Trapezoidal Area

Section 4.2A

Calculus AP/Dual, Revised ©2018

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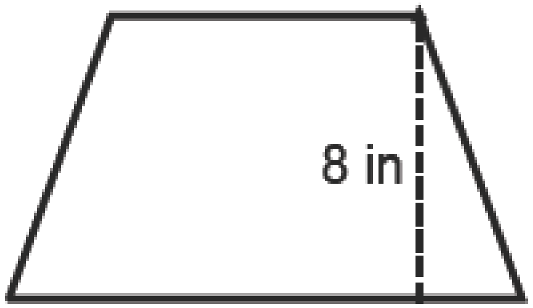
REVIEW

Find the area of the trapezoid.



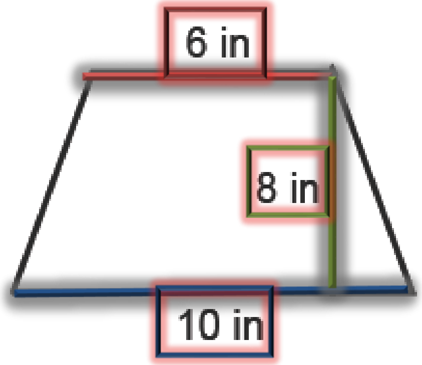
$$A = h\left(\frac{b_1 + b_2}{2}\right)$$

$$A = (8)\left(\frac{6+10}{2}\right)$$

$$A = 64\ units^2$$

TRAPEZOIDAL AREA (UNEVEN AREA)

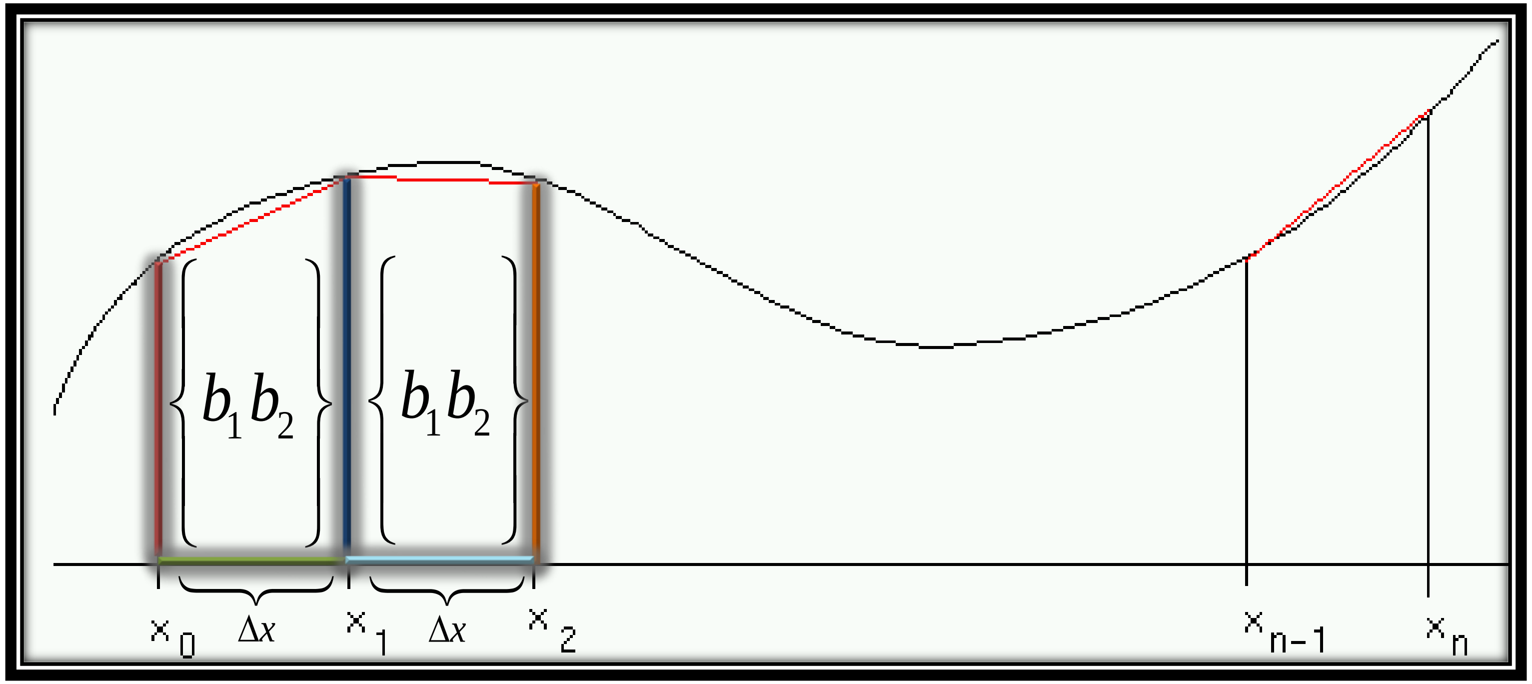
Find the area of the curve using the trapezoid sum.



$$A = h\left(\frac{b_1 + b_2}{2}\right)$$

$$A = (8)\left(\frac{6+10}{2}\right)$$

$$A = 64\ units^2$$



$$A = \frac{1}{2}\Delta x\left(f\left(x_0\right) + f\left(x_1\right)\right) + \frac{1}{2}\Delta x\left(f\left(x_1\right) + f\left(x_2\right)\right) + \frac{1}{2}\Delta x\left(f\left(x_2\right) + f\left(x_{n-1}\right)\right) + \frac{1}{2}\Delta x\left(f\left(x_{n-1}\right) + f\left(x_n\right)\right)$$

TRAPEZOIDAL RULE

A. To evaluate a definite integral involving a function whose anti-derivative cannot be found

B. Integral: $\int_a^b f(x)dx = \frac{1}{2}\left(\frac{b-a}{n}\right)(L_n + R_n)$

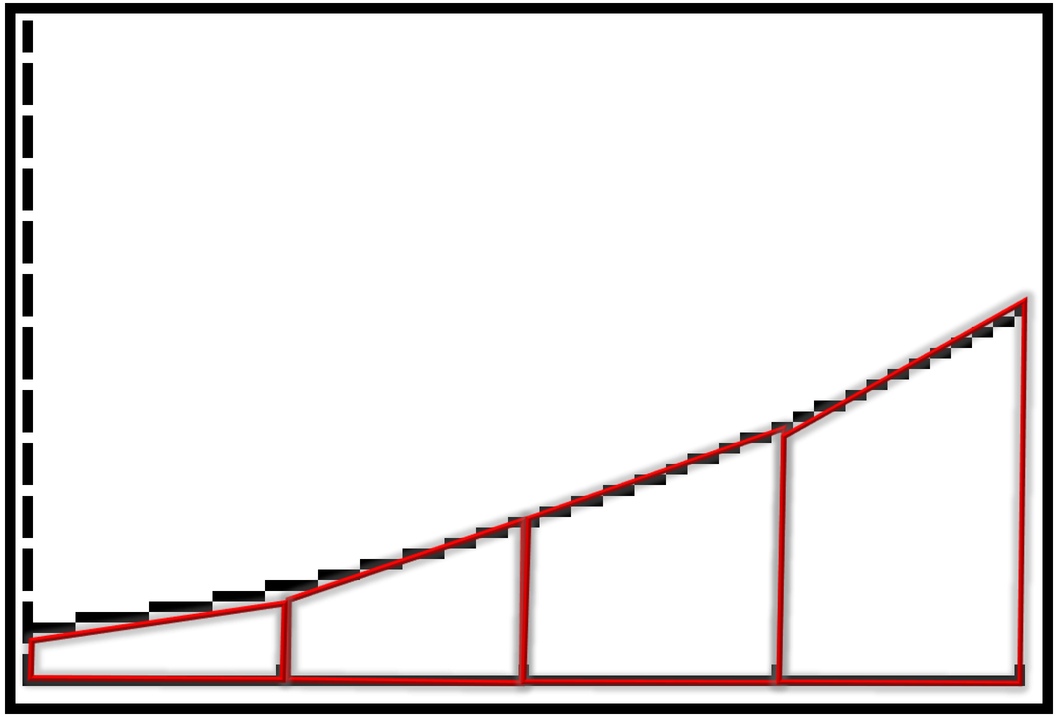
C. Area of Trapezoidal Rule:

$$\frac{1}{2}\Delta x[f(x_0 + x_1) + f(x_1 + x_2) + \cdots + f(x_{n-1} + x_n)]$$

D. $\Delta x = \frac{b-a}{n}$

Example 1

Use the Trapezoidal Rule to approximate the value of $\int_1^5 (x^2+3)\,dx$ for $n=4$.



$$\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$$

$$A = \frac{1}{2}\Delta x \begin{pmatrix} \left(f(1)+f(2)\right)+\left(f(2)+f(3)\right)+ \\ \left(f(3)+f(4)\right)+\left(f(4)+f(5)\right) \end{pmatrix}$$

$$\int_1^5 \left(x^2+3\right)dx \approx$$

EXAMPLE 1

Use the Trapezoidal Rule to approximate the value of $\int_1^5 (x^2+3)\,dx$ for $n=4$.

$$A=\frac{1}{2}\Delta x\Big(\big(f(1)+f(2)\big)+\big(f(2)+f(3)\big)+\big(f(3)+f(4)\big)+\big(f(4)+f(5)\big)\Big)$$

$$\int_1^5 (x^2+3)\,dx \approx \left(\frac{1}{2}\right)(1)\Big(\big(f(1)+f(2)\big)+\big(f(2)+f(3)\big)+\big(f(3)+f(4)\big)+\big(f(4)+f(5)\big)\Big)$$

$$\left(\frac{1}{2}\right)\Big[(4+7)+(7+12)+(12+19)+(19+28)\Big]$$

$$54\ units^2$$

EXAMPLE 1

Use the Trapezoidal Rule to approximate the value of $\int_1^5 (x^2 + 3)\, dx$ for $n = 4$.

Solve using FTC:

$$F(5) - F(1)$$

$$53.3333\ units^2$$

Example 2

Use the Trapezoidal Rule to approximate the value of $\int_1^3 (4-x)^2\,dx$ for $n = 4$.

$8.75\ units^2$

YOUR TURN

Use the Trapezoidal Rule to approximate the value of $\int_1^9 x^2\,dx$ for $n = 4$.

$248\ units^2$

REVIEW

Use the <u>Left Hand Sum</u> to approximate the value of $\int_1^7 f(x)\,dx$.

x	1	2	5	6	7
$f(x)$	2	3	−1	4	8

$$LHS = \left(\frac{2-1}{1}\right)(f(1)) + \left(\frac{5-2}{1}\right)(f(2)) + \left(\frac{6-5}{1}\right)(f(5)) + \left(\frac{7-6}{1}\right)(f(6))$$

$$LHS = (1)(2) + (3)(3) + (1)(-1) + (1)(4)$$

$$LHS = 14\ units^2$$

EXAMPLE 3

Use the Trapezoidal Rule to approximate the value of $\int_1^7 f(x)\,dx$

x	1	2	5	6	7
$f(x)$	2	3	-1	4	8

$$A=\frac{1}{2}\left[\Delta x\left(f\left(x_1\right)+f\left(x_2\right)\right)+\Delta x\left(f\left(x_2\right)+f\left(x_5\right)\right)+\Delta x\left(f\left(x_5\right)+f\left(x_6\right)\right)+\Delta x\left(f\left(x_6\right)+f\left(x_7\right)\right)\right]$$

$$A=\frac{1}{2}\left[(1)(2+3)+(3)(3+-1)+(1)(-1+4)+(1)(4+8)\right]$$

$$13\ units^2$$

YOUR TURN

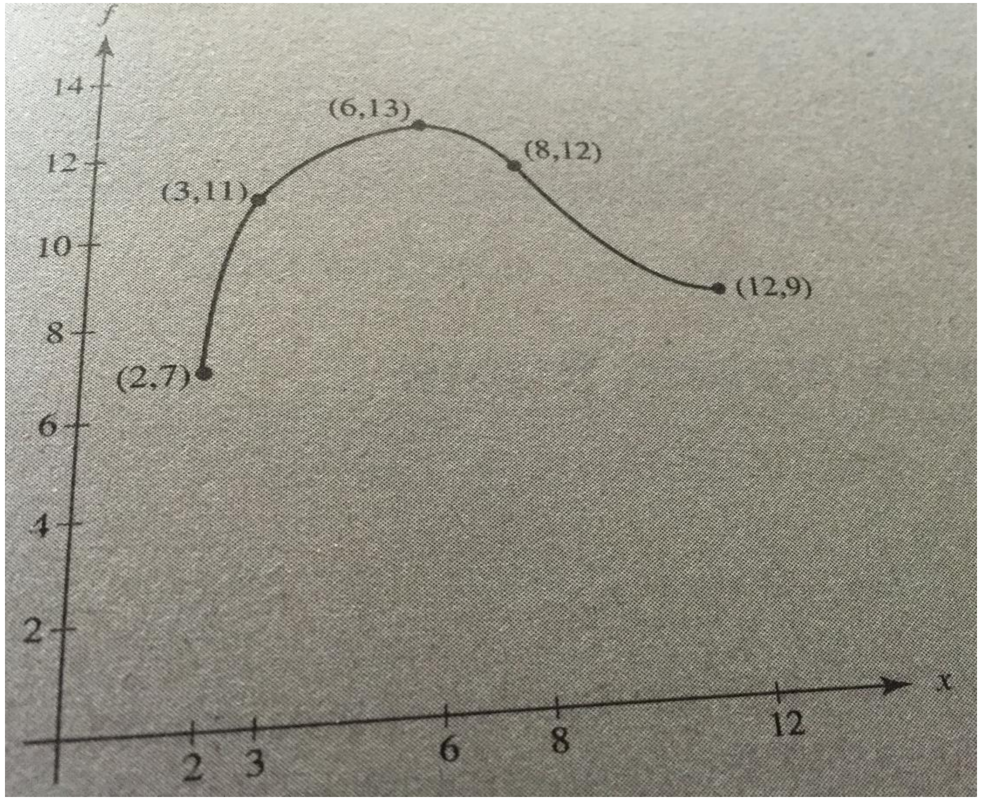
Use the Trapezoidal Rule to approximate the value of $\int_{1}^{13} f(x)\,dx$.

x	1	3	8	12	13
$f(x)$	2	5	3	1	4

$\frac{75}{2}$ $units^2$

AP Multiple Choice Practice Question 1 (non-calculator)

The graph of f is shown below. Approximate $\int_2^{12} f(x)\,dx$ using the trapezoid approximation.



(A) 44

(B) 88

(C) 112

(D) 224

AP Multiple Choice Practice Question 1 (non-calculator)

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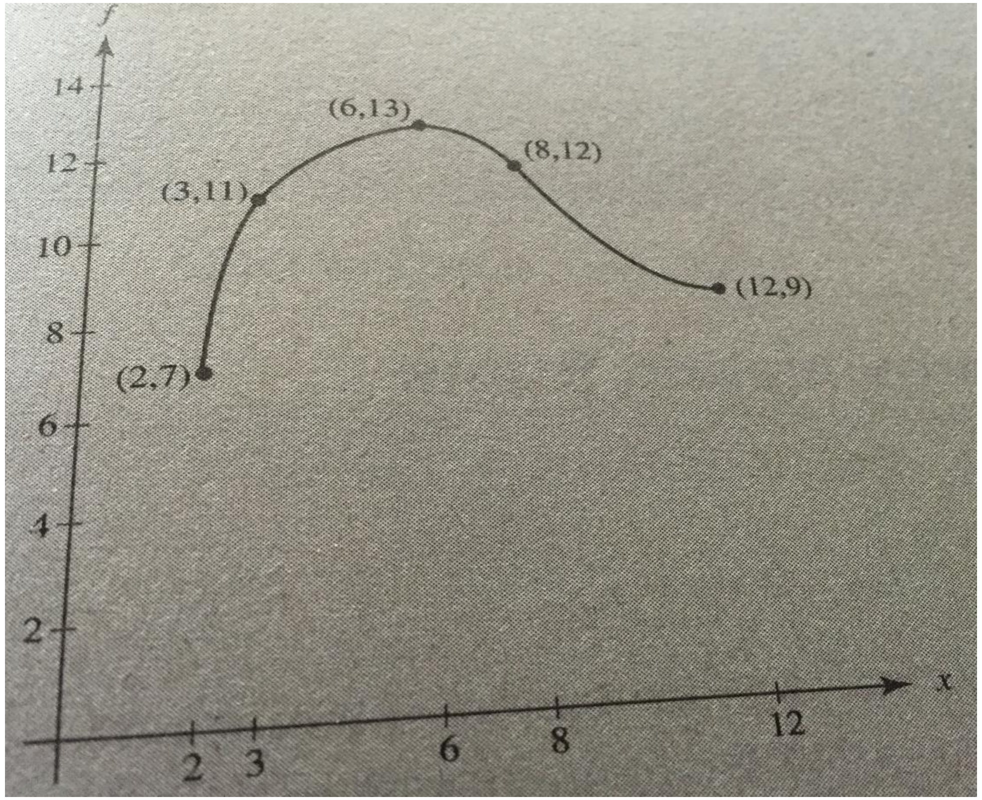

Vocabulary	Process and Connections	Answer
Trapezoid Riemann's Sum Unequal Subintervals	$\frac{1}{2}\left[\frac{3-2}{1}(f(2)+f(3))+\frac{6-3}{1}(f(3)+f(6))+\frac{8-6}{1}(f(6)+f(8))+\frac{12-8}{1}(f(8)+f(12))\right]$ $\frac{1}{2}\left[1(7+11)+3(11+13)+2(13+12)+4(12+9)\right]$ $\frac{1}{2}\left[1(18)+3(24)+2(25)+4(21)\right]$ $\frac{1}{2}(18+72+50+84)$ $\frac{1}{2}(224)=112$	C

ASSIGNMENT

Worksheet