Riemann's Sum

Section 4.2

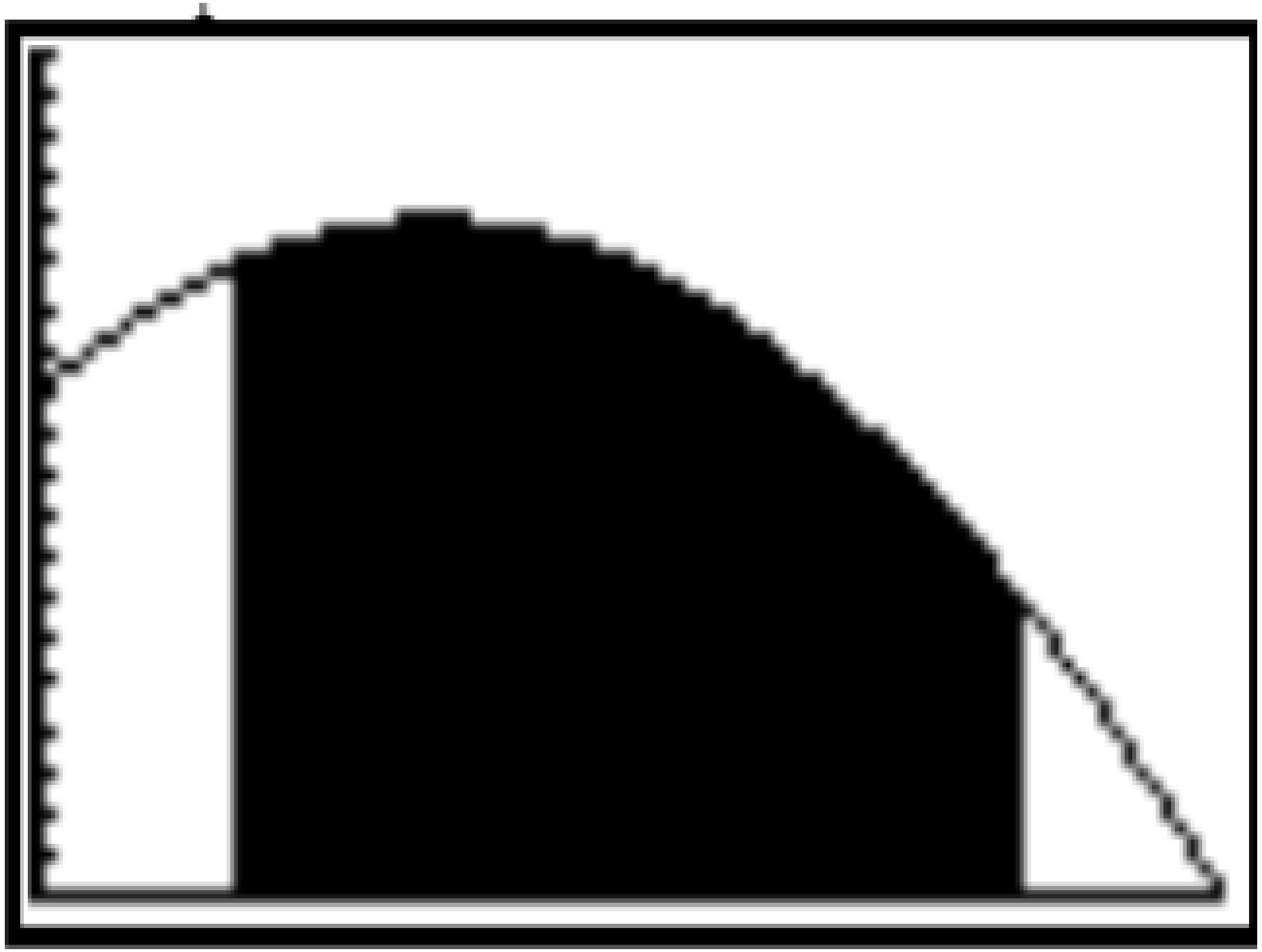
Calculus AP/Dual, Revised ©2018

viet.dang@humbleisd.net

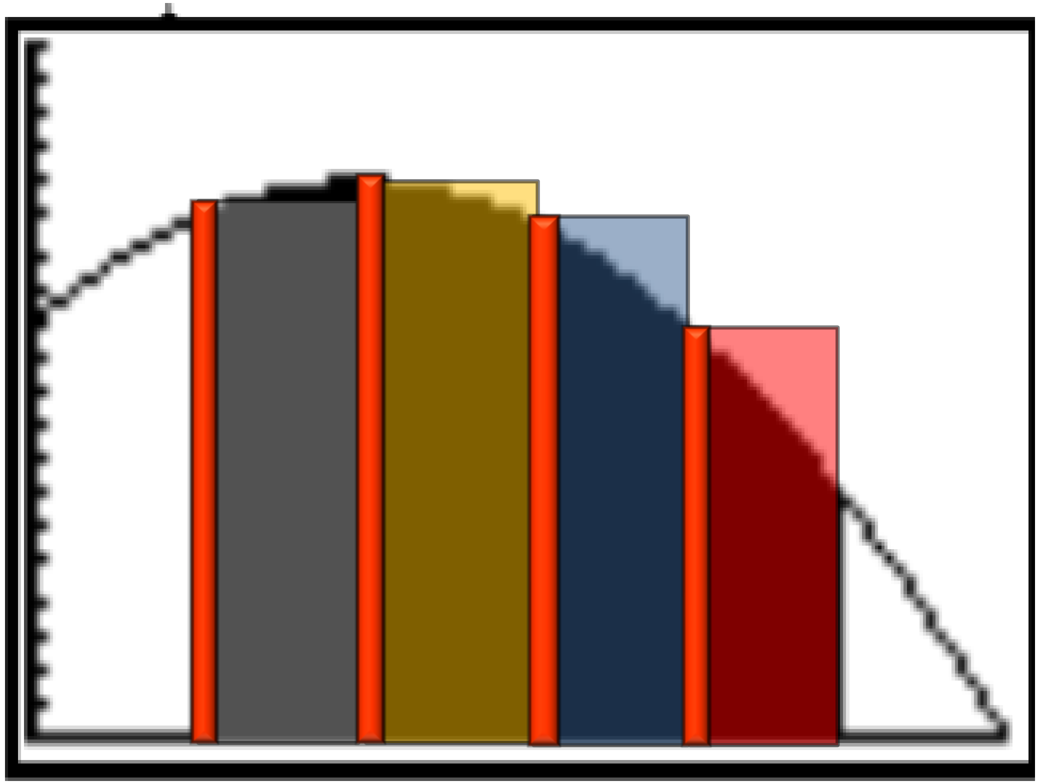
HISTORY OF BERNHARD RIEMANN

- 1826-1866
- German Mathematician and math professor
- Big on Number Theory and geometry
- Discovered an estimation of area under the curve through geometry
- As an unformal way of estimating area under the curve

How Do We Approximate the Area Under the Curve?



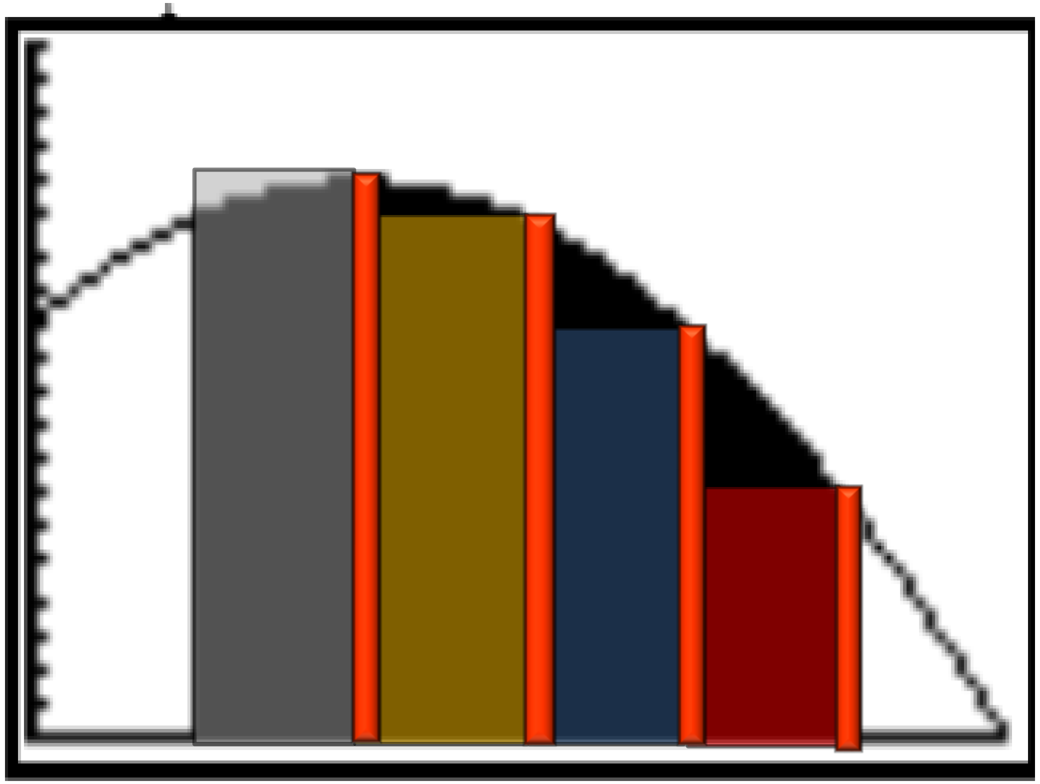
How Do We Approximate the Area Under the Curve?



Left Handed Rule

It is an overestimation

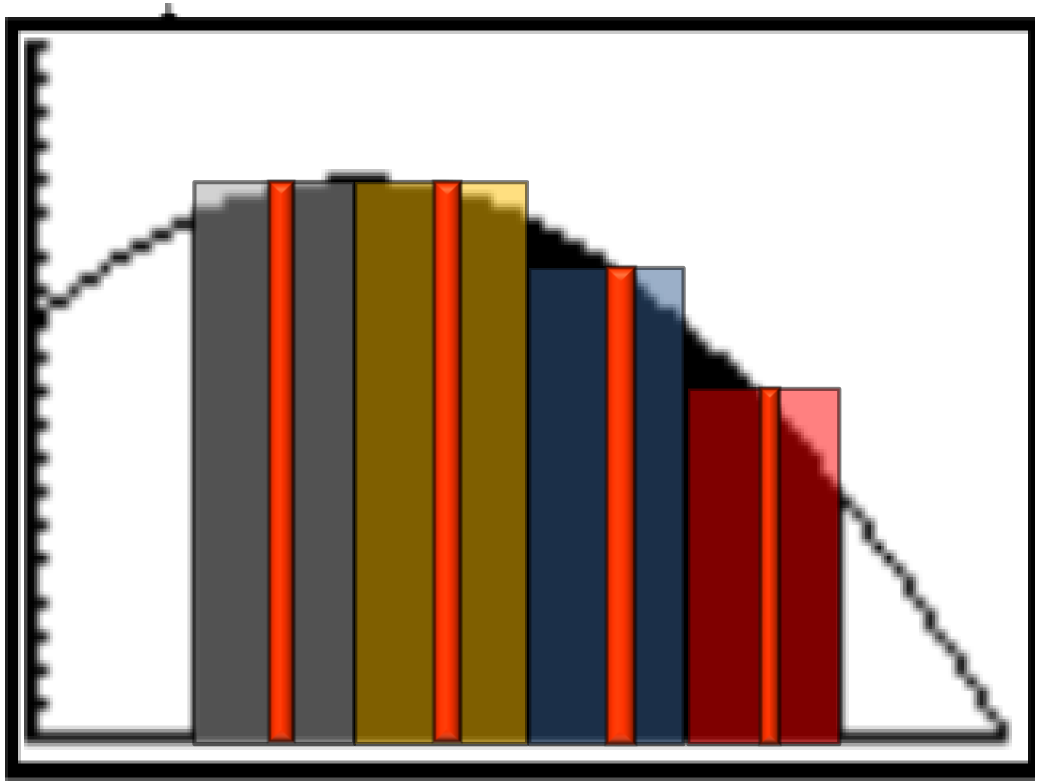
How Do We Approximate the Area Under the Curve?



Right Handed Sum

It is an underestimation

How Do We Approximate the Area Under the Curve?



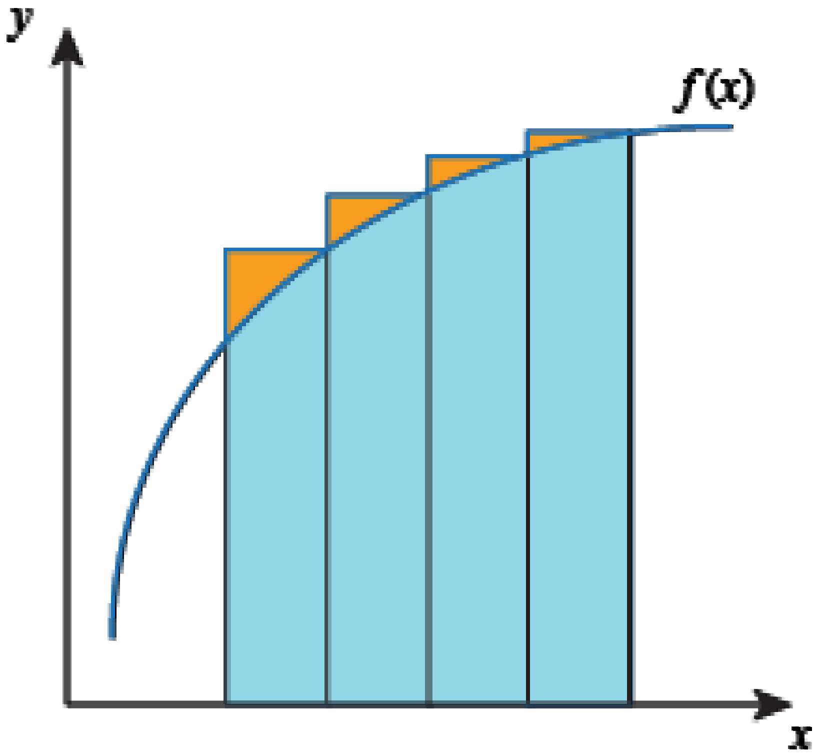
Midpoint Rule

Breakdown of Riemann's Sum

A. <u>Left side approximation</u> is the left of the sub-interval is used to determine the height of the rectangle.
B. <u>Right side approximation</u> is the right of the sub-interval is used to determine the height of the rectangle.
C. <u>Midpoint approximation</u> is the midpoint of each sub-interval us used to determine the height of each rectangle.

EXAMPLE 1

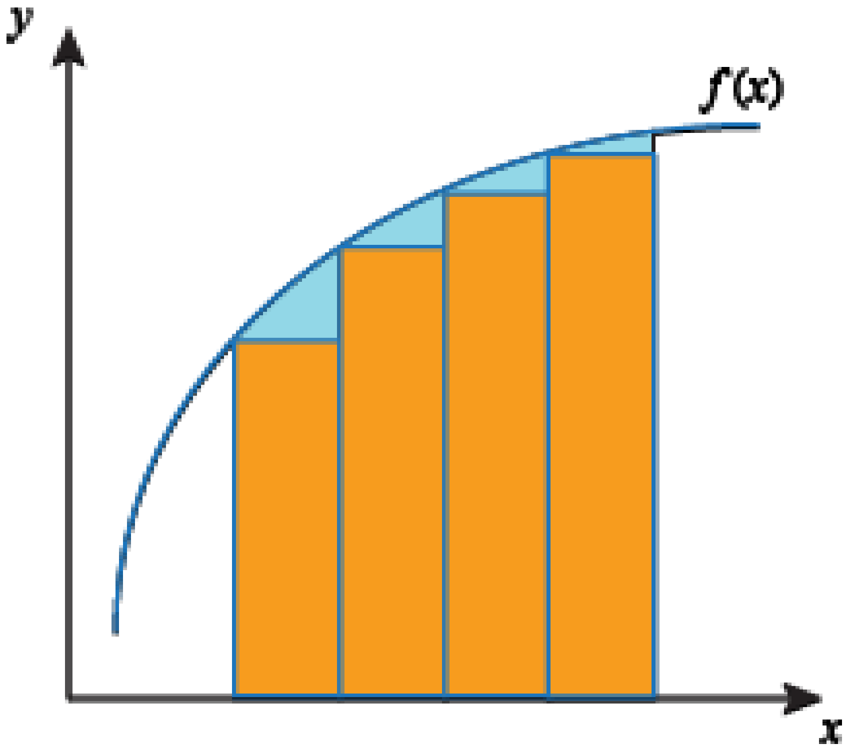
Given the graph below, is the graph of right Riemann's Sum an overestimation or underestimation? Explain the answer.



The right Riemann's Sum is an overestimation since $f(x)$ is an increasing function.

EXAMPLE 2

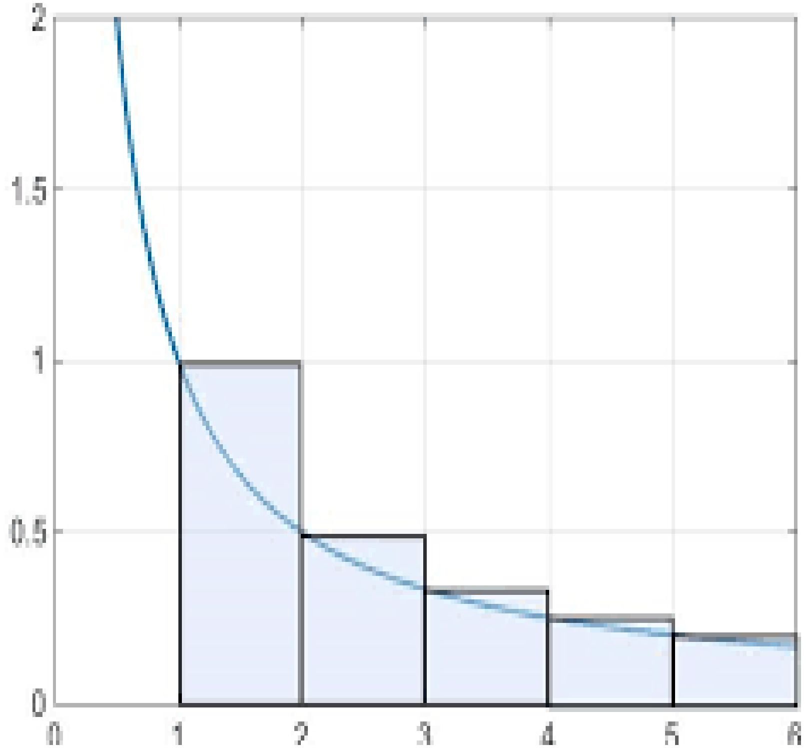
Given the graph below, is the graph of left Riemann's Sum an overestimation or underestimation? Explain the answer.



The left Riemann's Sum is an underestimation since $f(x)$ is an increasing function.

EXAMPLE 3

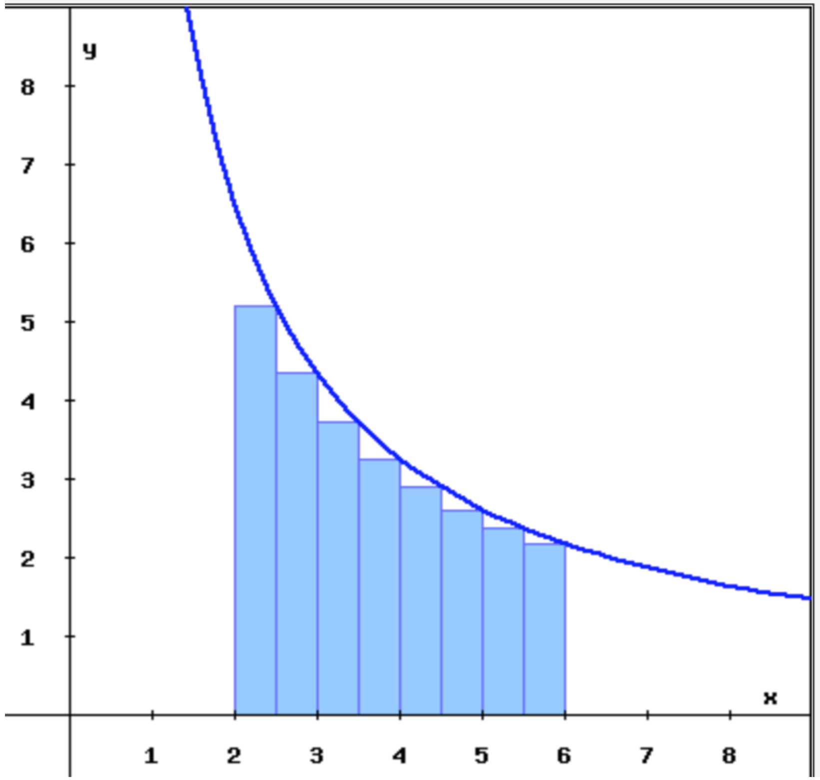
Given the graph $f(x)$ below, is the graph of left Riemann's Sum an overestimation or underestimation? Explain the answer.



The left Riemann's Sum is an overestimation since $f(x)$ is a decreasing function.

YOUR TURN

Given the graph below, is the graph of right Riemann's Sum an overestimation or underestimation? Explain the answer.



The right Riemann's Sum is an underestimation since $f(x)$ is an decreasing function.

RIEMANN'S SUM

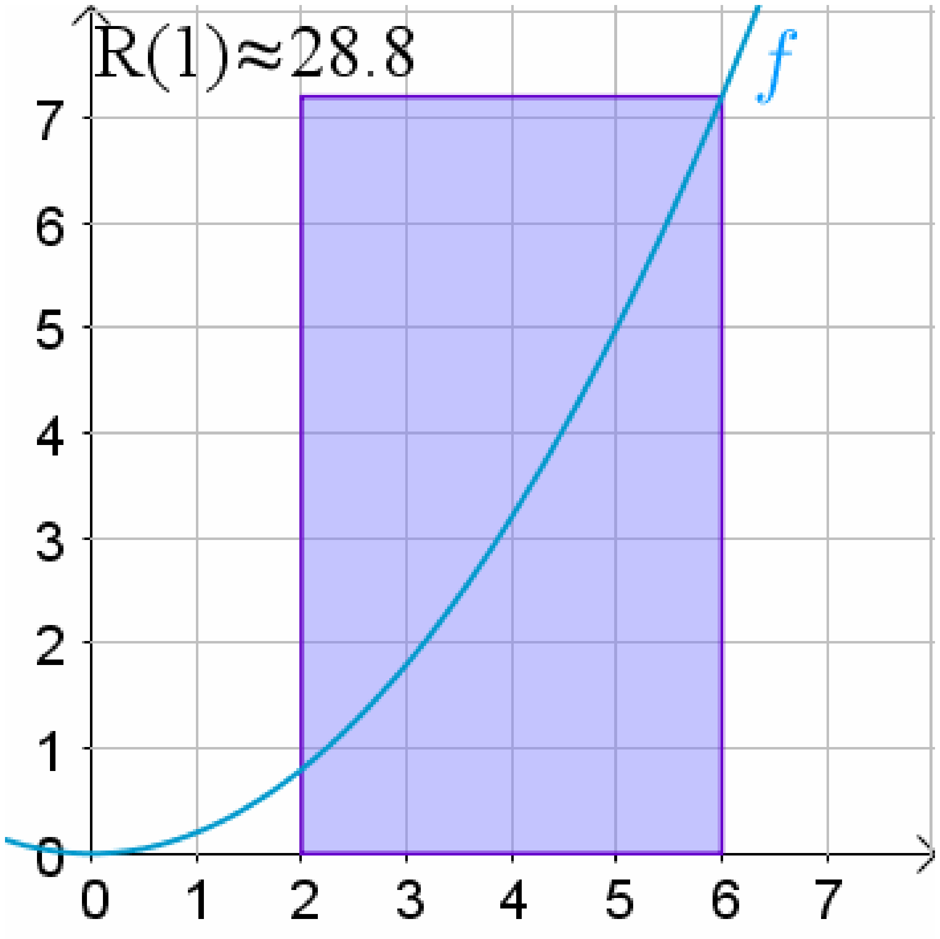
A. In mathematics, a Riemann sum is a method for <u>APPPROXIMATION the total area underneath a curve on a graph</u>

B. If f is closed on the interval from $[a, b]$, Then f in integrable on $[a, b]$

1. Where $f(x_K)$ is the height of the function at the value, xi
2. Partitions: $\Delta x = \frac{(b-a)}{n}$
3. $x_K = (\Delta x) f(x_K)$

File.

VISUAL



STEPS

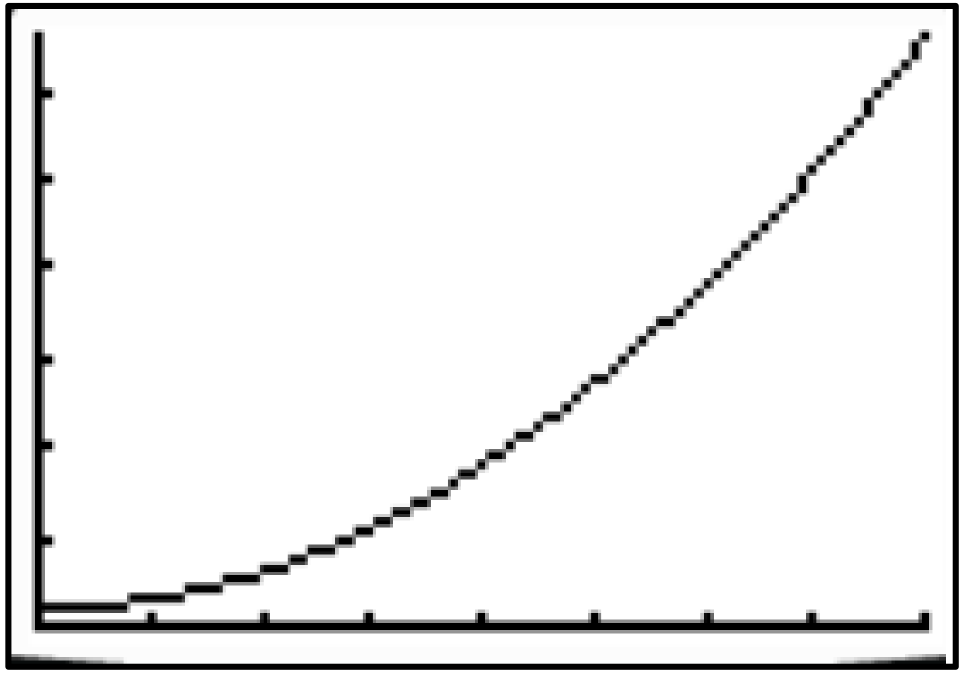
A. Identify the missing parts of $[a, b]$ and the amount of subintervals, n

B. Establish the amount of rectangles width (partitions) used to approximating an integral

C. Identify which summation is asked and apply the equation and repeat the process

EXAMPLE 4

Calculate the left & right side sum and midpoint sum for $y = x^2 + 2$ with 2 equal intervals from $[0, 8]$.



$a = 0$

$b = 8$

$n = 2$

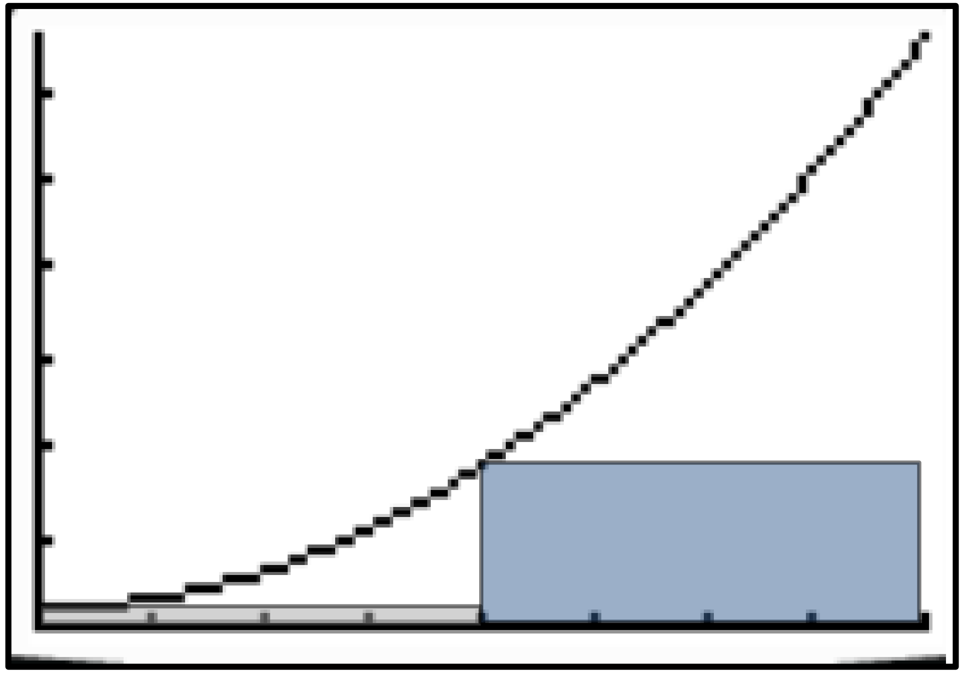
$\Delta x = 4$

Establish the TWO Intervals:

$$(0,4),(4,8)$$

EXAMPLE 4

Calculate the left & right side sum and midpoint sum for $y = x^2 + 2$ with 2 equal intervals from $[0, 8]$.



Establish the TWO LEFT Intervals:

$$(0, 4), (4, 8)$$

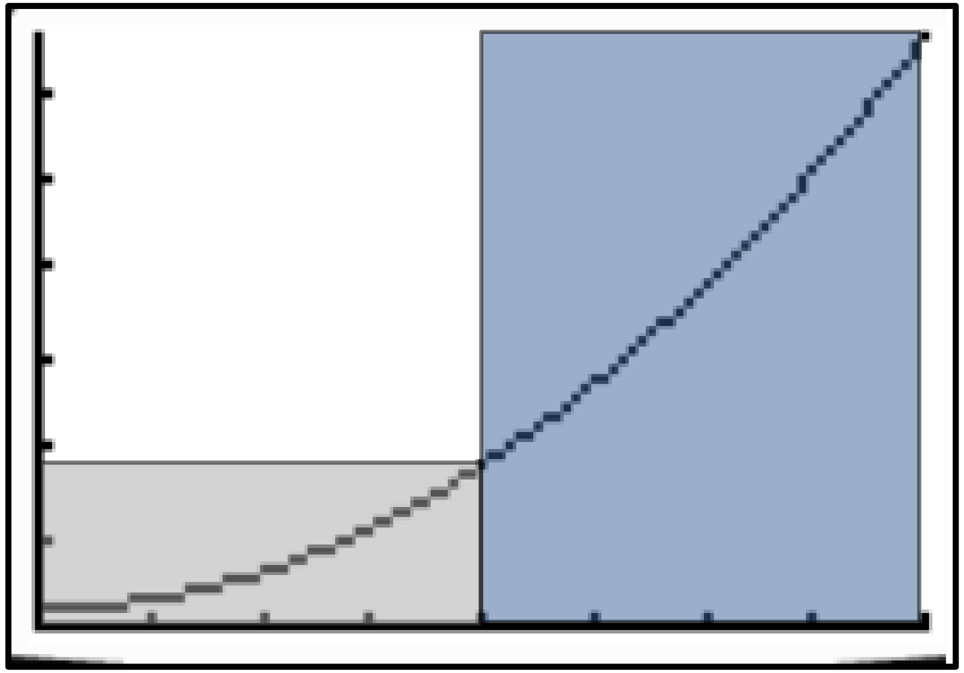
$$(4)\left[f(0) + f(4) \right]$$

$$(4)[2 + 18]$$

$$LRS = 80 \text{ units}^2$$

EXAMPLE 4

Calculate the left & right side sum and midpoint sum for $y = x^2 + 2$ with 2 equal intervals from $[0, 8]$.



Establish the TWO RIGHT Intervals:

$$(0,\boxed{4}),(4,\boxed{8})$$

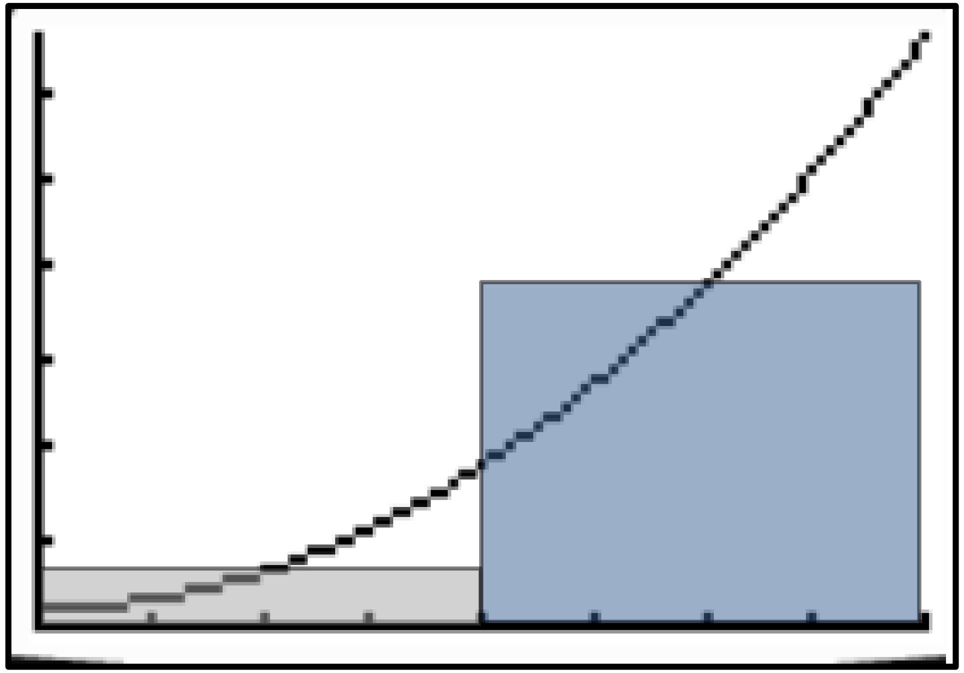
$$(4)\left[f(4)+f(8)\right]$$

$$(4)[18+66]$$

$$RRS = 336\,\text{units}^2$$

EXAMPLE 4

Calculate the left & right side sum and midpoint sum for $y = x^2 + 2$ with 2 equal intervals from $[0, 8]$.



Establish the TWO MIDPOINT Intervals:

$$(0,4),(4,8)$$

$$(4)\left[f(2)+f(6)\right]$$

$$(4)[6+38]$$

$$MPS = 176\,\text{units}^2$$

Example 4

Calculate the left & right side sum and midpoint sum for $y = x^2 + 2$ with 2 equal intervals from $[0, 8]$.

Left Side Sum of the Interval

$$LRS = 80\,\text{units}^2$$

Right Side Sum of the Interval

$$RRS = 336\,\text{units}^2$$

Midpoint Sum of the Interval

$$MPS = 176\,\text{units}^2$$

$$\int_0^8 \left(x^2 + 2\right) dx = 186.6667\,\text{units}^2$$

Example 5

Calculate the left, right, and midpoint sum for $\int_0^2 x^2 + 1\, dx$ with 4 equal intervals from $[0, 2]$.

$$\Delta x = \frac{(2)-(0)}{(4)}$$

$$a = 0$$

$$b = 2$$

$$n = 4$$

$$\Delta x = \frac{1}{2}$$

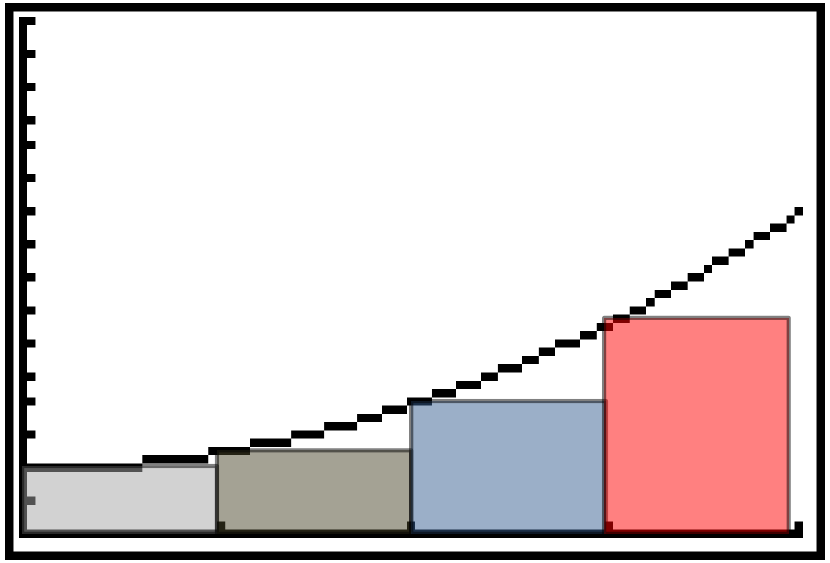
Establish the Four Intervals: $\left(0, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \left(1, \frac{3}{2}\right), \left(\frac{3}{2}, 2\right)$

EXAMPLE 5A

Calculate the left, right, and midpoint sum for $\int_0^2 x^2 + 1\, dx$ with 4 equal intervals from $[0, 2]$.

$$\left(0,\frac{1}{2}\right),\left(\frac{1}{2},1\right),\left(1,\frac{3}{2}\right),\left(\frac{3}{2},2\right)$$

Left Hand Sum of the Interval



$$\frac{1}{2}\left[f(0)+f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)\right]$$

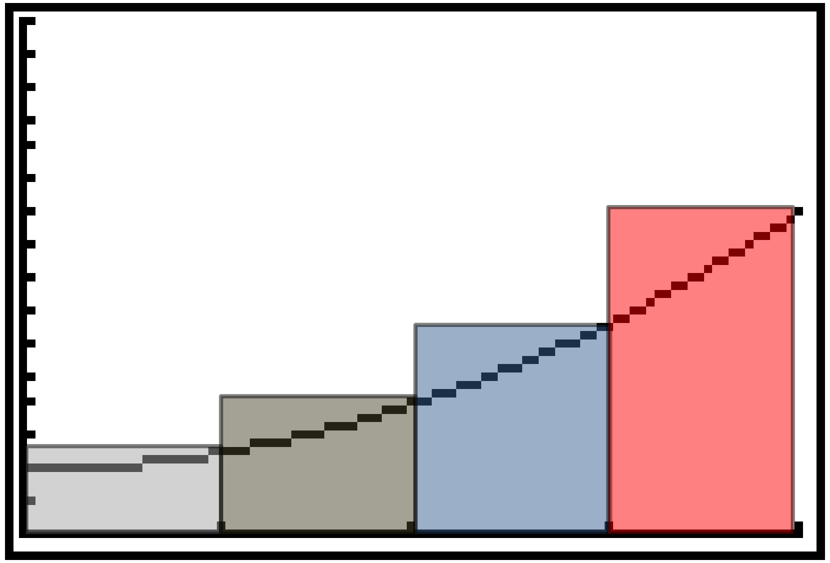
$$\frac{1}{2}\left[1+\frac{5}{4}+2+\frac{13}{4}\right]$$

$$LRS = 3.75\,\text{units}^2$$

Example 5b

Calculate the left, right, and midpoint sum for $\int_0^2 x^2 + 1\, dx$ with 4 equal intervals from $[0, 2]$.

$$\left(0,\frac{1}{2}\right),\left(\frac{1}{2},1\right),\left(1,\frac{3}{2}\right),\left(\frac{3}{2},2\right)$$



Right Hand Sum of the Interval

$$\frac{1}{2}\left[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)\right]$$

$$\frac{1}{2}\left[\frac{5}{4}+2+\frac{13}{4}+5\right]$$

$$RRS = 5.75\,\text{units}^2$$

EXAMPLE 5C

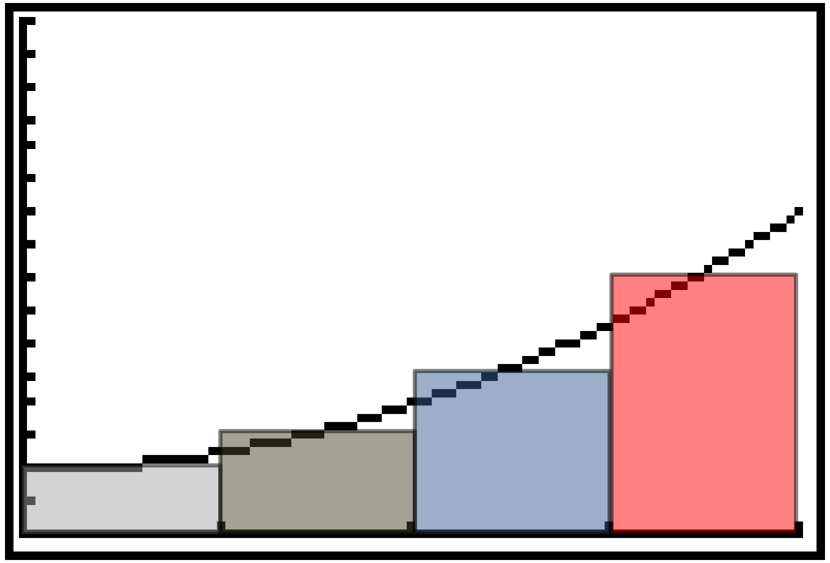
Calculate the left, right, and midpoint sum for $\int_0^2 x^2+1\,dx$ with 4 equal intervals from $[0,2]$.

$$\left(0,\frac{1}{2}\right),\left(\frac{1}{2},1\right),\left(1,\frac{3}{2}\right),\left(\frac{3}{2},2\right)$$

Midpoint Sum of the Interval

$$\frac{1}{2}\left[f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)+f\left(\frac{5}{4}\right)+f\left(\frac{7}{4}\right)\right]$$

$$\frac{1}{2}\left[\frac{17}{16}+\frac{25}{16}+\frac{41}{16}+\frac{65}{16}\right]$$



$$MPS = 4.625\,\text{units}^2$$

EXAMPLE 5

Calculate the left, right, and midpoint sum for $\int_0^2 x^2 + 1\, dx$ with 4 equal intervals from $[0, 2]$.

Left Hand Sum of the Interval

$$LHS = 3.75\,\text{units}^2$$

Right Hand Sum of the Interval

$$RHS = 5.75\,\text{units}^2$$

Midpoint Sum of the Interval

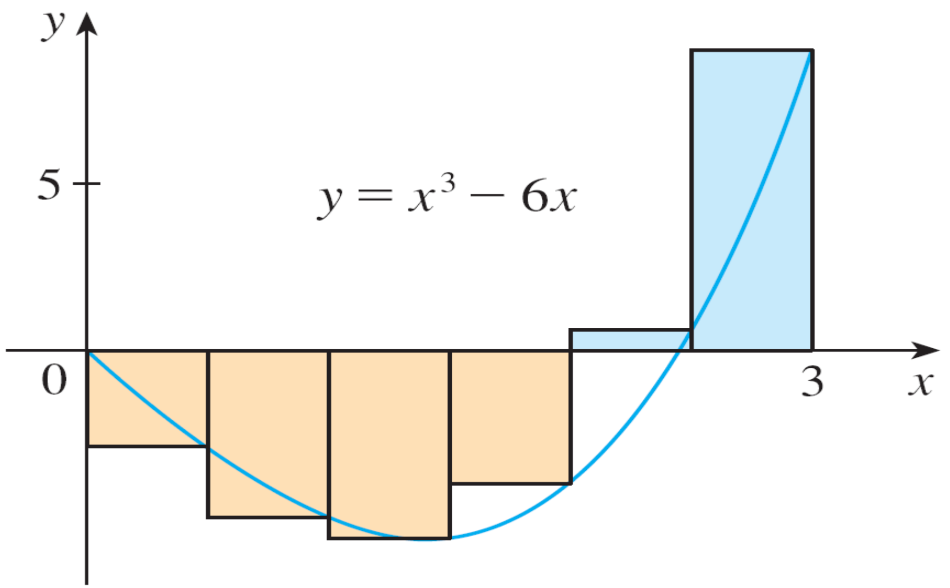
$$MPS = 4.625\,\text{units}^2$$

$$\int_0^2 \left(x^2 + 1\right) dx$$

$$= 4.667\,\text{units}^2$$

EXAMPLE 6

Evaluate the Riemann sum for $f(x) = x^3 - 6x$, taking the sample points to be right endpoints and $a = 0$, $b = 3$, and $n = 6$.



Example 6

Evaluate the Riemann sum for $f(x) = x^3 - 6x$, taking the sample points to be right endpoints and $a = 0$, $b = 3$, and $n = 6$.

$$a = 0$$

$$b = 3$$

$$n = 6$$

$$\Delta x = \frac{(3)-(0)}{(6)}$$

$$\Delta x = \frac{1}{2}$$

Establish the Intervals: $\left(0, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \left(1, \frac{3}{2}\right), \left(\frac{3}{2}, 2\right), \left(2, \frac{5}{2}\right), \left(\frac{5}{2}, 3\right)$

Example 6

Evaluate the Riemann sum for $f(x) = x^3 - 6x$, taking the sample points to be right endpoints and $a = 0$, $b = 3$, and $n = 6$.

Establish the Intervals: $\left(0,\frac{1}{2}\right),\left(\frac{1}{2},1\right),\left(1,\frac{3}{2}\right),\left(\frac{3}{2},2\right),\left(2,\frac{5}{2}\right),\left(\frac{5}{2},3\right)$

Right Hand Sum of the Interval: $\frac{1}{2}\left[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)+f\left(\frac{5}{2}\right)+f(3)\right]$

$$\frac{1}{2}\left[-2.875-5-5.625-4+0.625+9\right]$$

$$RRS = -3.9375\,\text{units}^2$$

YOUR TURN

Calculate the left, right, and midpoint sum for $\int_0^4 x^3\, dx$ with 2 equal intervals from $[0, 4]$.

$LRS = 16\,\text{units}^2$

$RRS = 144\,\text{units}^2$

$MPS = 56\,\text{units}^2$

EXAMPLE 7

The function f is continuous on the closed interval $[2, 14]$ and has the values as shown in the table below. Using the subintervals $[2, 5], [5, 10]$ and $[10, 14]$, what is the approximation of $\int_2^{14} f(x)\,dx$ found using the right Riemann Sum?

x	2	5	10	14
$f(x)$	12	28	34	30

$$\left(\frac{5-2}{1}\right)(28)+\left(\frac{10-5}{1}\right)(34)+\left(\frac{14-10}{1}\right)(30)$$

$$374\,\text{units}^2$$

Your Turn

The function f is continuous on the closed interval $[0, 10]$ and has the values as shown in the table below. What is the approximation of $\int_0^{10} f(x)dx$ found using the left Riemann Sum?

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

37 units^2

AP Multiple Choice Practice Question 1 (non-calculator)

Selected values of the continuous function, $f(x)$ are given in the table. Using 4 subintervals of Left Riemann's Sum, determine the approximation for $\int_0^{100} f(x)\,dx$

x	0	40	70	90	100
$f(x)$	150	180	195	184	172

(A) $17,140$

(B) $17,795$

(C) $18,425$

(D) $18,450$

AP Multiple Choice Practice Question 1 (non-calculator)

Selected values of the continuous function, $f(x)$ are given in the table. Using 4 subintervals of Left Riemann's Sum, determine the approximation for $\int_0^{100} f(x)\,dx$

x	0	40	70	90	100
$f(x)$	150	180	195	184	172

Vocabulary	Process and Connections	Answer and Justifications
Left Riemann's Sum Unequal Subintervals	$\frac{40-0}{1}[f(0)]+\frac{70-40}{1}[f(40)]+\frac{90-70}{1}[f(70)]+\frac{100-90}{1}[f(100)]$ $40(150)+30(180)+20(195)+10(184)$ $40(150)+30(180)+20(195)+10(184)$ $6000+5400+3900+1840$ $17,140$	A

Assignment

Worksheet