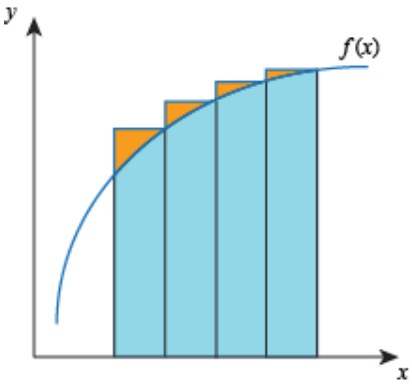
§4.2: Riemann's Sum
"I WILL …
…approximate and use the limit process using Riemann's Sum."

I. Approximations
   A. _____ approximation is the left of the sub-interval for height of rectangle.
   B. _____ approximation is the right of the sub-interval for height of rectangle.
   C. _____ approximation is the midpoint of each sub-interval for height of the rectangle.

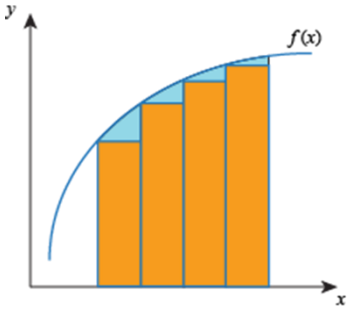
II. Breakdown of Riemann's Sum
   A. In mathematics, a Riemann sum is a method for approximating the total area underneath a curve on a graph
   B. If f is closed on the interval from $[a, b]$, then f in integrable on $[a, b]$ and the limit is denoted by:
      1. Where $f(x_i)$ is the height of the function at the value, x_i
      2. Partitions: $\Delta x =$ _____
      3. $x_i =$ _____

III. Steps
   A. Identify the missing parts of $[a, b]$ and the number of subintervals, n
   B. Establish the amount of rectangles width (partitions) used to approximating an integral
   C. Identify which summation is asked and apply the equation and repeat the process

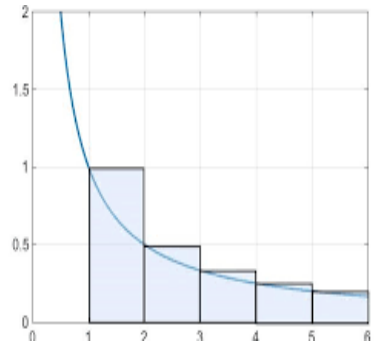
Ex 1: Given the graph below, is the graph of right Riemann's Sum an overestimation or underestimation? Explain the answer.



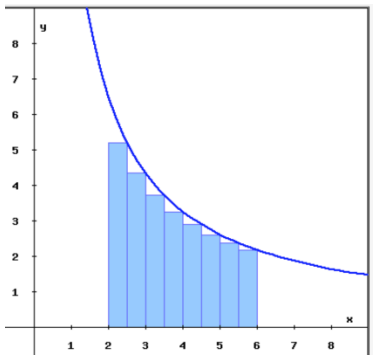
Ex 2: Given the graph below, is the graph of left Riemann's Sum an overestimation or underestimation? Explain the answer.



Ex 3: Given the graph $f(x)$ below, is the graph of left Riemann's Sum an overestimation or underestimation? Explain the answer.



Your Turn: Given the graph below, is the graph of right Riemann's Sum an overestimation or underestimation? Explain the answer.



Ex 4: Calculate the left & right end sum and midpoint sum for $y = x^2 + 2$ with 2 equal intervals from $[0,\ 8]$.

Ex 5: Calculate the left, right, and midpoint sum for $\int_0^2 x^2 + 1\ dx$ with 4 equal intervals from $[0,2]$.

Ex 6: Evaluate the Riemann sum for $f\,(x) = x^3 - 6x$, taking the sample points to be right endpoints and $a = 0$, $b = 3$, and $n = 6$.

Your Turn: Calculate the left, right, and midpoint sum for $\int_0^4 x^3\,dx$ with 2 equal intervals from $[0,\ 4]$.

Ex 7: The function f is continuous on the closed interval $[2,\ 14]$ and has the values as shown in the table below. Using the subintervals $[2,\ 5]$, $[5,\ 10]$ and $[10,\ 14]$, what is the approximation of $\int_2^{14} f\,(x)\,dx$ found using the right Riemann Sum?

x	2	5	10	14
$f(x)$	12	28	34	30

Your Turn: The function f is continuous on the closed interval $[0,\ 10]$ and has the values as shown in the table below. What is the approximation of $\int_0^{10} f\,(x)dx$ found using the left Riemann Sum?

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

AP 1) Selected values of the continuous function, $f(x)$ are given in the table. Using 4 subintervals of Left Riemann's Sum, determine the approximation for $\int_0^{100} f(x)\,dx$

x	0	40	70	90	100
$f(x)$	150	180	195	184	172

(A) 17,140 (B) 17,795 (C) 18,425 (D) 18,450

Vocabulary	Process and Connections	Answer and Justifications

Assignment: Worksheet