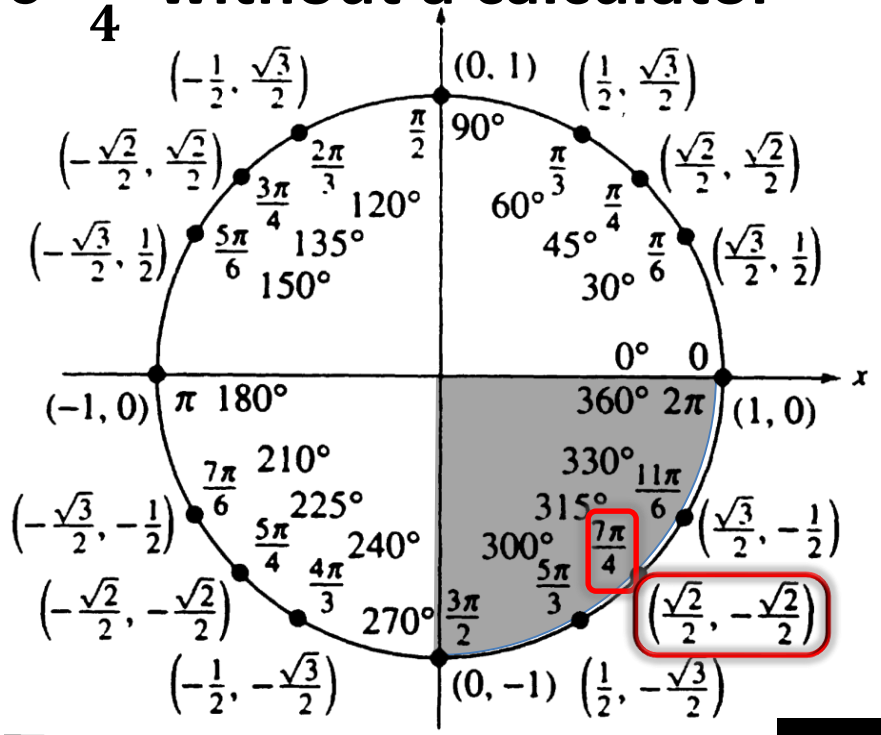
YOUR TURN

Solve $\tan \frac{9\pi}{4}$ without a calculator

1

EXAMPLE 5

Solve sec $\frac{7\pi}{4}$ without a calculator



$$\frac{1}{\cos\frac{7\pi}{4}} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)}$$

$$1 \div \frac{\sqrt{2}}{2} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$$

$$\sqrt{2}$$

EXAMPLE 6

Solve $\csc \frac{4\pi}{3}$ without a calculator

$$-\frac{2\sqrt{3}}{3}$$

EXAMPLE 7

Solve $\cot \frac{11\pi}{6}$ without a calculator

$-\sqrt{3}$

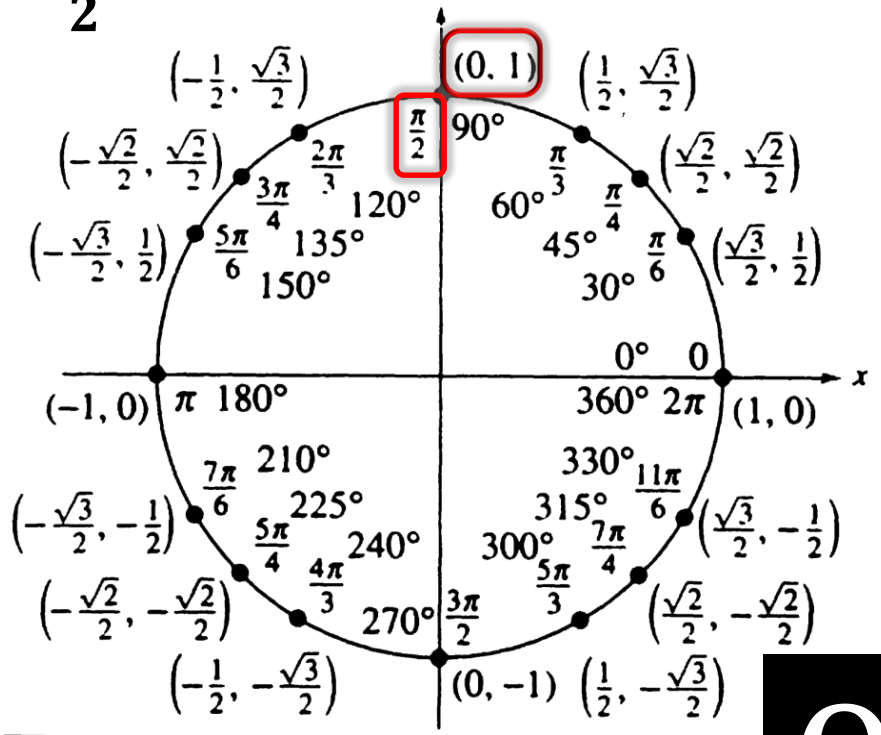
YOUR TURN

Solve $\csc\left(-\frac{11\pi}{6}\right)$ without a calculator

2

EXAMPLE 8

Solve $\cot \frac{\pi}{2}$ without a calculator



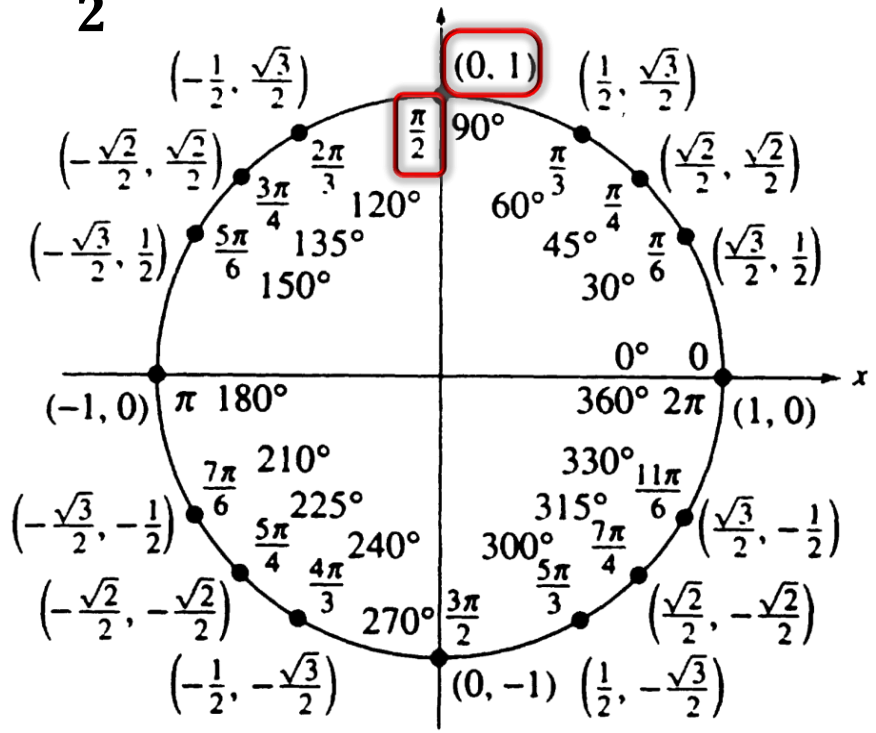
$$\frac{1}{\tan\frac{\pi}{2}} = \frac{1}{\left(\frac{1}{0}\right)}$$

$$1 \div \frac{1}{0} = 1 \cdot \frac{0}{1} =$$

0

EXAMPLE 9

Solve $\tan \frac{\pi}{2}$ without a calculator



$$\tan \frac{\pi}{2} = \frac{1}{\left(\frac{0}{1}\right)}$$

$$1 \div \frac{0}{1} = \quad 1 \cdot \frac{1}{0} =$$

undefined

YOUR TURN

Solve $\cot 4\pi$ without a calculator

undefined

ASSIGNMENT

Worksheet 2