

§3.4A: Second Derivative Test

“I WILL ...

...identify the relative extrema using the Second Derivative Test.”

I. First Derivative Test and Concavity

- A.  $f'$  positive then  $f$  is: \_\_\_\_\_
- B.  $f'$  negative then  $f$  is: \_\_\_\_\_
- C.  $f''$  positive then  $f'$  is Concave \_\_\_\_\_
- D.  $f''$  negative then  $f'$  is Concave \_\_\_\_\_
- E.  $f'(c) = 0$  or  $f'(c) = DNE$  to which  $c$  is  $x$  of a critical point, then the point is called \_\_\_\_\_
- F.  $f''(x) = 0$  or  $f''(x) = DNE$  and  $f''(x)$  changes signs, then the point is called \_\_\_\_\_

II. Second Derivative Test

- A. Let  $f$  be a function such that  $f'(c) = 0$  and  $f''$  exists on an open interval containing  $c$ :
  - 1. If  $f''(c) > 0$ , then  $f(x)$  has a \_\_\_\_\_ at  $(c, f(c))$  and  $f$  is concave up
  - 2. If  $f''(c) < 0$ , then  $f(x)$  has a \_\_\_\_\_ at  $(c, f(c))$  and  $f$  is concave down
  - 3. If  $f'(x) = 0$  and  $f''(x) = 0$ , then  $f(x)$  is \_\_\_\_\_. Use the first derivative test.

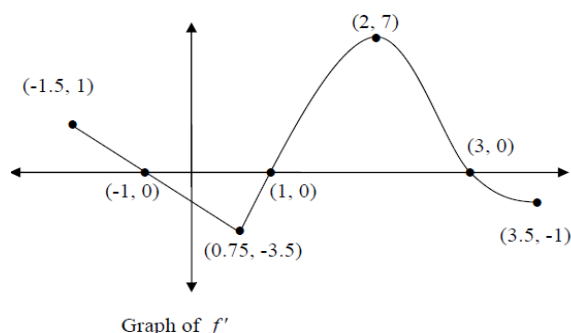
Ex 1: Using the Second Derivative Test to find the relative extrema of  $f(x) = x + \frac{2}{x}$  and justify.

Ex 2: Using the Second Derivative Test to find the relative extrema of  $f(x) = -3x^5 + 5x^3$  and justify.

Your Turn: Using the Second Derivative Test to find the relative extrema of  $f(x) = \frac{1}{3}x^3 - x^2 - 3x$  and justify.

Ex 3: Suppose that the function  $f$  has a continuous second derivative for all  $x$  and that  $f(-1) = 2$ ,  $f'(-1) = -3$ ,  $f''(-1) = 5$ . Let  $g$  be a function whose derivative is given by  $g'(x) = (x^4 - 6x^3)(3f(x) + 2f'(x))$  for all  $x$ . Write an equation of the tangent line to the graph of  $f$  at the point of where  $x = -1$ . Justify response.

Ex 4: The figure below shows the graph of the derivative of  $f$ ,  $f'$  on the closed interval  $[-1.5, 3.5]$ . The graph of  $f'$  has a horizontal tangent line at  $x = 2$  and is linear on the interval  $[-1.5, 0.75]$ .



- (a) Find the  $x$ -coordinates of the relative maxima of  $f$ . Justify your answer.
- (b) Find the  $x$ -coordinates of the points of inflection of  $f$ . Justify your answer.
- (c) On what intervals is  $f$  decreasing? Justify your answer.
- (d) Is the function  $f$  twice-differentiable? Justify your answer.

AP 1: Let  $h$  be twice differentiable.  $h'(x) > 0$  and  $h''(x) > 0$  for all reals. What is a possible value for  $h(3)$  if  $h(0) = 0$ ,  $h(1) = 2$  and  $h(2) = 7$  ?

(A) 12

(B) 7

(C) 4

(D) 14

Vocabulary	Connections and Process	Answer and Justifications