

# ARC LENGTH AND SECTOR AREA

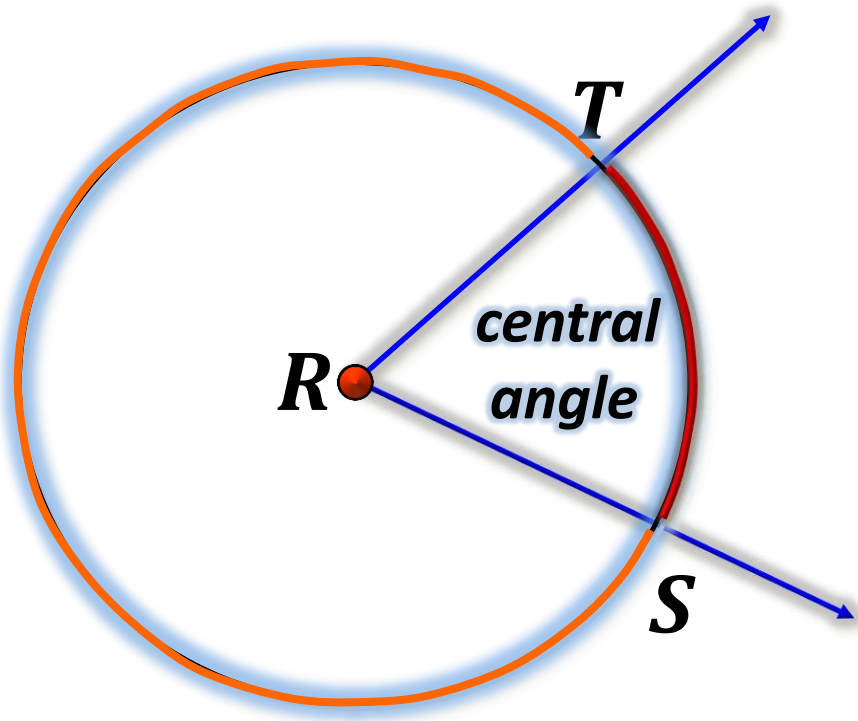
Section 4.1A

Precalculus PreAP/Dual, Revised ©2017

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# ARC LENGTH

How do we find the arc length? What is circumference of a circle?



$$C = (2\pi) r$$

$$C = \left( \frac{\theta}{2\pi} \right) r$$

$$S = \theta r$$

$$S = r\theta$$

# ARC LENGTH

- A. Sector is a region of the circle that bounded by two radii and an arc of a circle
- B. The Central Angle of a sector is the angle formed by the two radii
- C. Arc Length equation:  $s = r\theta$
- D. Degrees must be converted to Radians
- E. Do NOT forget the units

# EXAMPLE 1

Determine the Arc Length of a circle with the given radius of  $r = 4$  inches and  $\theta = \frac{\pi}{6}$ .

*Arc Length*

$$s = r\theta$$

$$s = (4)\left(\frac{\pi}{6}\right)$$

$$s = \frac{2\pi}{3} \text{ in}$$

## EXAMPLE 2

The second hand on a clock is 6 inches long. How far does the tip of the second hand move in 15 seconds? 1 full rotation is 60 seconds. Round to 4 decimal places.

*Arc Length*

$$s = r\theta$$

$$\theta = \frac{\text{minutes}}{\text{hour}} (2\pi)$$

$$\theta = \left( \frac{15}{60} \right) (2\pi)$$

## EXAMPLE 2

The second hand on a clock is 6 inches long. How far does the tip of the second hand move in 15 seconds? 1 full rotation is 60 seconds.

Round to 4 decimal places.  $\theta = \frac{\pi}{2}$

$$s = r\theta$$

$$s = (6)\left(\frac{\pi}{2}\right)$$

$$s = 3\pi$$

$$s = 9.4247in$$

# YOUR TURN

Determine the Arc Length of a circle with the given radius of  $r = 4$  inches and  $\theta = 240^\circ$ . Make sure to convert degrees to radians.

$$s = r\theta$$

$$s = (4) \left( \frac{\pi}{180} \right) (240^\circ)$$

$$s = (4) \left( \frac{4\pi}{3} \right)$$

$$s = \frac{16\pi}{3} \text{ in}$$

# AREA OF A SECTOR AREA

- A. Sector is a circle of the region bounded by two radii of the circle and their intercepted arc
- B. Equation:  $A = \frac{1}{2} r^2 \theta$  where radius is the base and  $\theta$  is the height
- C. Always leave answers in radian mode



# AREA OF A SECTOR

$$A = \frac{1}{2} r^2 \theta$$

$$C = 2\pi r$$

$$A = \pi \left( \frac{\theta}{360} \right) r^2$$

$$A = \pi \left( \frac{\theta}{2\pi} \right) r^2$$

$$A = \frac{1}{2} \theta r^2$$

## EXAMPLE 3

A sprinkler on a golf course fairway sprays water over a distance of 70 feet and rotates through an angle of  $120^\circ$ . Find the area of the fairway watered by the sprinkler.

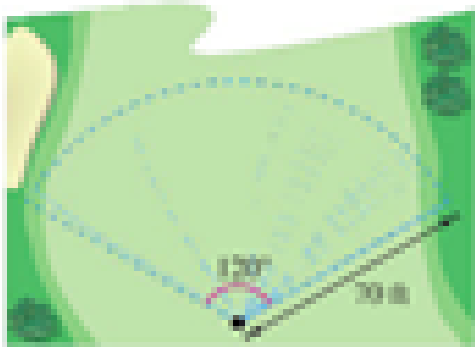


Figure 4.56

$$\theta = 120^\circ$$

$$A = \frac{1}{2} r^2 \theta$$

$$\theta = (120^\circ) \left( \frac{\pi}{180^\circ} \right) = \frac{2\pi}{3}$$

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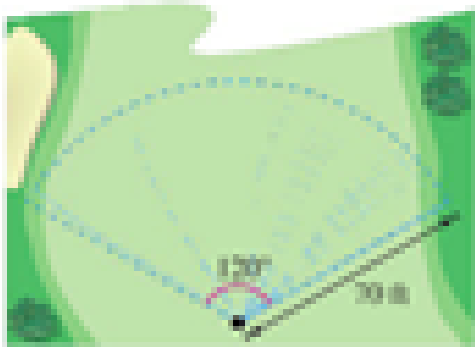


Figure 4.56

$$\theta = (120^\circ) \left( \frac{\pi}{180^\circ} \right) = \frac{2\pi}{3}$$

$$A = \frac{1}{2} (70)^2 \left( \frac{2\pi}{3} \right)$$

$$A = \frac{4900\pi}{3} \text{ feet}^2$$

## EXAMPLE 4

A sector of a circle of radius 24 m has an area of  $288 \text{ m}^2$ . Find the central angle of this sector.

$$\theta = 1 \text{ radian}$$



# YOUR TURN

**A sector of a circle has a central angle of  $60^\circ$ . Find the area of the sector if the radius of the circle is 3 miles.**

$$\frac{3\pi}{2} \text{ miles}^2$$

# ASSIGNMENT

## Worksheet