Particle Motion: Day 1

Section 3.6A

Calculus AP/Dual, Revised ©2018

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When you see… Think…

When you see…	Think…
Initially	$t = 0$
At rest	$v(t) = 0$
At the origin	$x(t) = 0$
Velocity is POSITIVE	Particle is moving RIGHT
Velocity is NEGATIVE	Particle is moving LEFT
Average Velocity (Given $x(t)$)	$\frac{s(b) - s(a)}{b - a}$
Instantaneous Velocity	Velocity at an exact moment
POSITIVE acceleration	Velocity is increasing
NEGATIVE acceleration	Velocity is decreasing
Instantaneous Speed	$\|v(t)\|$

What is Position, Velocity, and Acceleration?

www.nbclearn.com/nfl/cuecard/50770

MOTIONS

A. **<u>Position</u> is $s(t)$ or $x(t)$; also known as speed is the rate of motion**
   1. **Label could be known as meters**
   2. **"Initially" means when $t = 0$**
   3. **"At the origin" means $x(t) = 0$**

B. **<u>Velocity</u> is $v(t) = s'(t)$; absolute value rate of motion or known as <u>*SPEED*</u> and <u>*DIRECTION*</u>**
   1. **Label could be known as meters/second or speed/time**
   2. **"At rest" means $v(t) = 0$**
   3. **If the velocity of the particle is POSITIVE, then the particle is moving to the <u>right</u>**
   4. **If the velocity of the particle is NEGATIVE, then the particle is moving to the <u>left</u>**
   5. **If the order of the particle CHANGES SIGNS, the <u>velocity must change signs</u>**

Motions

C. <u>Acceleration</u> known as $a(t) = v'(t) = s''(t)$

1. **Label could be known as $\frac{units}{time^2}$**
2. **If the acceleration of the particle is POSITIVE, then the particle is <u>increasing</u>**
3. **If the acceleration of the particle is NEGATIVE, then the particle is <u>decreasing</u>**
4. **If a particle <u>*S*</u>lows <u>*D*</u>own, signs from $v'(t)$ and $s''(t)$ are different (<u>S</u>IGNS <u>D</u>IFFERENT)**

REMEMBER…

A. **All graphs are given in a two-dimensional plane**
B. **In particle motion, motion is two-directional form (one-dimensional x-axis)**

WEBSITE

http://www.cengage.com/math/discipline_content/stewartcalcet7/2008/14_cengage_tec/publish/deployments/transcendentals_7e/m3_7_sa.swf

http://phet.colorado.edu/simulations/sims.php?sim=The_Moving_Man

Formulas

A. **Total Distance = Candidates Test**

B. **Average Velocity =** $\frac{s(b)-s(a)}{b-a}$ **or divide the change in position by the change in time**

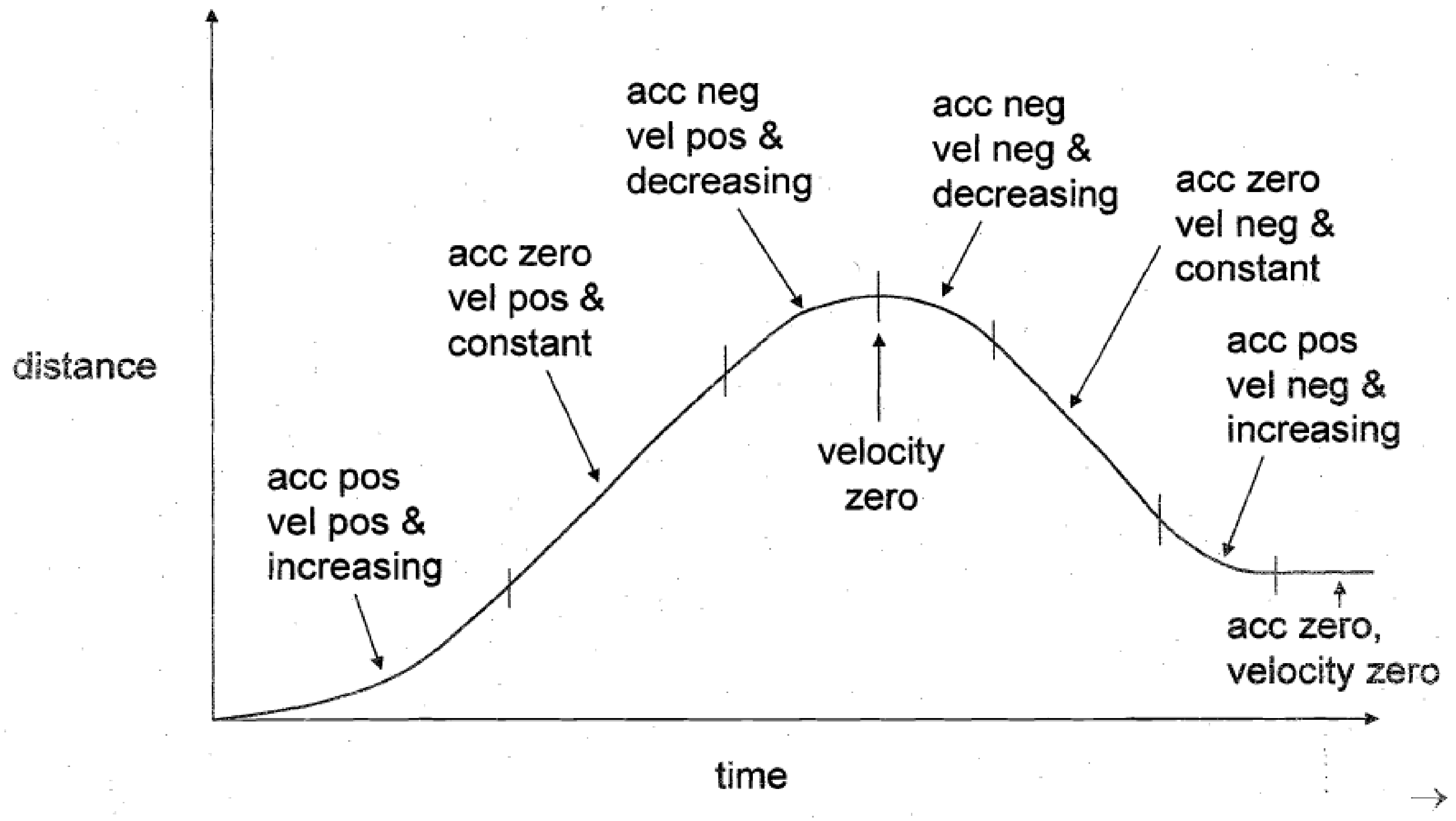
C. **Instantaneous Speed =** $|v(t)|$

$$Position$$

$$Velocity \left\langle \begin{array}{l} \text{Left} \\ \text{Right} \end{array} \right.$$

$$Acceleration \left\langle \begin{array}{l} \text{Increasing (Above } x\text{-axis)} \\ \text{Decreasing (Below } x\text{-axis)} \end{array} \right.$$

GRAPH



acc neg
vel pos &
decreasing
acc neg
vel neg &
decreasing
acc zero
vel neg &
constant
acc zero
vel pos &
constant
distance
acc pos
vel neg &
increasing
velocity
zero
acc pos
vel pos &
increasing
acc zero,
velocity zero
time

Techniques of Speeding Up/Slowing Down

SPEEDING UP SAME SIGNS	SLOWING DOWN SIGNS DIFFERENT
Positive Velocity Positive Acceleration	Positive Velocity Negative Acceleration
Negative Velocity Negative Acceleration	Negative Velocity Positive Acceleration

EXAMPLE 1A

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 10t^2 + 5$ where t is measured in seconds and s is in meters. Determine the a) position, b) instantaneous velocity, c) acceleration and d) speed of the particle all at $t = 1$.

$$s(t) = 2t^3 - 10t^2 + 5$$

$$s(1) = 2(1)^3 - 10(1)^2 + 5$$

$$s(1) = -3$$

$$s(1) = -3\ meters$$

Example 1b

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 10t^2 + 5$ where t is measured in seconds and s is in meters. Determine the a) position, b) instantaneous velocity, c) acceleration and d) speed of the particle all at $t = 1$.

$$s(t) = 2t^3 - 10t^2 + 5$$

$$v(t) = s'(t) = 6t^2 - 20t$$

$$v(1) = 6(1)^2 - 20(1)$$

$$v(1) = -14\ m/s$$

Example 1c

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 10t^2 + 5$ where t is measured in seconds and s is in meters. Determine the a) position, b) instantaneous velocity, c) acceleration and d) speed of the particle all at $t = 1$.

$$v(t) = 6t^2 - 20t$$

$$a(t) = v'(t) = 12t - 20$$

$$a(1) = 12(1) - 20$$

$$a(1) = -8\ m/s^2$$

EXAMPLE 1D

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 10t^2 + 5$ where t is measured in seconds and s is in meters. Determine the a) position, b) instantaneous velocity, c) acceleration and d) speed of the particle all at $t = 1$.

$$\left|v(t)\right|$$

$$v(1) = \left|6t^2 - 20t\right|$$

$$v(1) = \left|6(1)^2 - 20(1)\right|$$

$$v(1) = 14\ m/s$$

Example 2a

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest from the left, e) particle moving to the right, and f) slowing down?

Velocity at time *t*

$$x'(t) = v(t)$$

$$s(t) = 2t^3 - 19t^2 + 12t - 7$$

$$v(t) = 6t^2 - 38t + 12$$

EXAMPLE 2B

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest from the left, e) particle moving to the right, and f) slowing down?

Acceleration

$$x''(t) = v'(t) = a(t)$$

$$s(t) = 2t^3 - 19t^2 + 12t - 7$$

$$v(t) = 6t^2 - 38t + 12$$

$$a(t) = 12t - 38$$

EXAMPLE 2C

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest to the left, e) particle moving to the right, and f) slowing down?

At Rest

$$v(t) = 0$$

$$6t^2 - 38t + 12 = 0$$

$$2(3t^2 - 19t + 6) = 0$$

$$2(3t-1)(t-6) = 0$$

$$t = \frac{1}{3}\text{sec}, 6\,\text{secs}$$

Example 2D

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest to the left, e) particle moving to the right, and f) slowing down?

Relative Min

$$v(t) = 6t^2 - 38t + 12$$

$$2(3t-1)(t-6) = 0 \qquad t = \frac{1}{3}, t = 6$$

Moving furthest to the Left

$t = 0$	$\left(0, \frac{1}{3}\right)$	$t = \frac{1}{3}$	$\left(\frac{1}{3}, 6\right)$	$t = 6$	$(6, \infty)$
$f(0)$ $(0, -7)$	$f'\left(\frac{1}{4}\right)$ $(-)(-)$ POSITIVE RIGHT	$f\left(\frac{1}{3}\right)$ $\left(\frac{1}{3}, -\frac{136}{27}\right)$ Rel MAX	$f'(2)$ $(+)(-)$ NEGATIVE LEFT	$f(6)$ $(6, 187)$ Rel MIN	$f'(7)$ $(+)(+)$ POSITIVE RIGHT

The particle is moving furthest to the left at point $(6,187)$ because $v(t)$ changes signs from Negative to Positive

Example 2e

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest from the left, e) particle moving to the right, and f) slowing down?

Moving to the RIGHT

$t = 0$	$\left(0, \frac{1}{3}\right)$	$t = \frac{1}{3}$	$\left(\frac{1}{3}, 6\right)$	$t = 6$	$(6, \infty)$
$f(0)$ $(0, -7)$	$f'\left(\frac{1}{4}\right)$ $(-)(-)$ POSITIVE RIGHT	$f\left(\frac{1}{3}\right)$ $\left(\frac{1}{3}, -\frac{136}{27}\right)$ Rel MAX	$f'(2)$ $(+)(-)$ NEGATIVE LEFT	$f(6)$ $(6, 187)$ Rel MIN	$f'(7)$ $(+)(+)$ POSITIVE RIGHT

The particle is moving to the right at intervals $\left(0, \frac{1}{3}\right) \cup (6, \infty)$ because $v(t) > 0$

EXAMPLE 2F

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest from the left, e) particle moving to the right, and f) slowing down?

SLOWING DOWN (get critical values and POI)

$$v' = a(t) = 12t - 38 = 0$$

$$t = \frac{19}{6}$$

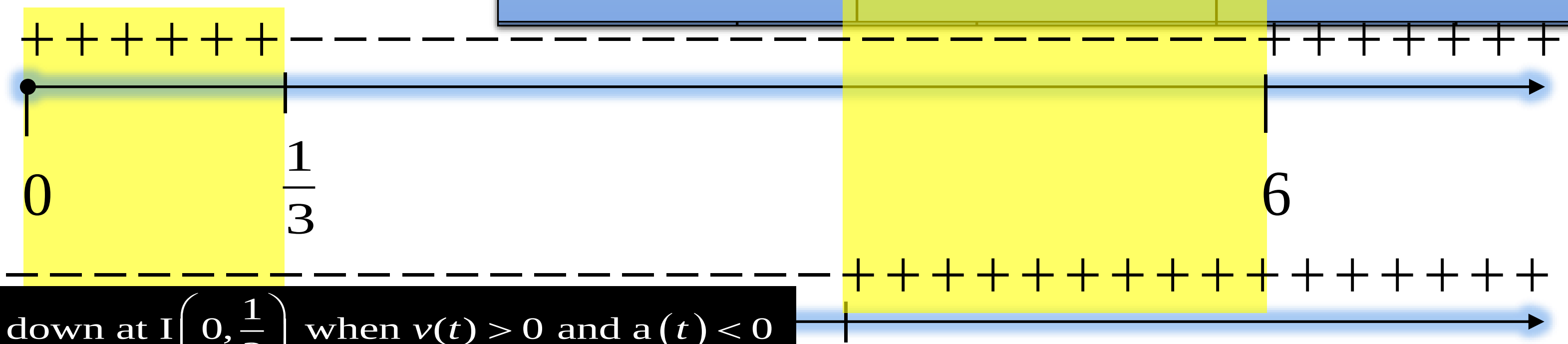
EXAMPLE 2F

The position function of a particle moving on a straight line is $s(t) = 2t^3 - 19t^2 + 12t - 7$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration, c) at rest, d) particle moving furthest to the left, e) particle moving to the right, and f) slowing down?

$t = 0$	$\left(0, \frac{19}{6}\right)$	$t = 19/6$	$\left(\frac{19}{6}, \infty\right)$
$f(0)$ $(0, 7)$ $f''(x) = 12t - 38$	$f''(1)$ $(-)$ NEGATIVE	$f''\left(\frac{19}{6}\right)$	$f''(4)$ $(+)$ POSITIVE

$v(t) = \frac{1}{3}, 6$

$a(t) = \frac{19}{}$



Particle is slowing down at I$\left(0, \frac{1}{3}\right)$ when $v(t) > 0$ and $a(t) < 0$

Particle is slowing down at I$\left(\frac{19}{6}, 6\right)$ when $v(t) < 0$ and $a(t) > 0$

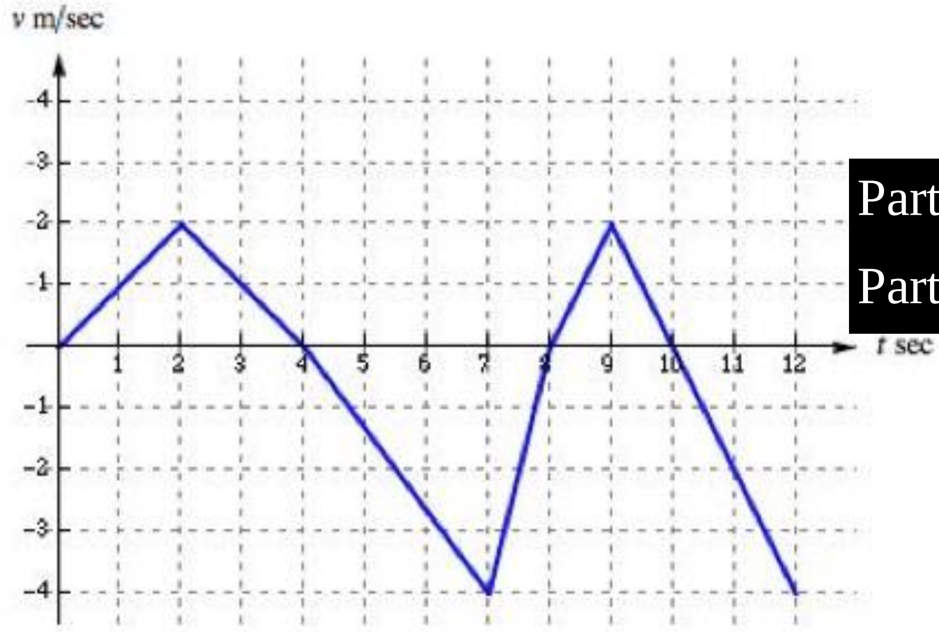
$\frac{19}{6} \approx 3.1667$

EXAMPLE 2F

SPEEDING UP SAME SIGNS	SLOWING DOWN SIGNS DIFFERENT
Positive Velocity Positive Acceleration	Positive Velocity Negative Acceleration
Negative Velocity Negative Acceleration	Negative Velocity Positive Acceleration

EXAMPLE 3

This problem deals with a particle in motion along an x-axis. For $0 \le t \le 12$, the particle's velocity $v(t)$ is given by a piecewise-linear function as shown in the figure below. For $0 < t < 12$, on which of the given t-intervals is the particle slowing down?

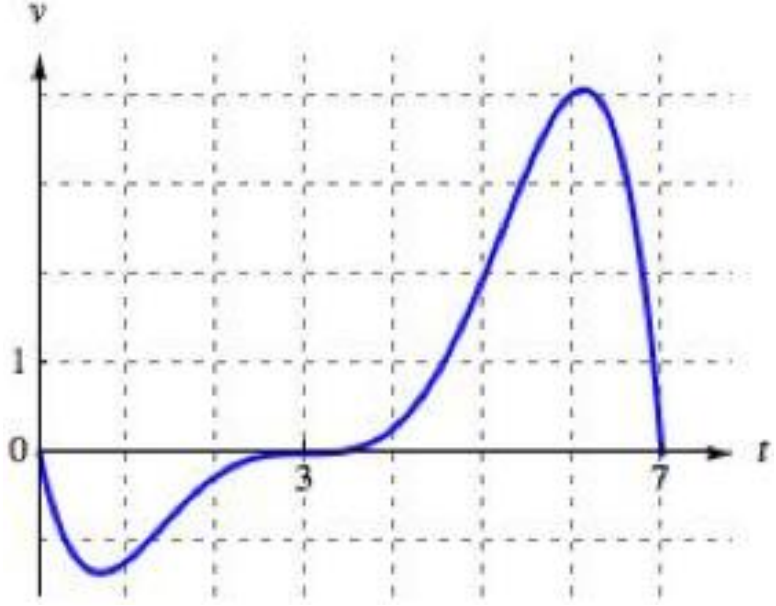


Particle is slowing down at $\text{I}(2,4)\cup(9,10)$ when $v(t)>0$ and $a(t)<0$

Particle is slowing down at $\text{I}(7,8)$ when $v(t)<0$ and $a(t)>0$

Example 4

The motion of a car moving along an east-west highway. Use eastward as the positive direction. The graph shows the velocity of a car moving along the highway at time t, where $0 \leq t \leq 7$. Which of the following statements is true? If the statement is untrue, explain why.



(A) The average acceleration of the car for $3 \leq t \leq 7$ is positive.

(B) The car travels in the same direction throughout the time interval $0 < t < 7$.

(C) The car has negative acceleration at $t = 5$.

(D) The car is slowing down at $t = 2$.

D

YOUR TURN

The position function of a particle moving on a straight line is $x(t) = 3t^4 - 16t^3 + 24t^2$ from $[0, 5]$ where t is measured in seconds and x is in feet. Determine the a) velocity at time t, b) acceleration at time t, c) at rest, d) particle slows down, e) identify the velocity when acceleration is first zero

$A)\, v(t) = 12t^3 - 48t^2 + 48t$

$B)\, a(t) = 36t^2 - 96t + 48$

$C)\, t = 0, t = 2$

$D)\, \left(\frac{2}{3}, 2\right)$

$E)\, v\left(\frac{2}{3}\right) = \frac{128}{9}$

AP Multiple Choice Practice Question 1 (non-calculator)

The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is

(A) 9

(B) $\frac{1}{9}$

(C) -9

(D) $-\frac{1}{9}$

AP Multiple Choice Practice Question 1
(Non-Calculator)

The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is:

Vocabulary	Connections and Process	Answer and Justifications
Velocity Acceleration Chain Rule	$x(t) = \sin(2t) - \cos(3t)$ $x'(t) = v(t) = 2\cos(2t) + 3\sin(3t)$ $x''(t) = v'(t) = a(t) = -4\sin(2t) + 9\cos(3t)$ $x''(\pi) = v'(\pi) = a(\pi) = -4\sin(2(\pi)) + 9\cos(3(\pi))$ $a(\pi) = -4(0) + 9(-1)$ $a(\pi) = -9$	*C*

Assignment

Worksheet