Particle Motion: Day 2

Section 3.6A

Calculus AP/Dual, Revised ©2018

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When you see… Think…

When you see…	Think…
Initially	$t = 0$
At rest	$v(t) = 0$
At the origin	$x(t) = 0$
Velocity is POSITIVE	Particle is moving RIGHT
Velocity is NEGATIVE	Particle is moving LEFT
Average Velocity (Given $x(t)$)	$\frac{s(b) - s(a)}{b - a}$
Instantaneous Velocity	Velocity at an exact moment
POSITIVE acceleration	Velocity is increasing
NEGATIVE acceleration	Velocity is decreasing
Instantaneous Speed	$\lvert v(t) \rvert$

Speed, Velocity, and Tangent Lines

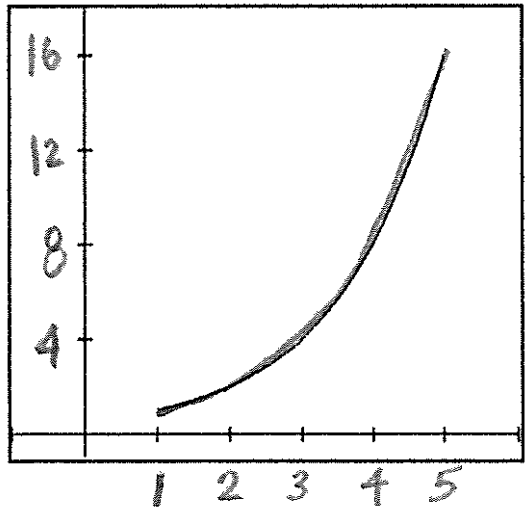
A. Speed is the absolute value of velocity. It is measured of how fast something is moving with the regard of direction
B. The effect of how an absolute value function has it on the graph is that it reflects all values that are below and above on the x-axis

IMAGINE...

- **Stepping on the gas,**
  - **Acceleration pulls into a direction that you are moving. Does your speed increase or decrease?**
  - **Going too fast is not good so you put on your breaks. Now, you accelerate in the opposite direction but you are moving forward. Does your speed increase or decrease?**
  - **Finally, you stop and shift into reverse and start moving backwards (negative velocity) and then you push the on the gas to accelerate in the negative direction. Does your speed increase or decrease?**
  - **Then, you pull on the breaks after all of that. Speed increases or decreases?**

EXAMPLE 1

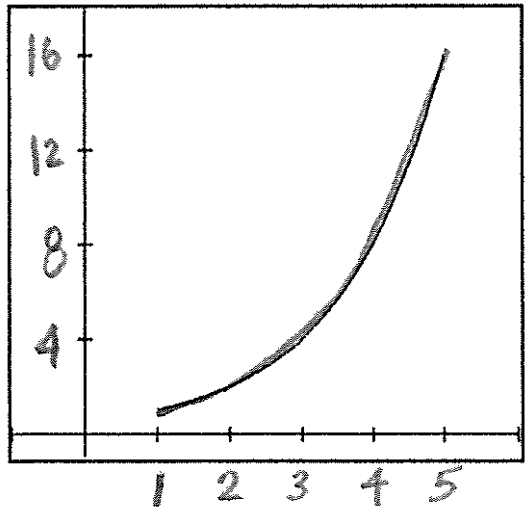
For each situation, the graph is differentiable when giving velocity as a function of time $[1, 5]$ along the selected values of the velocity. In this graph, each horizontal mark represents $1\ unit$ and each vertical mark represents $4\ units$. Plot the speed graph on the same coordinate plane as velocity.



Time	Velocity	Speed
1	1	1
2	2	2
3	4	4
4	8	8
5	16	16

EXAMPLE 1

For each situation, the graph is differentiable when giving velocity as a function of time $[1, 5]$ along the selected values of the velocity. In this graph, each horizontal mark represents $1\ unit$ and each vertical mark represents $4\ units$. Plot the speed graph on the same coordinate plane as velocity.



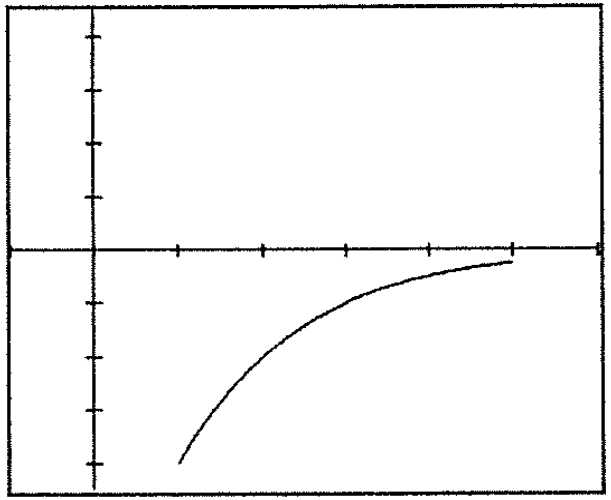
A) In this situation, the velocity is positive/negative and increasing/decreasing?

B) When velocity is increasing/decreasing, we know that acceleration is positive/negative?

C) When examining the graph of speed and table of values, the conclusion is that speed is increasing/decreasing?

EXAMPLE 2

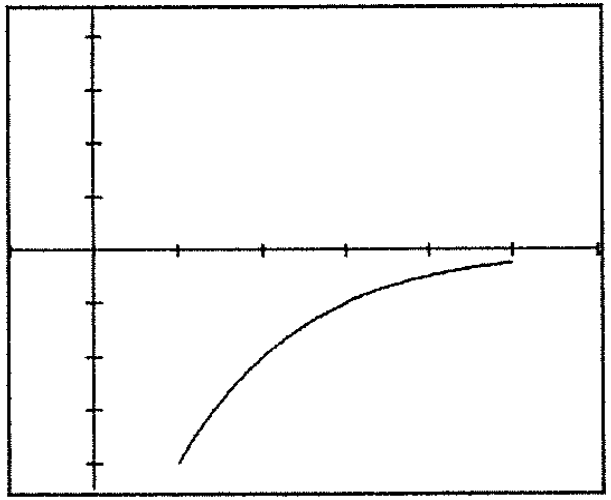
For each situation, the graph is differentiable when giving velocity as a function of time $[1, 5]$ along the selected values of the velocity. In this graph, each horizontal mark represents $1\ unit$ and each vertical mark represents $4\ units$. Plot the speed graph on the same coordinate plane as velocity.



Time	Velocity	Speed
1	–16	16
2	–8	8
3	–4	4
4	–2	2
5	–1	1

EXAMPLE 2

For each situation, the graph is differentiable when giving velocity as a function of time $[1, 5]$ along the selected values of the velocity. In this graph, each horizontal mark represents $1\ unit$ and each vertical mark represents $4\ units$. Plot the speed graph on the same coordinate plane as velocity.



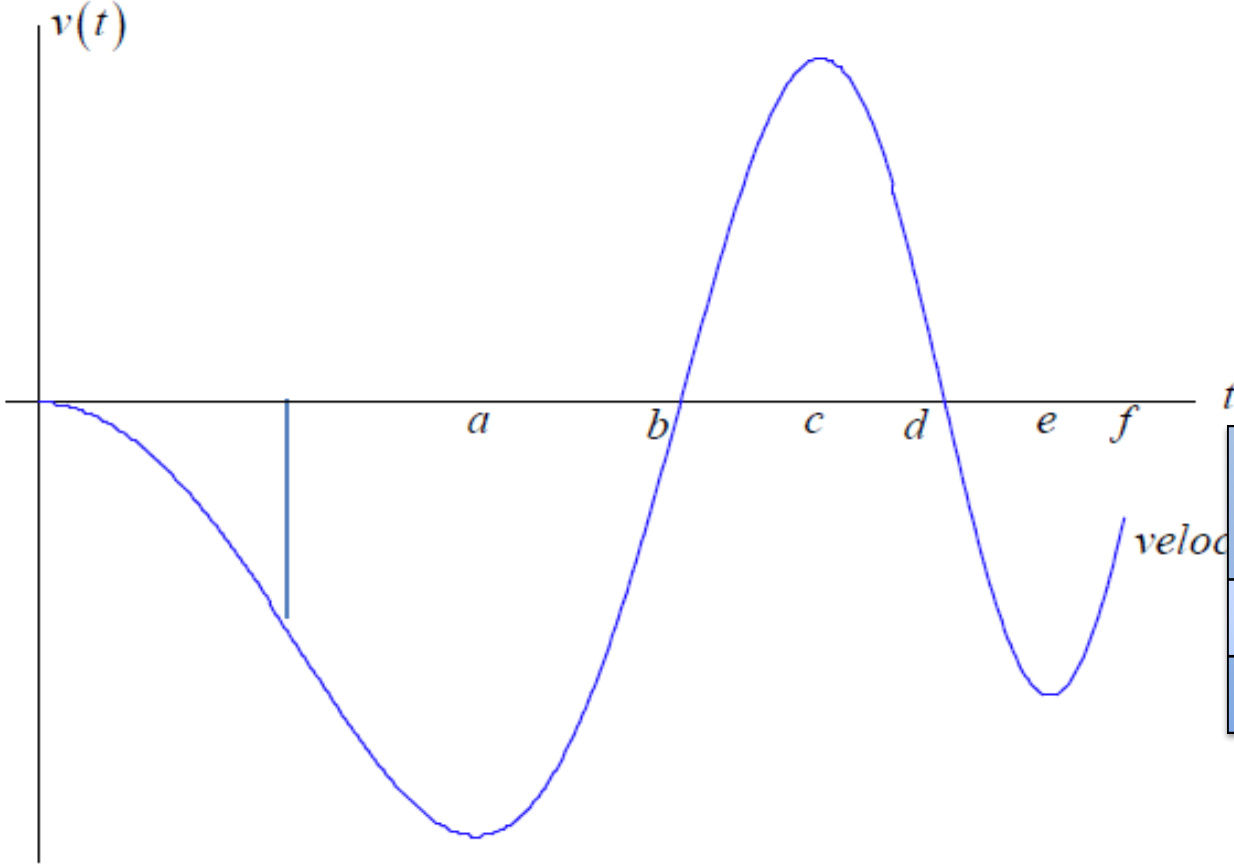
A) In this situation, the velocity is positive/negative and increasing/decreasing?

B) When velocity is increasing/decreasing, we know that acceleration is positive/negative?

C) When examining the graph of speed and table of values, the conclusion is that speed is increasing/decreasing?

EXAMPLE 3

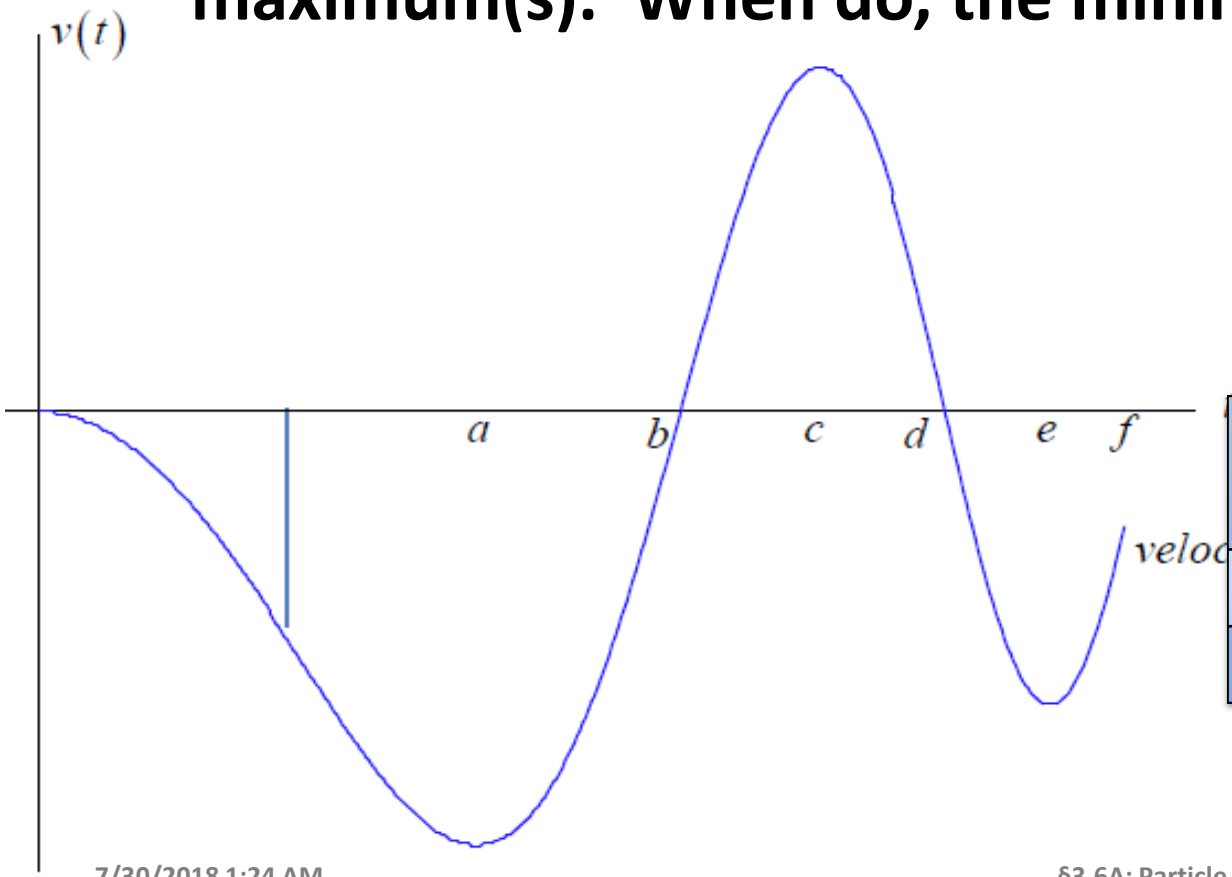
Given this graph below of $v(t)$, fill out the table below for intervals $[0, a]$ and $[c, d]$



Interval	Velocity Positive or Negative	Acceleration Positive or Negative	Speed Increasing or Decreasing
$[0, a]$	$-$	$-$	*Inc.*
$[c, d]$	$+$	$-$	*Dec.*

YOUR TURN

Given this graph below of $v(t)$, fill out the table below. Then, identify the values of t at which the speed obtains its local and absolute maximum(s). When do, the minimum speeds occur? When are they?



Local Maxs: $t = c, t = e$

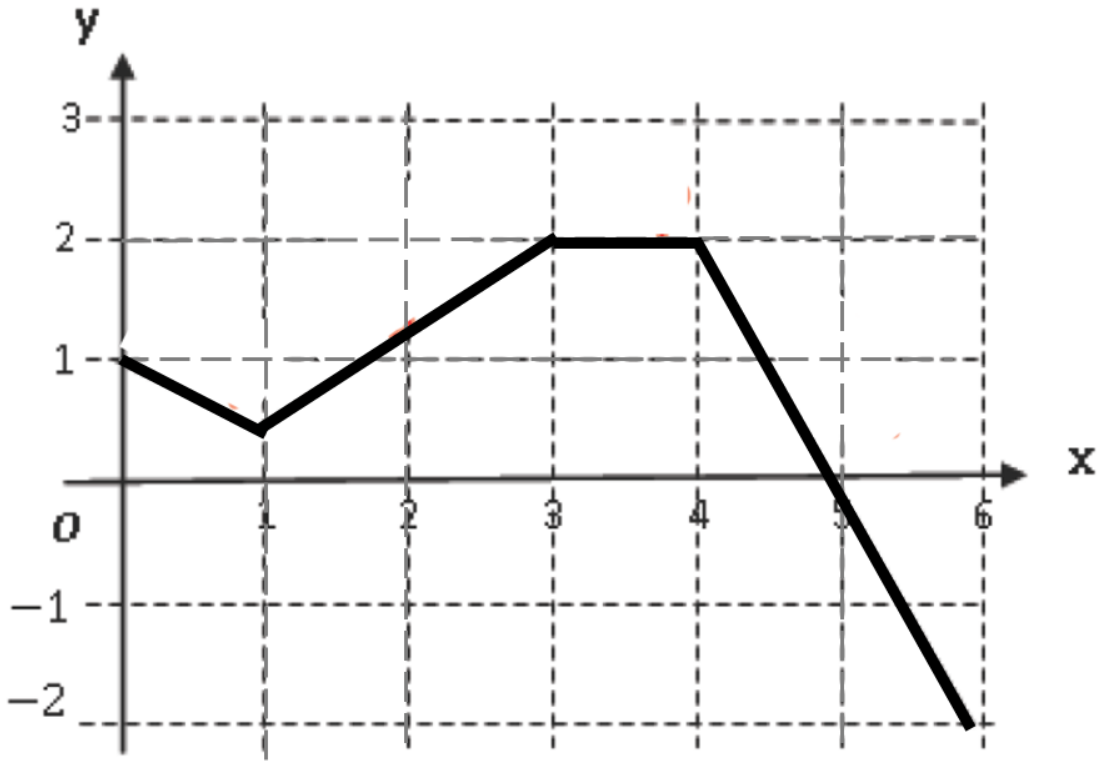
Absolute Max: $t = a$

Absolute Min of 0: $t = 0, t = b, t = d, t = f$

Interval	Velocity Positive or Negative	Acceleration Positive or Negative	Speed Increasing or Decreasing
$[a, b]$	$-$	$+$	*Dec.*
$[b, c]$	$+$	$+$	*Inc.*

EXAMPLE 4

A particle P (position) moves along the x-axis over the time interval from $t = 0$ and $t = 6$ seconds where x stands for seconds and y is in feet. P(t) graph given.



A) Over what time interval is the particle moving to the left? Explain.

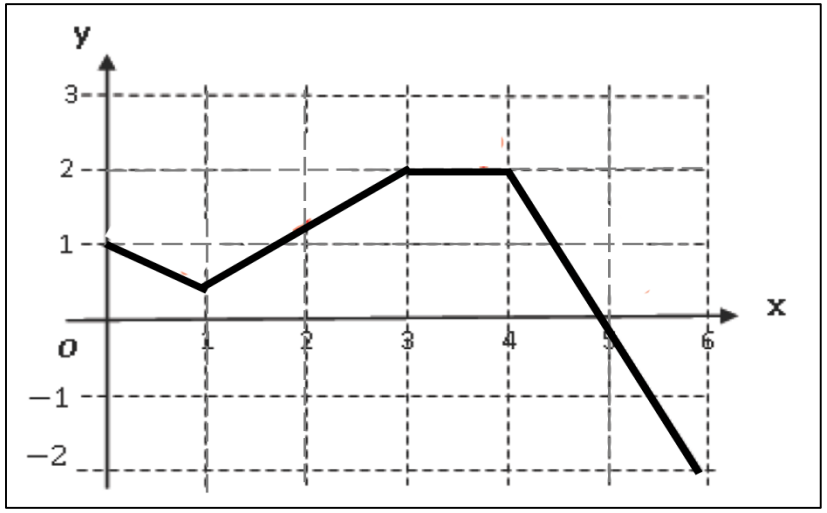
B) When is the first time that P reverses direction?

C) When does P move at its greatest speed? Explain.

D) Is there guaranteed to be a time t in the interval, $[0, 3]$ such $that\ v(t) = \frac{3}{8}\frac{ft}{sec}$? Justify answer.

EXAMPLE 4A

A particle P (position) moves along the x-axis over the time interval from $t = 0$ and $t = 6$ seconds where x stands for seconds and y is in feet.

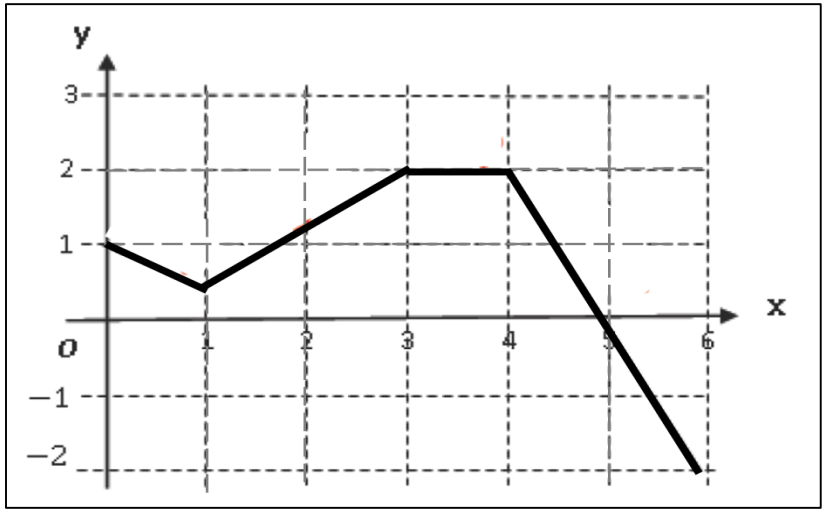


Over what time interval is the particle moving to the left? Explain.

The particle is moving to the left from $[0,1)$ and $(4,6]$ because $s'(t) < 0$

EXAMPLE 4B

A particle P (position) moves along the x-axis over the time interval from $t = 0$ and $t = 6$ seconds where x stands for seconds and y is in feet.

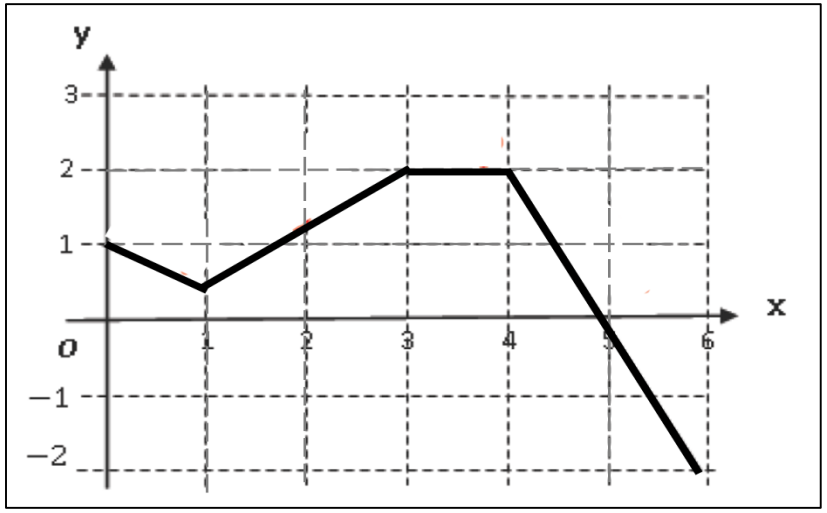


When is the first time that P reverses direction?

P reverses direction at $t = 1$ because direction changes from left to right.

EXAMPLE 4C

A particle P (position) moves along the x-axis over the time interval from $t = 0$ and $t = 6$ seconds where x stands for seconds and y is in feet.

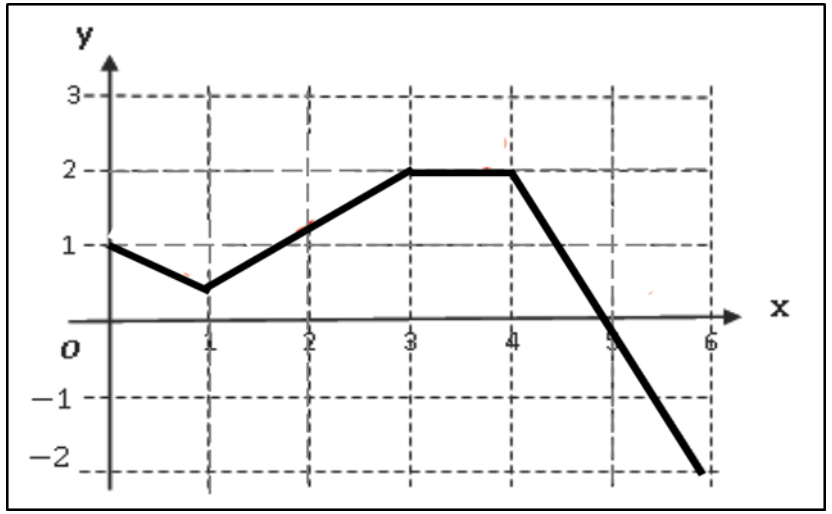


When does P move at its greatest speed? Explain.

The particle is at its greatest speed from $(4,6]$ because speed is the absolute value of velocity

EXAMPLE 4D

A particle P (position) moves along the x-axis over the time interval from $t = 0$ and $t = 6$ seconds where x stands for seconds and y is in feet.



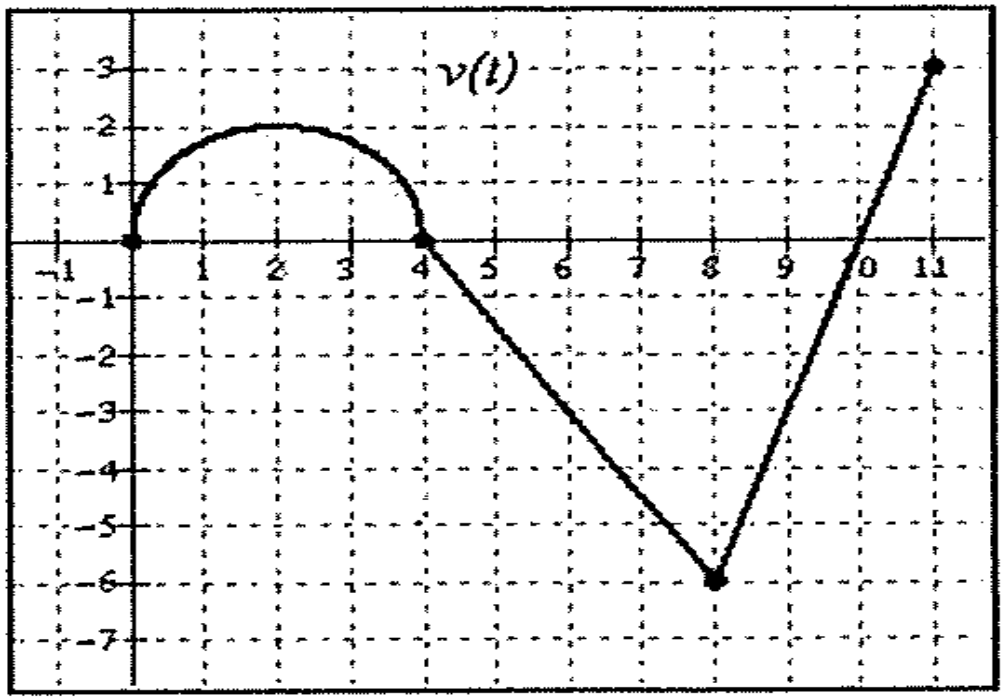
Is there guaranteed to be a time t in the interval, $[0, 3]$ such that $v(t) = \frac{3}{8}\frac{ft}{sec}$?

Justify answer.

MVT does not apply between 0 and 3 because $v(t)$ is not differentiable. Continuity does not validate differentiability.

EXAMPLE 5

The graph below represents the velocity, $v(t)$, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ & $t = 11$ seconds. It consists of a semicircle and two line segments.



A) At what time $[0, 11]$, is the speed of the particle the greatest?

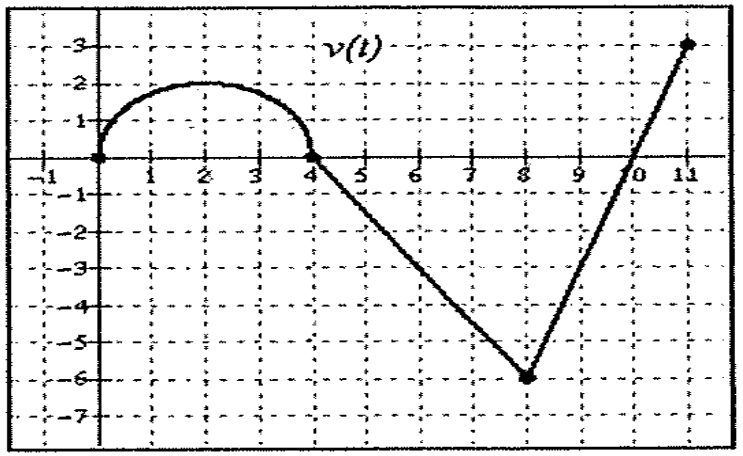
B) At which times, $t = 2$, $t = 6$, or $t = 9$ where the acceleration the greatest? Explain.

C) Over what time intervals is the particle moving left? Explain.

D) Over what time intervals is the speed of the particle decreasing? Explain.

EXAMPLE 5A

The graph below represents the velocity, $v(t)$, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ & $t = 11$ seconds. It consists of a semicircle and two line segments.

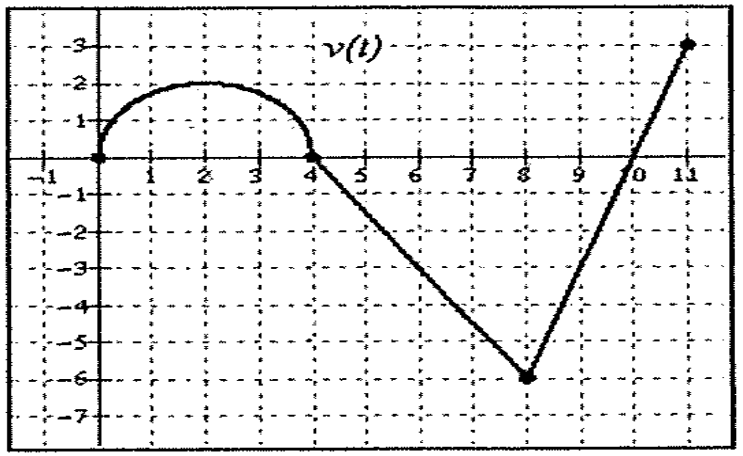


At what time $[0, 11]$, is the speed of the particle the greatest?

At t=8 when speed is 6 feet/sec

EXAMPLE 5B

The graph below represents the velocity, $v(t)$, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ & $t = 11$ seconds. It consists of a semicircle and two line segments.

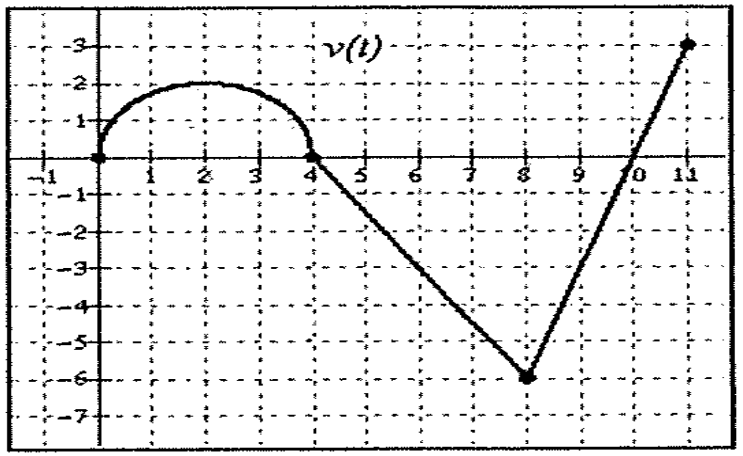


At which times, $t = 2$, $t = 6$, or $t = 9$ where the acceleration the greatest? Explain.

At $t = 9$ when acceleration is at 3 seconds whereas when $t = 2$ slope is $m = 0$ and where $t = 6$, slope is $-\frac{3}{2}$.

EXAMPLE 5C

The graph below represents the velocity, $v(t)$, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ & $t = 11$ seconds. It consists of a semicircle and two line segments.

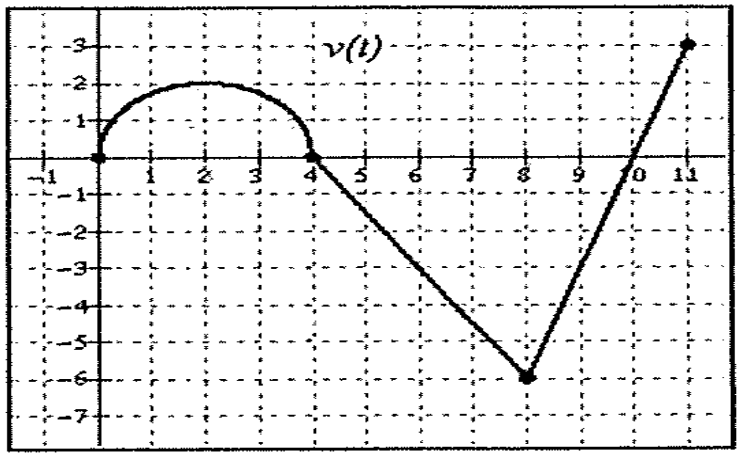


Over what time intervals is the particle moving left? Explain.

$(4,10)$ because $v(t) < 0$

EXAMPLE 5D

The graph below represents the velocity, $v(t)$, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ & $t = 11$ seconds. It consists of a semicircle and two line segments.

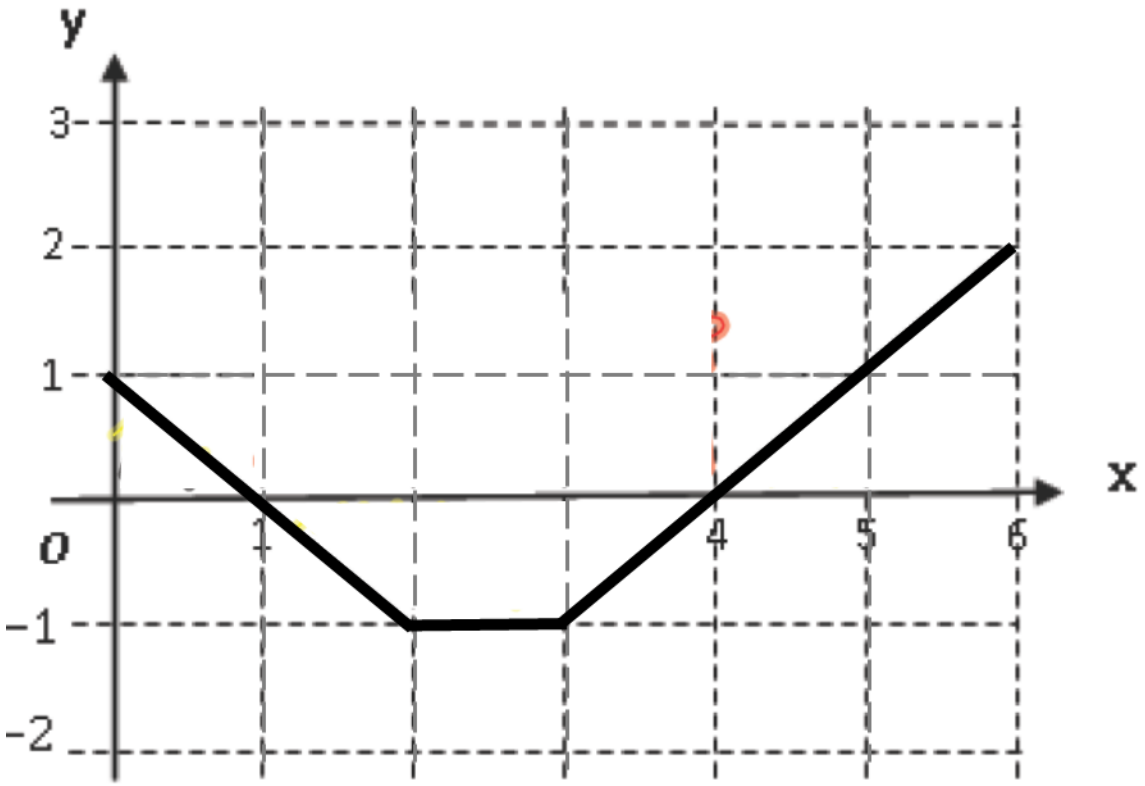


Over what time intervals is the speed of the particle decreasing? Explain.

At $(2,4)$ because $v(t) > 0$ and $a(t) < 0$ and at $(8,10)$ because $v(t) < 0$ and $a(t) > 0$ when the signs different of $v(t)$ and $a(t)$, the particle slows down.

YOUR TURN

A particle P (position) moves along the x-axis over the time interval from $t = 0$ and $t = 6$ seconds where x stands for seconds and y is in feet. This graph shows velocity.



A) Over what time interval is the particle moving to the left? Explain.

$At\,(1,4)$ because $v(t) < 0$

B) Over what time intervals is the speed of the particle decreasing? Explain.

$At\ [0,1)$ because $v(t) > 0$ and $a(t) < 0$ and at $(3,4)$ because $v(t) < 0$ and $a(t) > 0$
When the signs change of $v(t)$ and $a(t)$, the particle slows down.

C) Is there guaranteed to be a time *t* in the interval, $[0, 2]$ where the particle is at rest? Explain.

Yes, between $t = 0$ and $t = 2$ where the velocity must be 0.
Therefore, the IVT must exist and differentiability implies continuity

EXAMPLE 6

The data below in the table gives the selected values of velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is differentiable function of time, t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ $(meters/min)$	-3	2	3	5	7	5

A) If $t = 0$, is the particle moving to the right or left? Explain the answer.

B) Is there a time during the interval $[0, 12]$ minutes when the particle is at rest? Explain answer.

C) Use the data to the table to approximate $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle.

Example 6A

The data below in the table gives the selected values of velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is differentiable function of time, t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ $(meters/min)$	-3	2	3	5	7	5

If $t = 0$, is the particle moving to the right or left? Explain the answer.

Left because $v(t) < 0$

Example 6b

The data below in the table gives the selected values of velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is differentiable function of time, t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ $(meters/min)$	-3	2	3	5	7	5

Is there a time during the interval $[0, 12]$ minutes when the particle is at rest? Explain answer.

Yes, between $t = 0$ and $t = 2$ where the velocity must be 0.

Therefore, the IVT must exist and differentiability implies continuity

EXAMPLE 6C

The data below in the table gives the selected values of velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is differentiable function of time, t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ $(meters/min)$	-3	2	3	5	7	5

Use the data to the table to approximate $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle.

$$v'(t) \approx \frac{s(12)-s(8)}{12-8} \approx \frac{5-7}{12-8} \approx \frac{-2}{4} \text{ meters / min}^2$$

$v'(10)$ is the instantaneous acceleration of the particle when t=10 min

YOUR TURN

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 $feet$ at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table.

t $(secs)$	0	10	20	30	40	50	60	70	80
$v(t)$ $(ft/sec.)$	5	14	22	29	35	40	44	47	49

A) Find the average acceleration of rocket A over the interval $[0, 80]$ seconds. Indicate the units of measurement.

B) Is there a time during the interval $[0, 12]$ seconds when the particle is at rest? Explain answer.

C) Use the data to the table to approximate $v'(45)$ and explain the meaning of $v'(45)$ in terms of the motion of the particle.

YOUR TURN

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 $feet$ at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table.

t $(secs)$	0	10	20	30	40	50	60	70	80
$v(t)$ $(ft/sec.)$	5	14	22	29	35	40	44	47	49

A) Find the average acceleration of rocket A over the interval $[0, 80]$ seconds. Indicate the units of measurement.

B) Is there a time during the interval $[0, 12]$ seconds when the particle is at rest? Explain answer.

C) Use the data to the table to approximate $v'(45)$ and explain the meaning of $v'(45)$ in terms of the motion of the particle.

$A) \frac{11}{20}\ ft/\sec^2$

$B) No$ because $v(0)$ never falls below zero.

$C) v'(45) \approx \frac{v(50)-v(40)}{50-40} \approx \frac{40-35}{10} \approx \frac{1}{2}$ feet $/ \sec^2$

$v'(45)$ is the Instantaneous acceleration of the particle when t=45 secs

ASSIGNMENT

Worksheet