Optimization

Section 3.7A

Calculus AP/Dual, Revised ©2017

viet.dang@humbleisd.net

STEPS

A. Read the problem carefully, and identify what's given and what you need to find.
B. Organize the info: draw a diagram, construct a table, etc.
C. Identify the unknown variables; add to diagram or table.
D. Write at least two equations to relate the given and find.
    A. Constraint Equation
    B. Optimization equation in terms of one variable
E. Find the Derivative and Critical Numbers. Think of OPTIMUS PRIME
F. Test the critical numbers for max or min, using 1st derivative or 2nd derivative test, and state solution.
G. Check the solution: Is “to find” found? Does solution make sense? Do numbers fit?

Example 1

Find two positive numbers whose sum is 20 and whose product is as large as possible.

$$x = First$$

$$y = Second$$

$$\begin{cases} y = 20 - x \\ P = xy \end{cases}$$

$$x + y = 20$$

$$y = 20 - x$$

$$P = xy$$

$$P = x(20 - x)$$

$$P = 20x - x^2$$

$$P' = 20 - 2x = 0$$

$$20 = 2x$$

$$x = 10$$

$$y = 20 - x$$

$$y = 20 - (10)$$

$$y = 10$$

The two positive numbers are x=10 and y =10

YOUR TURN

Find two positive numbers such that the sum of the first and twice the second is 64 and whose product is a maximum.

$$x = First$$

$$y = Second$$

$$\begin{cases} x = 64 - 2y \\ P = xy \end{cases}$$

$$x + 2y = 64$$

$$x = 64 - 2y$$

$$P = xy$$

$$P = (64 - 2y)\,y$$

$$P = 64y - 2y^2$$

$$P' = 64 - 4y = 0$$

$$4y = 64$$

$$x = 32$$

$$y = 16$$

$$x = 64 - 2y$$

$$x = 64 - (32)$$

The two positive numbers are x=32 and y =16

Example 2

A rectangular plot is to be bounded on one side by a straight river and enclosed on the other three sides by a fence. With $800\ m$ of fencing at your disposal, what the largest area you can enclose?

$$x = Length$$

$$y = Width$$

$$\begin{cases} x = 800 - 2y \\ A = xy \end{cases}$$

$$x + 2y = 800$$

$$x = 800 - 2y$$

$$A = xy$$

$$A = (800 - 2y)\,y$$

$$A = 800y - 2y^2$$

$$A' = 800 - 4y = 0$$

$$A' = 4y = 800$$

$$y = 200$$

EXAMPLE 2

A rectangular plot is to be bounded on one side by a straight river and enclosed on the other three sides by a fence. With $800\ m$ of fencing at your disposal, what the largest area you can enclose?

$$x = Length$$

$$y = Width$$

$$\begin{cases} x = 800 - 2y \\ A = xy \end{cases}$$

$$x + 2y = 800$$

$$x = 800 - 2y$$

$$y = 200$$

$$x = 800 - 2y$$

$$x = 800 - 2(200)$$

$$x = 400$$

The maximum dimension of a rectangle plot is 400×200 meters

YOUR TURN

Find the dimensions of a rectangle having maximum area and a perimeter of $36\ in$.

$$x = First$$

$$y = Second$$

$$\begin{cases} y = 18 - x \\ A = xy \end{cases}$$

$$2x + 2y = 36$$

$$y = 18 - x$$

$$A = xy$$

$$A = x(18 - x)$$

$$A = 18x - x^2$$

$$A' = 18 - 2x = 0$$

$$x = 9$$

$$y = 18 - x$$

$$y = 18 - (9)$$

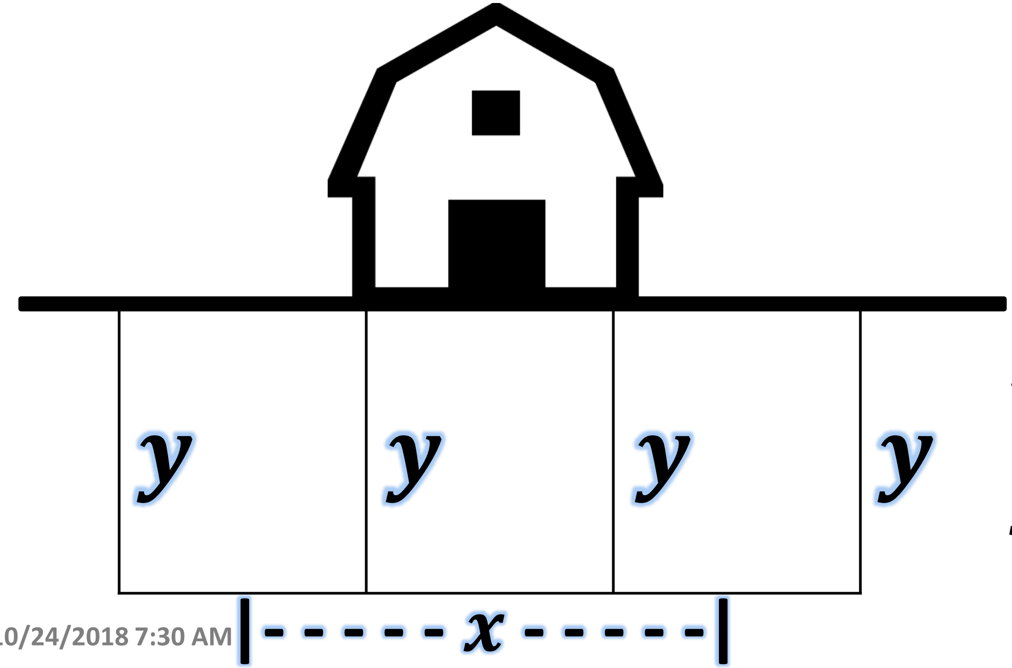
$$y = 9$$

The maximum dimension of a rectangle for perimeter and area is 9×9 in

EXAMPLE 3

A rancher has $400\ ft$ of fence for constructing a rectangular corral. One side of the corral will be formed by a barn and requires no fence. Three exterior fences and two vertical interior fences partition the corral into three rectangular regions. What are the dimensions of the corral to maximize the enclosed area and determine the area of the corral?

$$\begin{cases} x = 400 - 4y \\ A = xy \end{cases}$$



$x = Length$

$y = Width$

$$x + 4y = 400$$

$$x = 400 - 4y$$

Example 3

A rancher has $400\ ft$ of fence for constructing a rectangular corral. One side of the corral will be formed by a barn and requires no fence. Three exterior fences and two interior fences partition the corral into three rectangular regions. What are the dimensions of the corral to maximize the enclosed area and determine the area of the corral?

$$\begin{cases} x = 400 - 4y \\ A = xy \end{cases}$$

$$x + 4y = 400$$

$$x = 400 - 4y$$

$$A = xy$$

$$A = (400 - 4y)\,y$$

$$A = 400y - 4y^2$$

$$A' = 400 - 8y = 0$$

$$A' = 8y = 400$$

$$y = 50\ ft$$

$$x = 400 - 4y$$

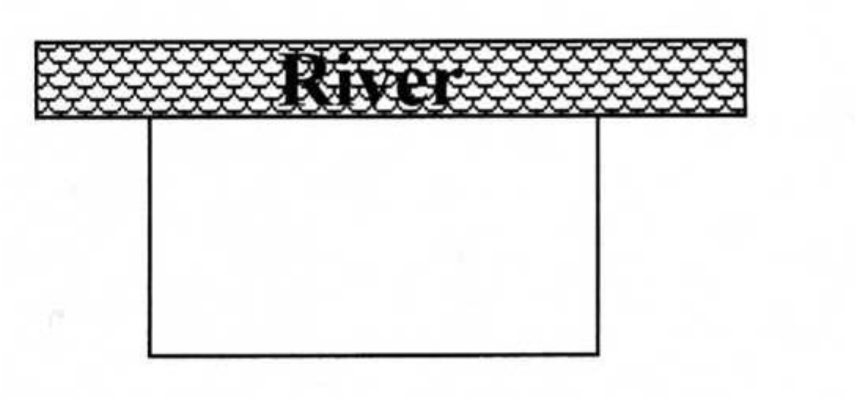
$$x = 400 - 4(50)$$

$$x = 200\ ft$$

The maximum area of a rectangle plot is 50×200 feet

YOUR TURN

A farmer has $2,400\ ft$ of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river. What are the dimensions of the field that has the largest area?



The maximium dimensions of fencing are 1200×600 ft

EXAMPLE 4

The manager of an 80-unit apartment complex is trying to decide what rent to charge. Experience has shown that at a rent of $200, all the units will be full. On the average, one additional unit will remain vacant for each $20 increase in rent. Find the rent to charge to maximize revenue.

$$\text{Current}:(\text{apt units})(\text{Rent \$}) = Total$$

$$\text{Increase}:(\text{apt units} - x)(\text{Rent \$+\$20}x) = Total$$

$$x = vacancies$$

$$I(x) = \text{Income}$$

$$\text{I}(x) = (Units)(\text{Re}\,venue)$$

$$I(x) = (80 - x)(\$200\text{+}\$20x)$$

$$I'(x) = (-1)(\$200\text{+}\$20x) + (80 - x)(20)$$

$$-200 - 20x + 1600 - 2x = 0$$

$$40x = 1400$$

$$x = 35$$

EXAMPLE 4

The manager of an 80-unit apartment complex is trying to decide what rent to charge. Experience has shown that at a rent of $200, all the units will be full. On the average, one additional unit will remain vacant for each $20 increase in rent. Find the rent to charge to maximize revenue.

$$R(x) = (\$200 + \$20x),\ x = 35$$

$$R(35) = (\$200 + \$20(35))$$

$$R(35) = (\$200 + \$700)$$

$$R(35) = \$900$$

$$\text{Current}: (\text{apt units})(\text{Rent }\$) = Total$$

$$\text{Increase}: (\text{apt units} - x)(\text{Rent }\$ + \$20x) = Total$$

$$x = vacancies$$

$$I(x) = \text{Income}$$

$$\text{I}(x) = (Units)(\text{Re}venue)$$

Example 5

A concert promoter knows that 5,000 people will attend an event with tickets set at \$50. For each dollar, less in ticket price, an additional 1,000 tickets will be sold. What should the price of a ticket be to maximize the total receipts?

$$\text{Revenue} = (\text{Price}) \cdot (\text{Number Sold})$$

$$\mathrm{R}(x) = (\text{Total} - \text{Dollar Less } x)\ (\text{Attendance} + Add.\text{Tickets } x)$$

$$\mathrm{R}(x) = (\$50 - \$1x)(5000 + 1000x)$$

$$R'(x) = (50 - x)(1000) + (5000 + 1000x)(-1)$$

EXAMPLE 5

A concert promoter knows that 5,000 people will attend an event with tickets set at \$50. For each dollar, less in ticket price, an additional 1,000 tickets will be sold. What should the price of a ticket be to maximize the total receipts?

$$R'(x) = (50 - x)(1000) + (5000 + 1000x)(-1)$$

$$R'(x) = 45000 - 2000x = 0 \qquad x = 22.5$$

Price

$$50 - 22.5 = \$27.50$$

$$\$27.50$$

Your Turn

A pro golf operator has found that he can sell 500 sets a year of his top grade clubs at $300 per set. For each $5.00 he drops the price, he will sell 10 more sets of clubs. What price would give him the largest income?

$$I(x) = (\mathrm{Re}\,venue)(Sets\ Sold)$$

$$\mathrm{I}(x) = (\$300 - \$5x)(500 + 10x)$$

$$I'(x) = (300 - 5x)(10) + (500 + 10x)(-5)$$

$$I'(x) = (3000 - 50x) + (-2500 - 50x)$$

$$I'(x) = 500 - 100x = 0 \qquad -100x = -500$$

$$x = 5$$

$$\mathrm{R}(\$5) = (\$300 - \$5(5)) = \$275$$

$\$275$

EXAMPLE 6

Farmer Tate's apple orchard now has 30 trees per acre, and the average yield is 400 apples per tree. For each additional tree that he plants per acre, the average yield per tree is reduced by approximately 10 apples. How many trees per acre will give Farmer Tate the largest crop of apples?

$$\text{Current}: (30 \text{ trees})(400 \text{ apples/tree})$$

$$Y(t) = (30+t)(400-10t)$$

$$Y(t) = (30+t)(400-10t)$$

$$Y(t) = 12000 - 300t + 400t - 10t^2$$

$$Y' = 100 - 20t$$

$$t = 5$$

$$y = 30 + 5$$

35 trees per acre

Your Turn

Normally, a pear tree will produce 30 bushels of pears per tree when 20 (or fewer) pear trees are plated per acre. However, for each additional pear tree planted above 20 trees per acre, the yield per tree will fall by one bushel per tree. How many trees should be planted per acre to maximize the total yield?

$$\text{Revenue} = (\text{Price})\cdot(\text{Number Sold})$$

$$\text{R}(x) = (20+x)(30-x)$$

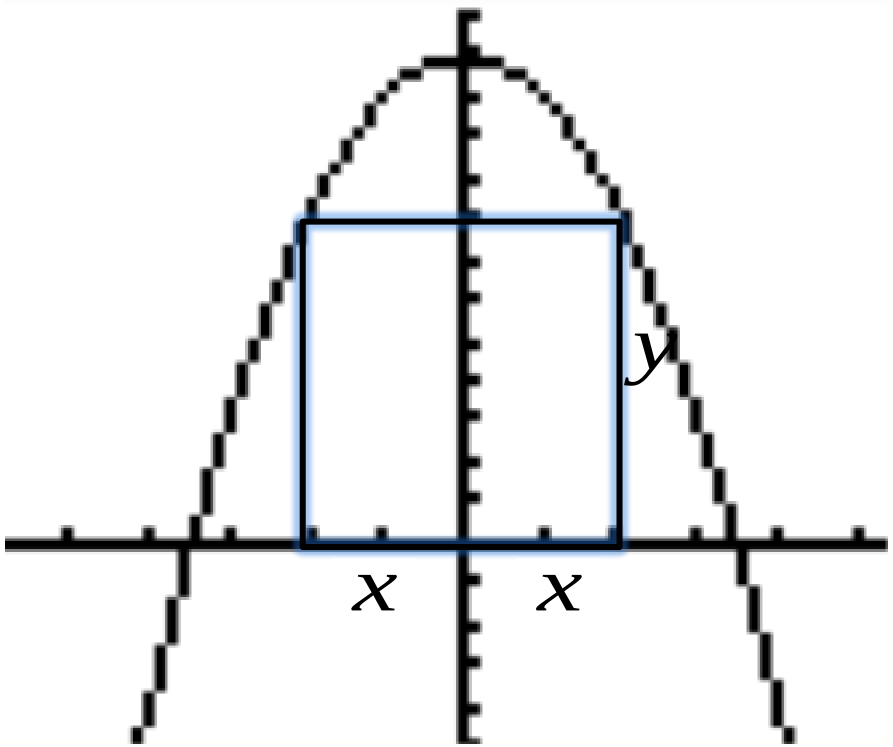
$$R'(x) = 10 - 2x = 0$$

$$x = 5$$

$$25 trees$$

Example 7

Find the area of the largest rectangle with lower base on the x-axis and upper vertices on the parabola of $y = 12 - x^2$.



$$x = Width$$

$$y = Height$$

$$\begin{cases} y = 12 - x^2 \\ A = 2xy \end{cases}$$

$$A = 2xy$$

$$A = 2x\left(12 - x^2\right)$$

$$A = 24x - 2x^3$$

$$A' = 24 - 6x^2 = 0$$

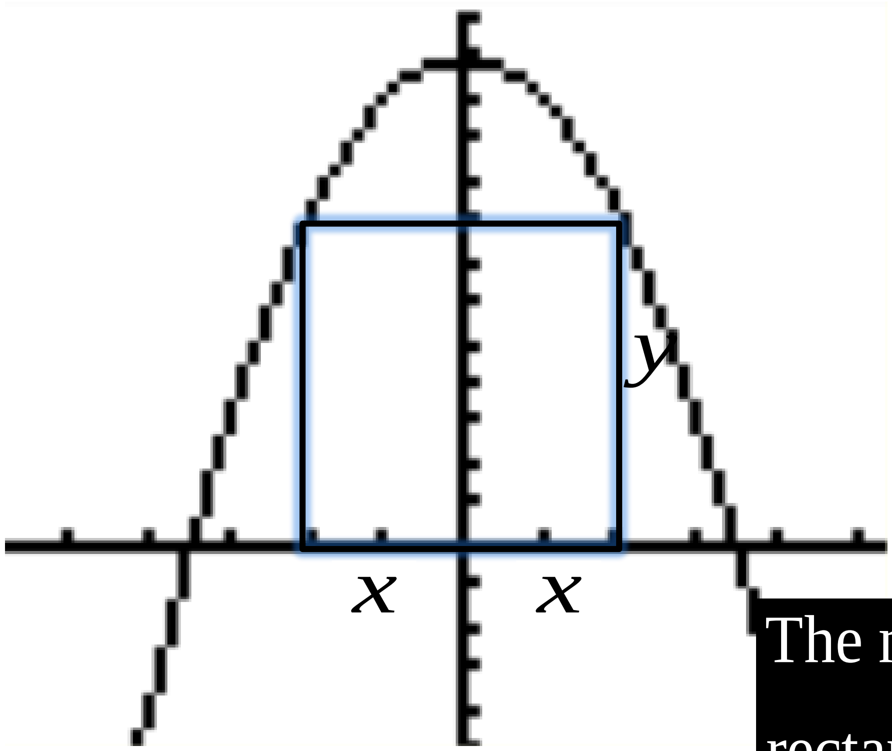
$$A' = 6\left(4 - x^2\right) = 0$$

$$A' = 6\left(2 - x\right)\left(2 + x\right) = 0$$

$$x = \pm 2$$

Example 7

Find the area of the largest rectangle with lower base on the x-axis and upper vertices on the parabola of $y = 12 - x^2$.



$$x = Width$$

$$y = Height$$

$$\begin{cases} y = 12 - x^2 \\ A = 2xy \end{cases}$$

$$y = 12 - x^2$$

$$y = 12 - (2)^2$$

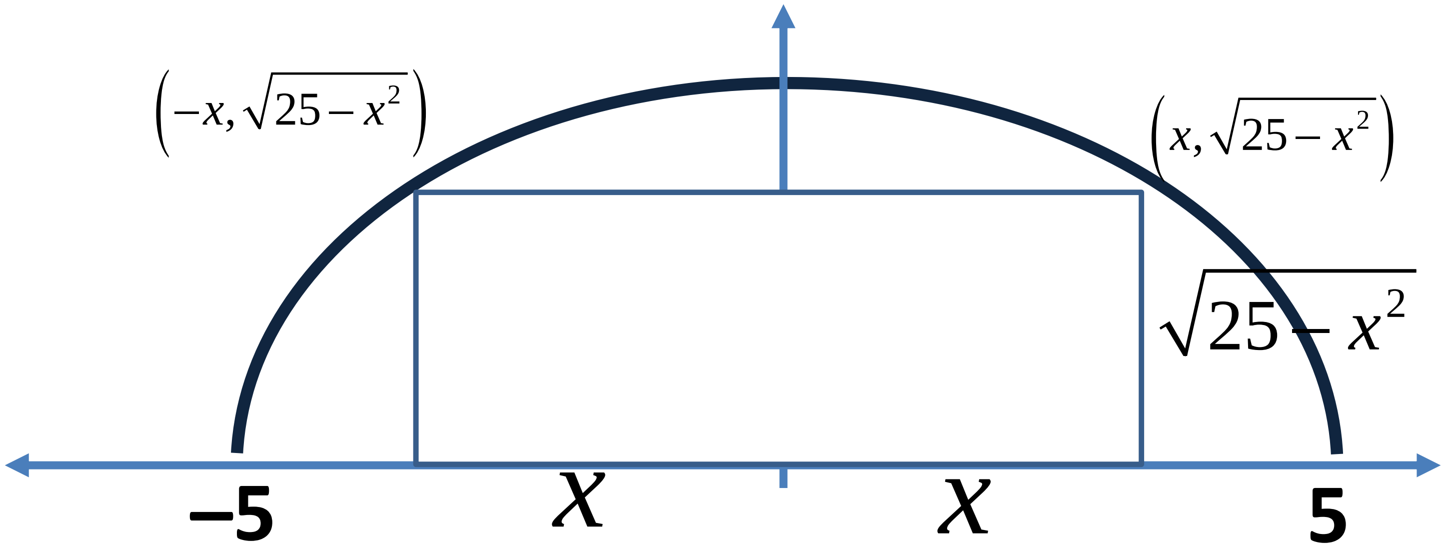
$$y = 12 - 4$$

$$y = 8 units$$

The maximum dimensions of a rectanglar area is 32 units^2

Example 8

A rectangle is bounded by the x-axis and the semicircle, $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?



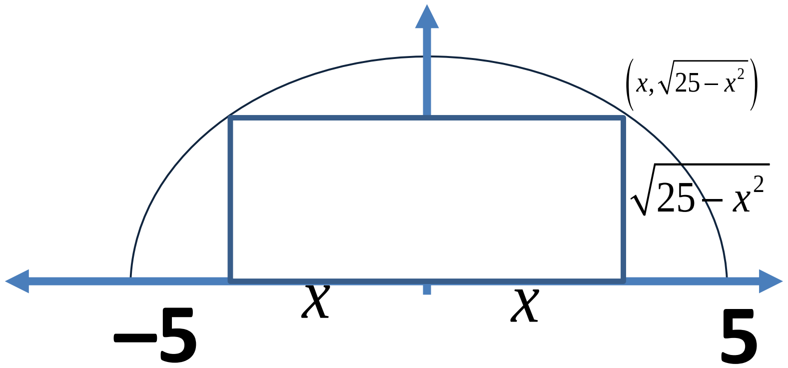
$x = Length$

$y = Width$

$$\begin{cases} y = \sqrt{25 - x^2} \\ A = 2xy \end{cases}$$

EXAMPLE 8

A rectangle is bounded by the x-axis and the semicircle, $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?



$$x = Length$$

$$y = Width$$

$$\begin{cases} y = \sqrt{25 - x^2} \\ A = 2xy \end{cases}$$

$$A = 2xy$$

$$A = (2x)\left(\sqrt{25 - x^2}\right)$$

$$A = (2x)\left(25 - x^2\right)^{1/2}$$

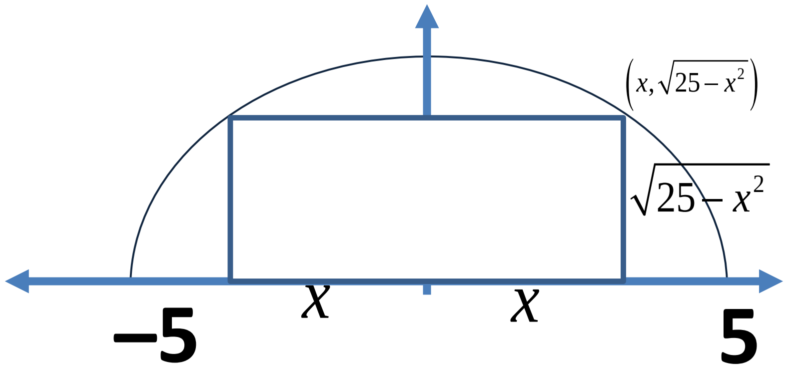
$$A = fg' + gf'$$

$$\frac{dA}{dx} = (2x)\left[\left(\frac{1}{2}\left(25 - x^2\right)^{-1/2}\right)(-2x)\right] + \left(\sqrt{25 - x^2}\right)(2)$$

Example 8

A rectangle is bounded by the x-axis and the semicircle, $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?

$$\frac{dA}{dx} = (2x)\left[\left(\frac{1}{2}(25 - x^2)^{-1/2}\right)(-2x)\right] + \left(\sqrt{25 - x^2}\right)(2)$$



$$\frac{dA}{dx} = \frac{-2x^2}{\sqrt{25 - x^2}} + 2\sqrt{25 - x^2}$$

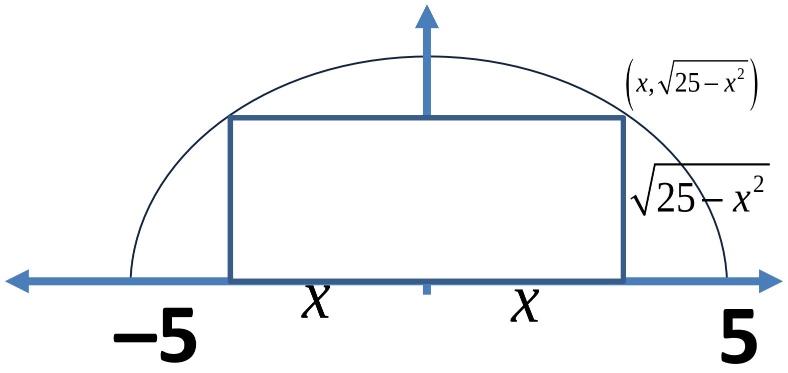
$$\frac{dA}{dx} = \frac{-2x^2}{\sqrt{25 - x^2}} + \frac{2\left(\sqrt{25 - x^2}\right)\left(\sqrt{25 - x^2}\right)}{\sqrt{25 - x^2}}$$

$$\frac{dA}{dx} = \frac{-2x^2}{\sqrt{25 - x^2}} + \frac{2(25 - x^2)}{\sqrt{25 - x^2}}$$

$$\frac{dA}{dx} = \frac{-4x^2 + 50}{\sqrt{25 - x^2}}$$

Example 8

A rectangle is bounded by the x-axis and the semicircle, $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?



$$0 = \frac{-4x^2 + 50}{\sqrt{25 - x^2}}$$

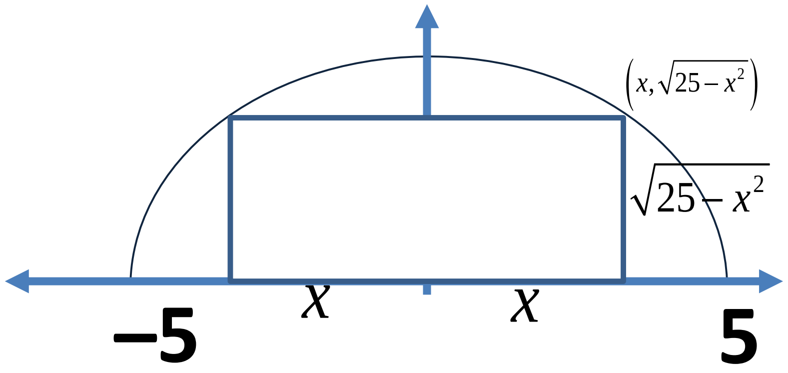
$$0 = -4x^2 + 50$$

$$x^2 = \frac{50}{4} = \frac{25}{2}$$

$$x = \frac{5}{\sqrt{2}}$$

EXAMPLE 8

A rectangle is bounded by the x-axis and the semicircle, $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?



$$x = \frac{5}{\sqrt{2}}$$

$$y = \sqrt{25 - x^2}$$

$$y = \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2}$$

$$y = \sqrt{25 - \left(\frac{25}{2}\right)}$$

$$y = \frac{5}{\sqrt{2}}$$

$$Length : x = 2\left(\frac{5}{\sqrt{2}}\right) = \frac{10}{\sqrt{2}}$$

$$Width : y = \frac{5}{\sqrt{2}}$$

YOUR TURN

A rectangle is bounded by the x-axis and the semicircle, $y = \sqrt{16 - x^2}$. What length and width should the rectangle have so that its area is a maximum?

$$Length : x = 4\sqrt{2}$$

$$Width : y = 2\sqrt{2}$$

EXAMPLE 9

Find the shortest distance from $y^2 = 2x$ to the point $(2, 0)$.

$$d = \sqrt{(x-2)^2 + (y-0)^2}$$

$$d = \sqrt{x^2 - 4x + 4 + y^2}$$

$$d = \sqrt{x^2 - 4x + 4 + 2x}$$

$$d = \sqrt{x^2 - 2x + 4}$$

$$d' = \frac{d}{dx}\sqrt{x^2 - 2x + 4}$$

Example 9

Find the shortest distance from $y^2 = 2x$ to the point $(2, 0)$.

$$d' = \frac{d}{dx}\left(x^2 - 2x + 4\right)^{1/2}$$

$$d' = \frac{1}{2}\left(x^2 - 2x + 4\right)^{-1/2}(2x - 2)$$

$$d' = \frac{2x - 2}{2\sqrt{x^2 - 2x + 4}}$$

$$0 = \frac{2x - 2}{2\sqrt{x^2 - 2x + 4}}$$

$$0 = 2x - 2$$

$$x = 1$$

X-coordinate of the optimization point

EXAMPLE 9

Find the shortest distance from $y^2 = 2x$ to the point $(2, 0)$.

$$y^2 = 2x$$

$$y^2 = 2(1)$$

$$y^2 = 2$$

$$y = \pm\sqrt{2}$$

$$d = \sqrt{(x-2)^2 + (y-0)^2}$$

$$d = \sqrt{((1)-2)^2 + \left(\left(\pm\sqrt{2}\right)-0\right)^2}$$

EXAMPLE 9

Find the shortest distance from $y^2 = 2x$ to the point $(2, 0)$.

$$d = \sqrt{\left((1) - 2\right)^2 + \left(\left(\pm\sqrt{2}\right) - 0\right)^2}$$

$$d = \sqrt{(-1)^2 + \left(\left(\pm\sqrt{2}\right)\right)^2}$$

$$d = \sqrt{3}$$

The shortest distance of $y^2 = 2x$ is $\sqrt{3}$ to the point $(2,0)$

EXAMPLE 10

Find the shortest distance from $y = x^2$ to the point $\left(2, \frac{1}{2}\right)$.

The shortest distance of $y = x^2$ is $\frac{\sqrt{5}}{2}$ to the point $\left(2, \frac{1}{2}\right)$

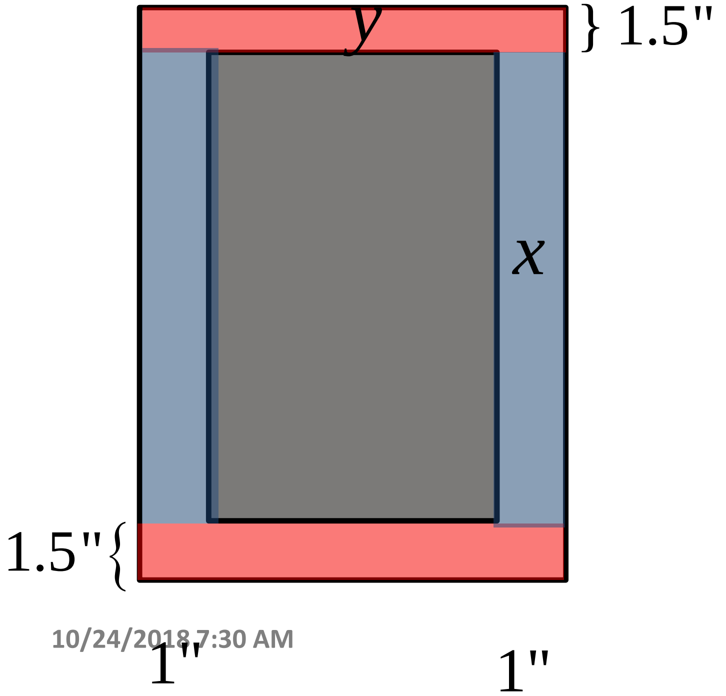
YOUR TURN

Find the point on the graph of the function $f(x) = \sqrt{x}$ to the point $(4, 0)$.

The point on the graph of is $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

Example 11

A rectangular page is to contain $24\ in^2$ of print. The margins at the top and bottom of the page are to be $1.5\ in$, and the margins on the left and the right are to be $1\ in$. What should the dimensions of the page be so that the least amount of paper is used? Prove that the dimensions are at a minimum. Define x as height and y as width.



$$x = Height$$

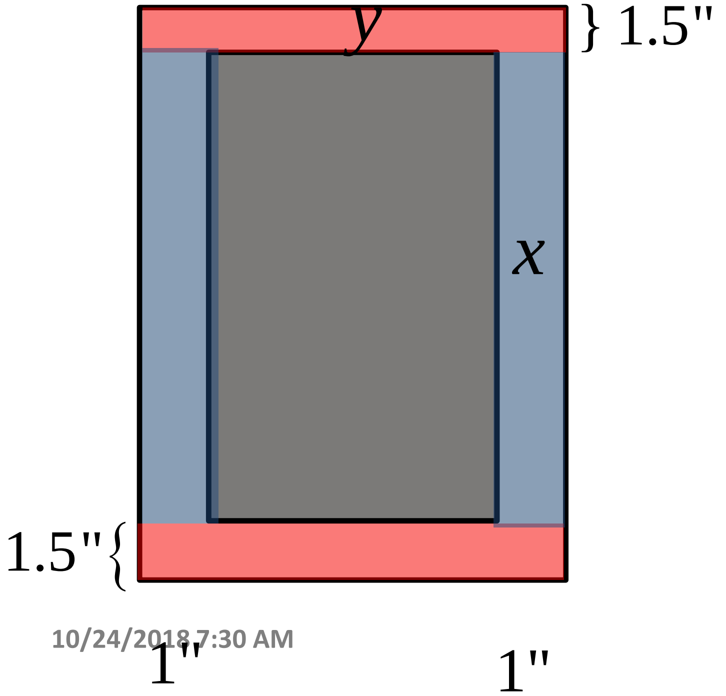
$$y = Width$$

$$\begin{cases} y = \dfrac{24}{x} \\ A = (x+3)(y+2) \end{cases}$$

$$A = (x+3)\left(\frac{24}{x} + 2\right)$$

Example 11a

A rectangular page is to contain $24\ in^2$ of print. The margins at the top and bottom of the page are to be $1.5\ in$, and the margins on the left and the right are to be $1\ in$. What should the dimensions of the page be so that the least amount of paper is used? Prove that the dimensions are at a minimum. Define x as length and y as width.



$$A=(x+3)\left(\frac{24}{x}+2\right)$$

$$A=24+2x+\frac{72}{x}+6$$

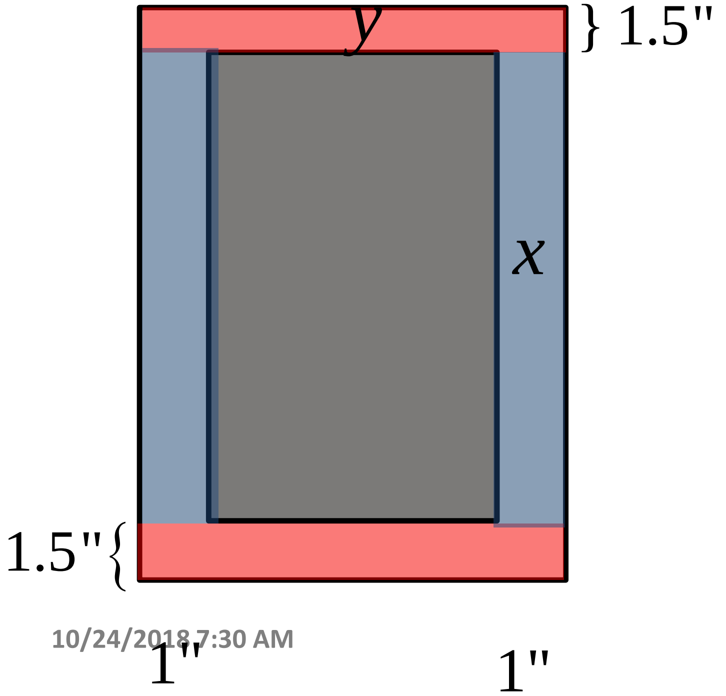
$$A=30+2x+72x^{-1}$$

$$A=30+2x+\frac{72}{x}$$

$$A'=2-72x^{-2}=2-\frac{72}{x^2}$$

Example 11A

A rectangular page is to contain $24\ in^2$ of print. The margins at the top and bottom of the page are to be $1.5\ in$, and the margins on the left and the right are to be $1\ in$. What should the dimensions of the page be so that the least amount of paper is used? Prove that the dimensions are at a minimum. Define x as length and y as width.



$$\frac{72}{x^2} = 2$$

$$2x^2 = 72$$

$$x^2 = 36$$

$$x = 6in.$$

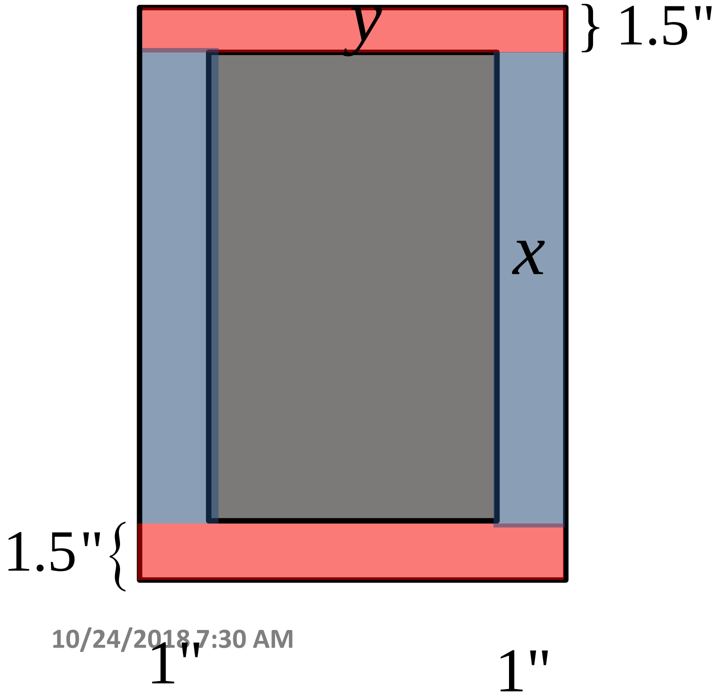
$$y = \frac{24}{x}$$

$$y = \frac{24}{6}$$

$$y = 4in.$$

Example 11a

A rectangular page is to contain $24\ in^2$ of print. The margins at the top and bottom of the page are to be $1.5\ in$, and the margins on the left and the right are to be $1\ in$. What should the dimensions of the page be so that the least amount of paper is used? Prove that the dimensions are at a minimum. Define x as length and y as width.



$$A=(y+2)$$

$$(4)+2=6''$$

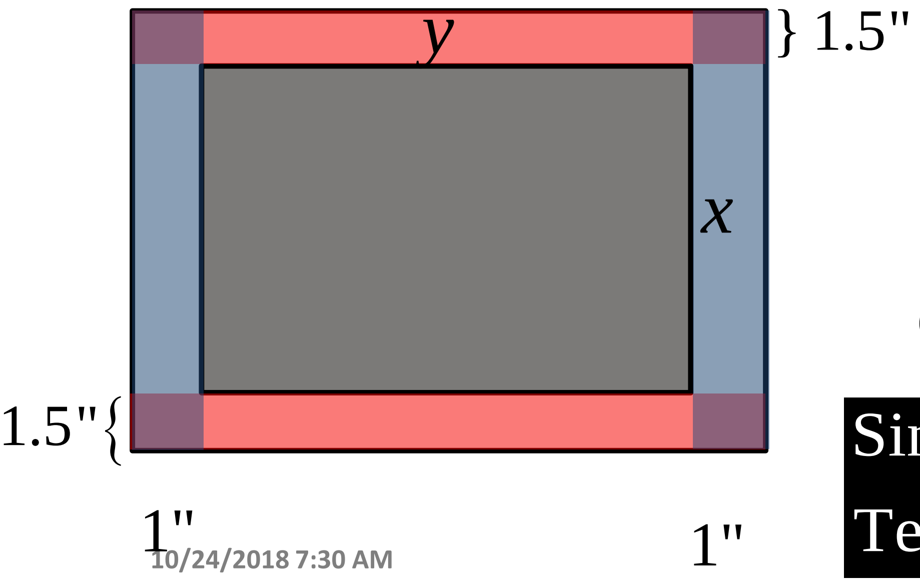
$$A=(x+3)$$

$$(6)+3=9''$$

$$\textit{Dimensions}: 9''\times 6''$$

Example 11b

A rectangular page is to contain $24\ in^2$ of print. The margins at the top and bottom of the page are to be $1.5\ in$, and the margins on the left and the right are to be $1\ in$. What should the dimensions of the page be so that the least amount of paper is used? Prove that the dimensions are at a minimum. Define x as length and y as width.



Prove $x = 6$ is a minimum, apply the 2nd Derivative Test

$$A' = 2 - 72x^{-2}$$

$$A'' = \frac{144}{x^3} \qquad A''(6) = \frac{144}{6^3} > 0$$

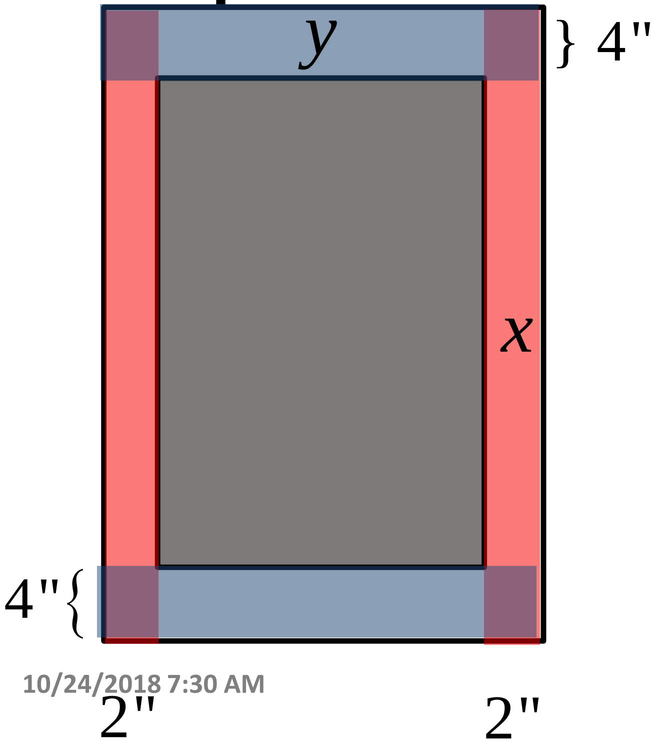
Concaves Up - Relative Min

$\text{Dimensions}: 9'' \times 6''$

Since $A'' > 0$, the Second Derivative Test proves $x = 6$ is a minimum.

YOUR TURN

You are designing a poster to contain $50\ in^2$ of printing with margins of $4\ in$ each at the top and bottom and $2\ in$ at each side. What overall dimensions will minimize the amount of paper used? Justify response.



$$x = Height$$

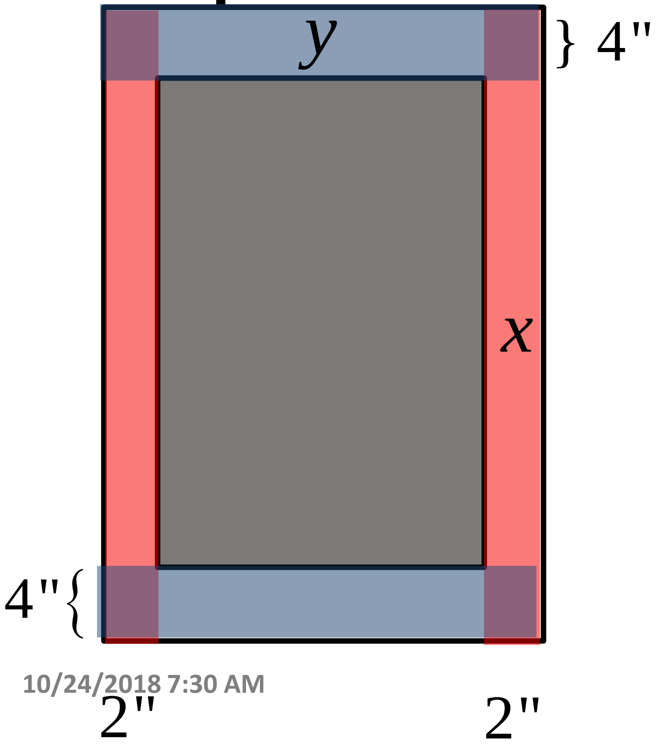
$$y = Width$$

$$\begin{cases} y = \dfrac{50}{x} \\ A = (x+4)(y+8) \end{cases}$$

$$A = (x+4)\left(\frac{50}{x} + 8\right)$$

Your Turn

You are designing a poster to contain $50\ in^2$ of printing with margins of $4\ in$ each at the top and bottom and $2\ in$ at each side. What overall dimensions will minimize the amount of paper used? Justify response.



$$A=(x+4)\left(\frac{50}{x}+8\right)$$

$$A=50+8x+200x^{-1}+32$$

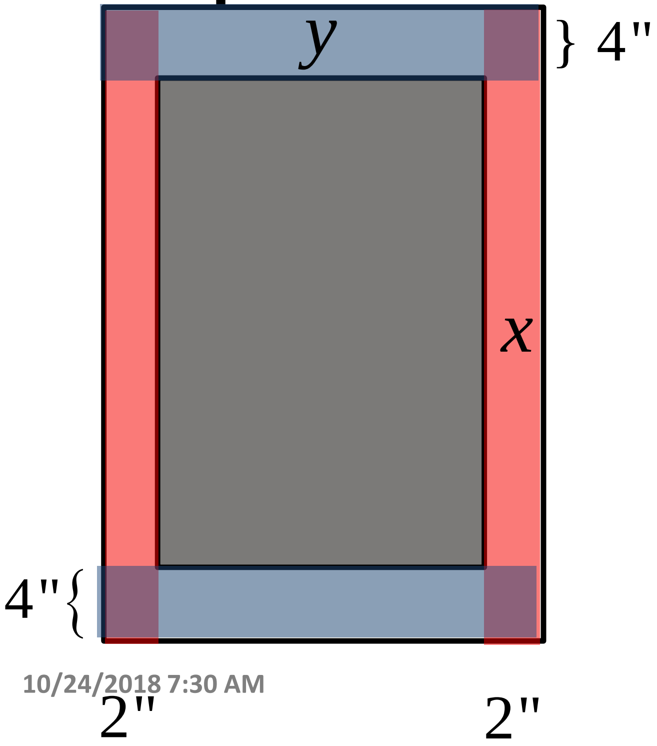
$$A=82+8x+\frac{200}{x}$$

$$A=82+8x+200x^{-1}$$

$$A'=8-200x^{-2}=8-\frac{200}{x^2}$$

YOUR TURN

You are designing a poster to contain $50\ in^2$ of printing with margins of $4\ in$ each at the top and bottom and $2\ in$ at each side. What overall dimensions will minimize the amount of paper used? Justify response.



$$\frac{200}{x^2} = 8$$

$$8x^2 = 200$$

$$x^2 = 25$$

$$x = 5in$$

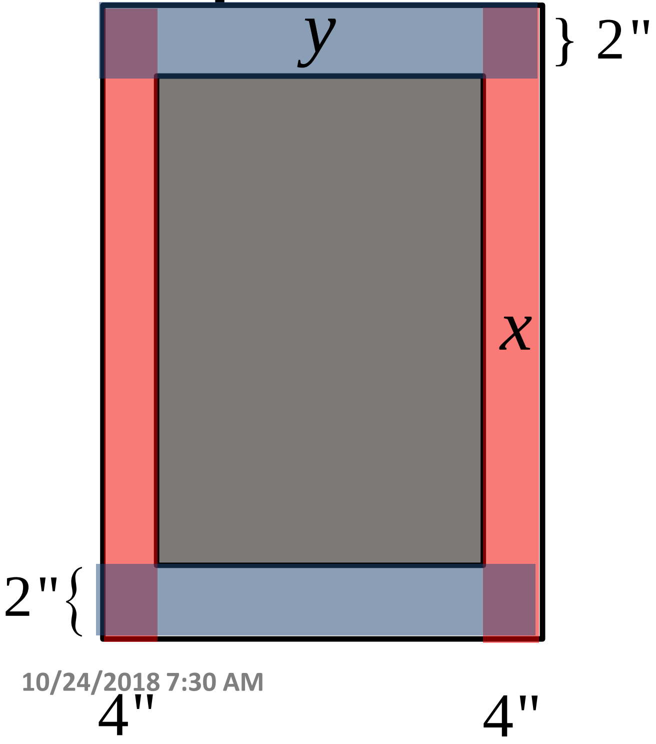
$$y = \frac{50}{x}$$

$$y = \frac{50}{5}$$

$$y = 10in$$

Your Turn

You are designing a poster to contain $50\ in^2$ of printing with margins of $4\ in$ each at the top and bottom and $2\ in$ at each side. What overall dimensions will minimize the amount of paper used? Justify response.



$$A=(x+4)$$

$$(5)+4=9''$$

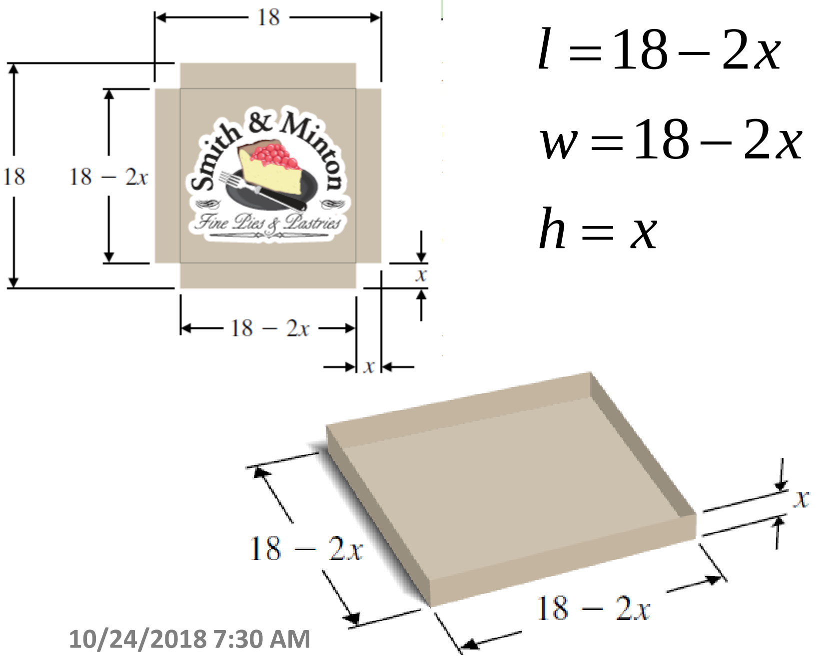
$$A=(y+8)$$

$$(10)+8=18''$$

$$Dimensions: 9''\times 18''$$

EXAMPLE 12

A square sheet of cardboard 18 on a side is made into an open box (i.e., there's no top), by cutting squares of equal size out of each corner and folding up the sides along the dotted lines. Find the dimensions of the box with the maximum volume. Volume = $\boldsymbol{l \times w \times h}$



$$l = 18 - 2x$$

$$w = 18 - 2x$$

$$h = x$$

$$V(x) = lwh$$

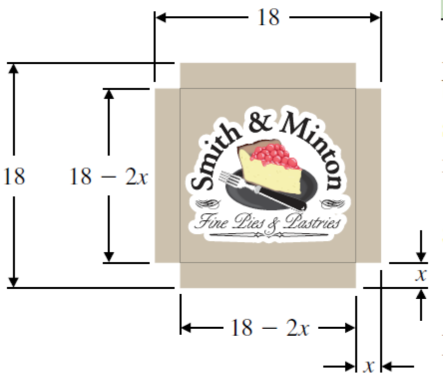
$$V(x) = (18 - 2x)(18 - 2x)(x)$$

$$V(x) = (18 - 2x)^2(x)$$

$$V(x) = 4(9 - x)^2(x)$$

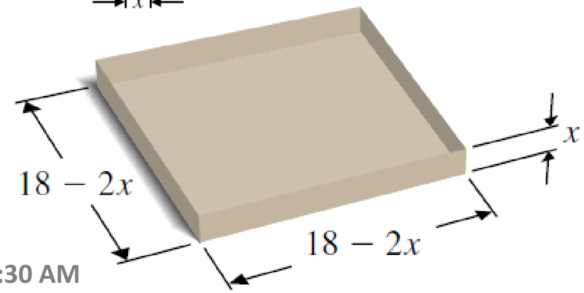
Example 12

A square sheet of cardboard 18 on a side is made into an open box (i.e., there's no top), by cutting squares of equal size out of each corner and folding up the sides along the dotted lines. Find the dimensions of the box with the maximum volume. Volume = $l \times w \times h$



$$l = 18 - 2x$$
$$w = 18 - 2x$$
$$h = x$$

$$V(x) = 4x(9-x)^2$$
$$V'(x) = 4(9-x)^2 + 4x(2)(9-x)(-1)$$
$$V'(x) = 4(9-x)\left[(9-x) - 2x\right]$$
$$0 = 4(9-x)(9-3x)$$



$x = 3$, ~~$x = 9$~~

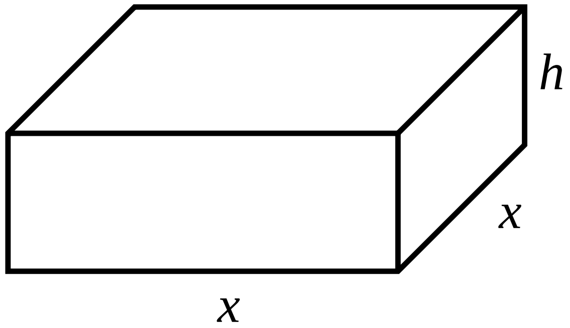
$$Dimensions: 12'' \times 12'' \times 3''$$

EXAMPLE 13

A manufacture wants to design an open box having a square base and a surface area of $108\ in^2$. What dimensions will produce the box with maximum volume?

$$\begin{cases} x = side \\ h = height \end{cases}$$

$$\begin{cases} V = x^2h \\ SA = x^2 + 4xh \end{cases}$$



$$SA = x^2 + 4xh$$

$$108 = x^2 + 4xh$$

$$108 - x^2 = 4xh$$

$$h = \frac{108 - x^2}{4x}$$

EXAMPLE 13

A manufacture wants to design an open box having a square base and a surface area of $108\ in^2$. What dimensions will produce the box with maximum volume?

$$\begin{cases} x = side \\ h = height \end{cases}$$

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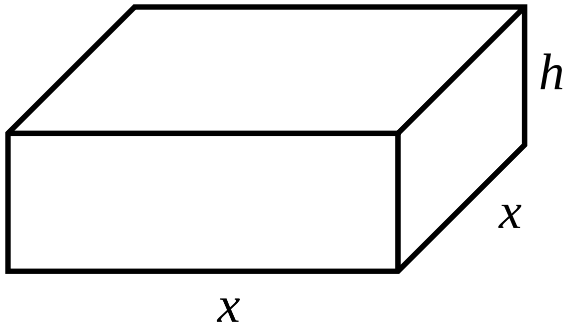


$$V = 27x - \frac{x^3}{4}$$

$$\frac{dV}{dx} = 27 - \frac{3}{4}x^2$$

$$0 = 27 - \frac{3}{4}x^2$$

$$\frac{3}{4}x^2 = 27$$

$$x^2 = 36$$

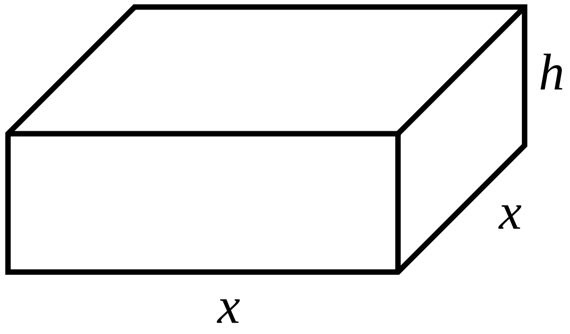
$$x = 6\ inches$$

EXAMPLE 13

A manufacture wants to design an open box having a square base and a surface area of $108\ in^2$. What dimensions will produce the box with maximum volume?

$$\begin{cases} x = side \\ h = height \end{cases}$$

$$\begin{cases} V = x^2 h \\ SA = x^2 + 4xh \end{cases}$$



$$108 = (6)^2 + 4(6)h$$

$$108 = 36 + 24h$$

$$72 = 24h$$

$$h = 3\ inches$$

$$6'' \times 6'' \times 3''$$

AP Multiple Choice Practice Question 1 (non-calculator)

The point on the curve $2y = x^2$ that is nearest the point $(4, 1)$ is:

(A) $(0, 0)$

(B) $(2, 2)$

(C) $(\sqrt{2}, 1)$

(D) $(2\sqrt{2}, 4)$

AP Multiple Choice Practice Question 1 (non-calculator)

The point on the curve $2y = x^2$ that is nearest the point $(4, 1)$ is:

Vocabulary	Connections and Process	Answer
Distance Optimization	$2y = x^2$ $y = \frac{1}{2}x^2$ $L = \sqrt{(x-4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2}$ $L^2 = \left(\sqrt{(x-4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2}\right)^2$ $L^2 = (x-4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2$ $2L\frac{dL}{dx} = 2(x-4) + 2\left(\frac{1}{2}x^2 - 1\right)(x)$ $\frac{dL}{dx} = \frac{2(x-4) + 2\left(\frac{1}{2}x^2 - 1\right)(x)}{2L} = \frac{2x - 8 + x^3 - 2x}{2L}$ $\frac{dL}{dx} = \frac{x^3 - 8}{2L} = \frac{1}{2}x^3 - 4 = 0$ $\frac{1}{2}x^3 = 4$ $x^3 = 8$ $x = 2$	B

ASSIGNMENT

Worksheet