Calculus AP/Dual Worksheet Rev 2017 **Name: ______________________________**

§4.2: Riemann's Sum **Date: __________________ Period ______**

Show all work. You can use a calculator but many of these problems can be done without it.

1) Use the table and the indicated technique to approximate $\int_0^{12} g(x)dx$

$\boldsymbol{x}$	0	1	2	3	4	5	6	7	8	9	10	11	12
$\boldsymbol{g(x)}$	7	9	11	12	14	18	21	20	18	22	25	27	26

(a) Left-Hand Riemann; 4 subintervals

(b) Right-Hand Riemann; 6 subintervals

(c) Midpoint Riemann; 2 subintervals

2) The rate at which water is being pumped into a tank is given by the function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 \leq t \leq 20$ minutes, is shown below.

t (min.)	0	4	9	17	20
$R(t)$ (gal/min)	25	28	33	42	46

(a) Use data from the table and four subintervals to find a left Riemann sum to approximate the value of $\int_0^{20} R(t)dt$. Is the answer an overestimation or underestimation?

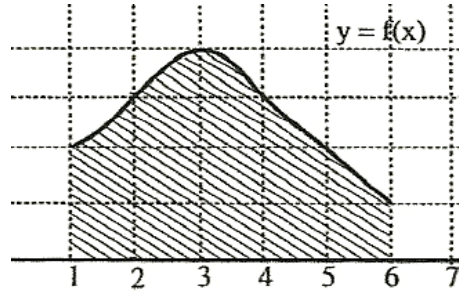
(b) Use data from the table and four subintervals to find a right Riemann sum to approximate the value of $\int_0^{20} R(t)dt$. Is the answer an overestimation or underestimation?

3) The rates $R(t)$, in cases per hour, are recorded at various times during a 10-hour period from 8:00 A.M. to 6:00 P.M. Use a Left Riemann approximation with an appropriate number of subintervals to approximate $\int_0^{10} R(t)dt$. You will need to label your answer with the appropriate units.

t	0	2	3	6	7	8	10
$R(t)$	120	115	119	120	115	112	120

4) Estimate the area bounded by the curve and x-axis on [1,6] using the 5 equal subintervals by:

(a) Left Riemann sum



(b) Right Riemann sum

(c) Midpoint Riemann sum

5) A hot cup of coffee is taken into a classroom and set on a desk to cool. When $t = 0$, the temperature of the coffee is $113°F$. The rate at which the temperature of the coffee is dropping is modeled by a differentiable function R for $0 \leq t \leq 8$, where $R(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. Values of $R(t)$ at selected values of time t are shown in the table below.

t (minutes)	0	3	5	8
$R(t)$ (°F/min.)	5.5	2.7	1.6	0.8

(a) Estimate the temperature of the coffee at $t = 8$ minutes by using a left Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

(b) Estimate the temperature of the coffee at $t = 8$ minutes by using a right Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

6) Use the graph of $f(x)$ below to approximate $\int_1^5 f(x)\,dx$. Use a Right Riemann sum and 4 subintervals for your approximation. Is the answer going to be greater or smaller than the actual value of $\int_1^5 f(x)\,dx$?