

§3.7: Optimization

Show all work on a separate sheet of paper. Show all steps and equations. Draw a diagram if possible, label appropriately and justify response.

- 1) The product of 192 and the sum of the first plus three times the second is a minimum. What are the numbers?

- 2) The sum of the first and twice the second is 100 and the product is a maximum. What are the numbers?

- 3) Find the maximum area of rectangle where the length and width have a given perimeter of 100 meters.

- 4) Find the minimum perimeter of rectangle where the length and width have a given area of 64 meters.

- 5) Find the shortest distance from $f(x) = x^2$ to the point $(2, \frac{1}{2})$.

- 6) Which points of the graph of $y = 4 - x^2$ are closest to the point of $(0, 2)$?

- 7) Find the points of the hyperbola $y^2 - x^2 = 4$ that are closest to the point $(2, 0)$.

- 8) Find the dimensions of the largest rectangle that can be inscribed in a semicircle of radius of 3.

- 9) Gary the Gardener wants to make a rectangular enclosure using a wall as one side and 120 meters of fencing for the other three sides. Express the area in terms of x and find the value of x that offers the greatest area.

- 10) A rectangle has a perimeter of 80 cm. If its width is x , express the length and area in terms of x , and find the maximum area.

- 11) A man is going to use 600 yards of fencing to enclose and subdivide a rectangular field into two plots with a fence parallel to a side. Of all the possible fields that can be fenced, what are the dimensions of the one of maximum area?
- 12) If 350 feet of fencing will be used to enclosed two adjacent rectangular pens. What dimensions should be used so that enclosed area will be a maximum? Then prove it is a maximum.
- 13) A rectangular page is to contain 30 square inches of print. The margins of each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used. Then prove it is a minimum.
- 14) You are designing a poster to contain 50 square inches of printing with margins of 4 inches each at the top and bottom and 2 inches at each side. What overall dimensions will minimize the amount of paper used? Then prove it is a minimum.
- 15) Midas Muffler charges \$28 to replace a muffler. At this rate, this company replaces 75,000 mufflers per week nationally. For each additional dollar that the company charges, it tends to lose 1,000 customers a week. For each dollar, the company subtracts from the \$28, the company gains 1,000 per week. How much should Midas charge to change a muffler to maximize their revenue? What would that revenue be?
- 16) Real estate office handles 50 apartment units. When the rent is \$540 per month, all units are occupied. However, on the average, for each \$30 increase in rent, one unit becomes vacant. Each occupied unit requires an average of \$36 per month for service and repairs. What rent should be charged to realize the most profit?

Answer Key:

- 1) (24,8) 2) (50,25) 3) (25,25) 4) (8 meters,8 meters)
- 5) $d = \frac{\sqrt{5}}{2}$ 6) $\left(\pm\sqrt{\frac{3}{2}}, \frac{5}{2}\right)$ 7) $(1, \sqrt{5})$ 8) $\frac{3}{\sqrt{2}} \times \frac{6}{\sqrt{2}}$
- 9) $x = 30$ feet 10) $A = 400 \text{ ft}^2$ 11) $A = 100 \times 150 \text{ yards}$ 12) $A = (58.333 \text{ ft.}, 43.75 \text{ ft.})$
- 13) $\left(\pm\sqrt{30} + 2", \frac{30}{\sqrt{30}} + 2"\right)$ 14) (18,9) 15) \$51.50 16) Rent: \$1050 per student