First Derivative Test

Section 3.3

Calculus AP/Dual, Revised ©2017

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DEFINITIONS

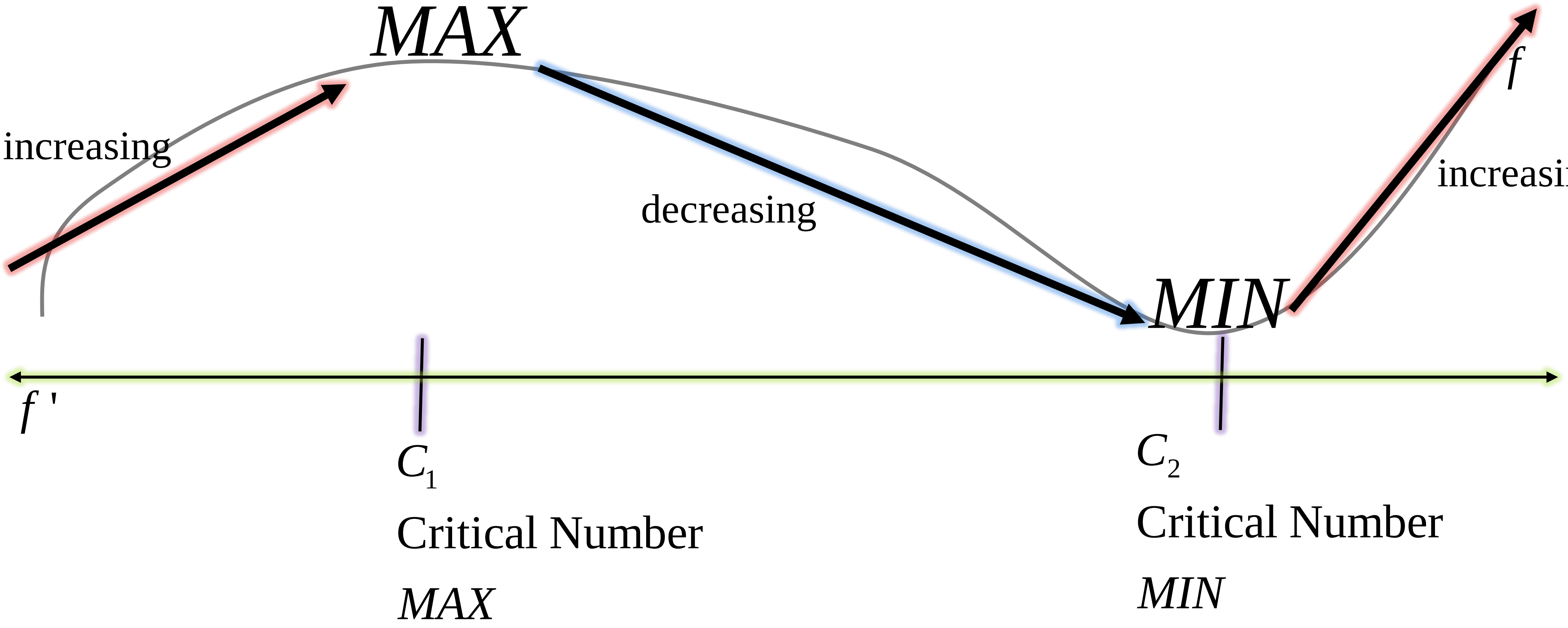
A. **First Derivative Test** is to find the intervals of increasing or decreasing and located any relative extrema

   1. If $f'(x) > 0$, then f is INCREASING
   2. If $f'(x) < 0$, then f is DECREASING

B. **Critical Point/Critical Number** can tell us whether we have a relative minimum or relative maximum when the derivative is equal to ZERO or UNDEFINED

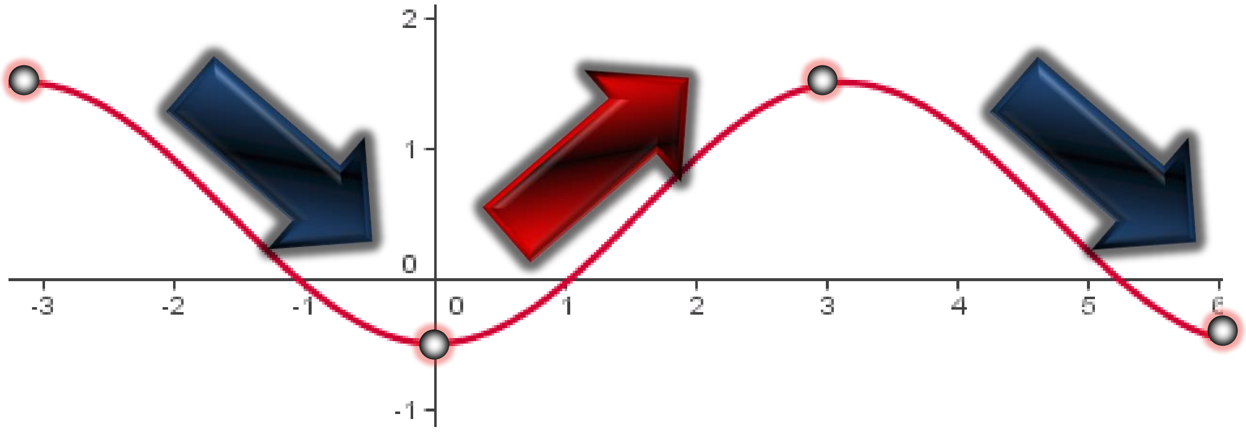
   1. If f' changes signs from positive to negative, then it is a RELATIVE MAXIMUM
   2. If f' changes signs from negative to positive, then it is a RELATIVE MINIMUM

VISUAL EXAMPLE



EXAMPLE 1

Identify the intervals where function $f(x)$ is increasing or decreasing.

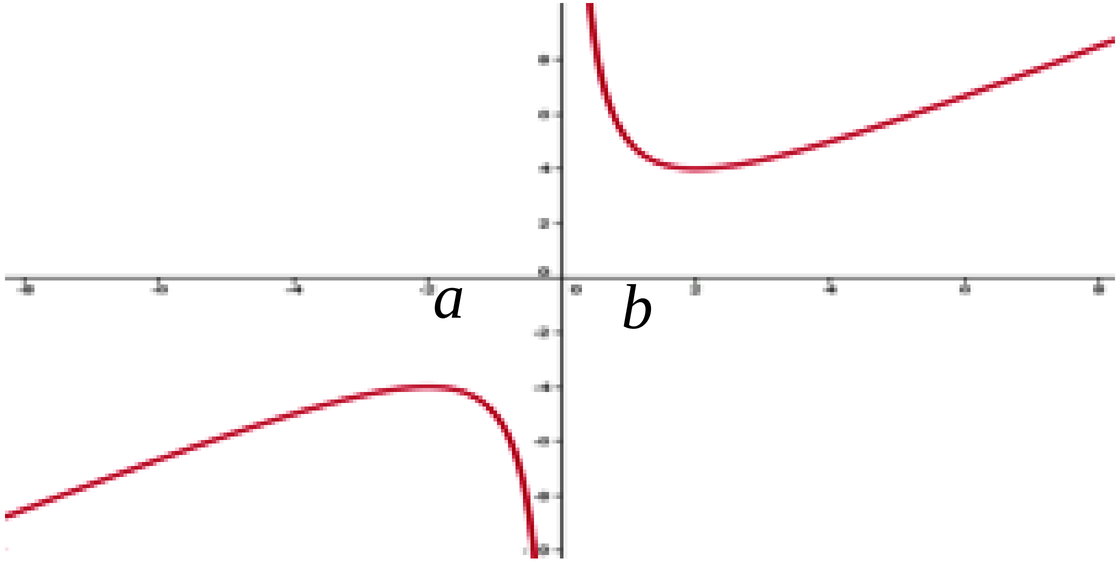


$f(x)$ is Increasing at $I(0,3)$ when $f' > 0$

$f(x)$ is Decreasing at $I(-3,0)\cup(3,6)$ when $f' < 0$

EXAMPLE 2

Identify the intervals where function $f(x)$ is increasing or decreasing.

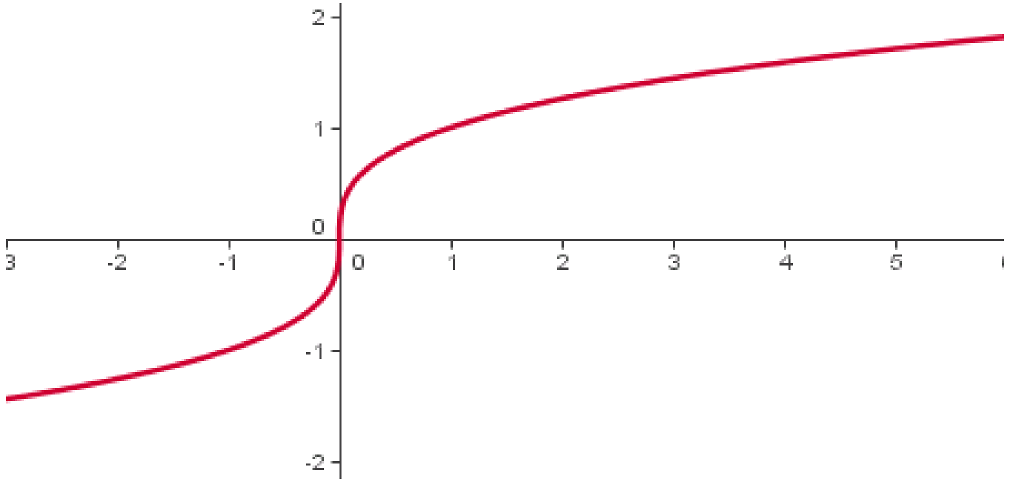


$f(x)$ is Increasing at $I(-\infty, a) \cup (b, \infty)$ when $f' > 0$

$f(x)$ is Decreasing at $I(a, 0) \cup (0, b)$ when $f' < 0$

Your Turn

Identify the intervals where function $f(x)$ is increasing or decreasing.



$f(x)$ is Increasing at $I(-\infty,0)\cup(0,\infty)$ at $f'>0$

$f(x)$ is Decreasing at no intervals

Identifying Increasing and Decreasing Functions

A. **Find the Critical Points by taking the derivative and write intervals**

B. **Substitute a value from each interval into to test it**

C. **Indicate how the function behaves (Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b))**

   1. **If $f'(x) > 0$ for all x in (a, b), then f is <u>increasing</u> on $[a, b]$**
   2. **If $f'(x) < 0$ for all x in (a, b), then f is <u>decreasing</u> on $[a, b]$**
   3. **If $f'(x) = 0$ for all x in (a, b), then f is <u>constant</u> on $[a, b]$**

D. **Apply to the ORIGINAL function to establish the maximums and minimums**

E. **Make sure to justify reasoning with explanations and math symbols/definitions**

Example 3

Find the open intervals on which $f(x) = x^2 + 8x + 10$ is increasing or decreasing and locate any Relative Extrema.

$$f(x) = x^2 + 8x + 10$$

$$f'(x) = 2x + 8$$

$$2(x+4) = 0$$

$$\text{Critical Number}: x = -4$$

EXAMPLE 3

Find the open intervals on which $f(x) = x^2 + 8x + 10$ is increasing or decreasing and locate any Relative Extrema.

$$f'(x) = 2x + 8$$

$$\text{Critical Point}: (-4, -6)$$

$(-\infty, -4)$	$x = -4$	$(-4, \infty)$
$f'(-5)$	$f(-4)$	$f'(0)$
$f'(-5) = 2x + 8$	$f(-4) = x^2 + 8x + 10$	$f'(0) = 2x + 8$
$f'(-5) = 2(-5) + 8$	$f(-4) = (-4)^2 + 8(-4) + 10$	$f'(0) = 2(0) + 8$
$f'(-5) = (-10) + (8)$	$f(-4) = 16 - 32 + 10$	
$f'(-5) = -2$	$(-4, -6)$	$f'(0) = 8$
$-$ DECREASING	$f(-4): - \to +$ Relative MIN	$+$ INCREASING

EXAMPLE 3

Find the open intervals on which $f(x) = x^2 + 8x + 10$ is increasing or decreasing and locate any Relative Extrema.

$f(x)$ is Increasing at $I(-4, \infty)$ when $f' > 0$

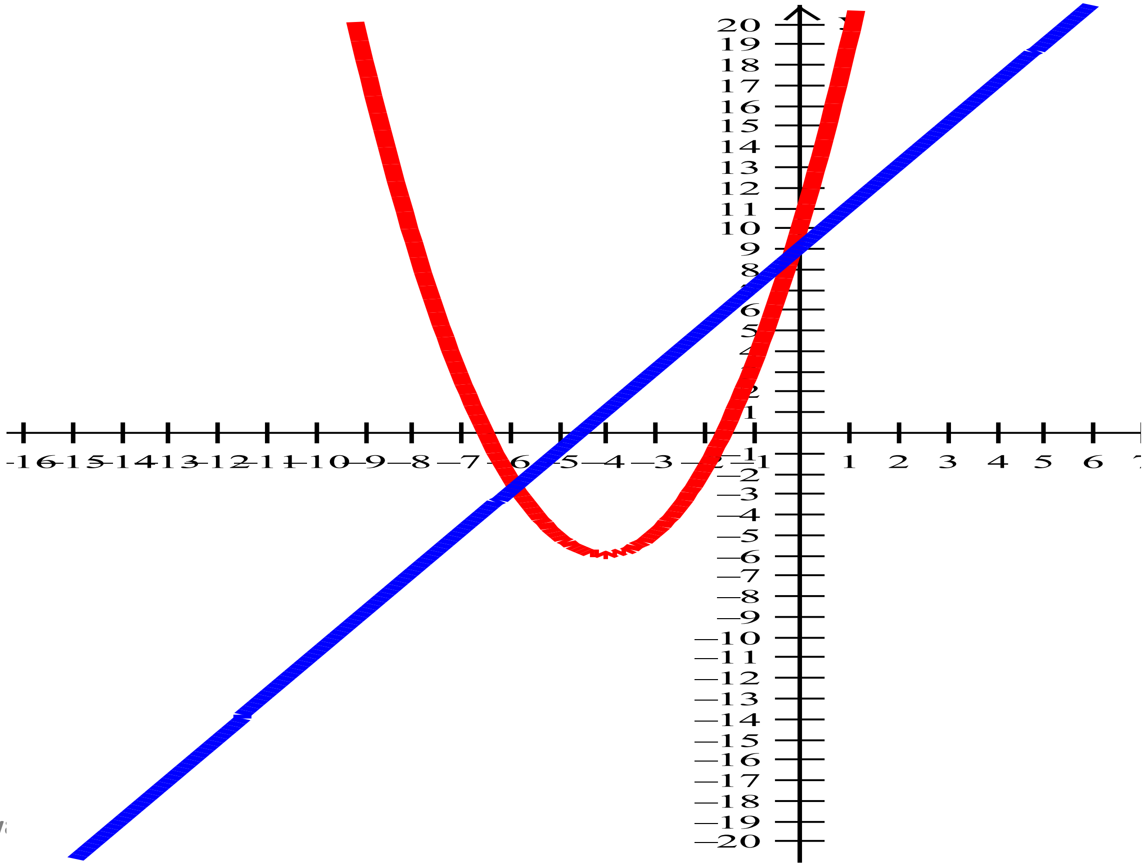
$f(x)$ is Decreasing at $I(-\infty, -4)$ when $f' < 0$

$f(x)$ *has* a Relative Minimum at $(-4, -6)$

when the sign of $f'(x)$ changes

from Negative to Positive.

$f(x)$ does not have a Relative M*aximum.*



EXAMPLE 4

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing and locate any Relative Extrema.

$$f(x) = x^3 - \frac{3}{2}x^2$$

$$f'(x) = 3x^2 - 3x$$

$$3x(x-1) = 0$$

$$\text{Critical Points}: x = 0, x = 1$$

Example 4

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing and locate any Relative Extrema.

$$f'(x) = 3x^2 - 3x$$

$$\text{Critical Points}: (0,0) \,\&\, \left(1, -\frac{1}{2}\right)$$

$(-\infty, 0)$	$x = 0$	$(0, 1)$	$x = 1$	$(1, \infty)$
$f'(-1)$	$f(0)$	$f'\left(\frac{1}{2}\right)$	$f(1)$	$f'(2)$
$f'(-1) = 3x^2 - 3x$	$f(0) = x^3 - \frac{3}{2}x^2$	$f'(1/2) = 3x^2 - 3x$	$f(1) = x^3 - \frac{3}{2}x^2$	$f'(2) = 3x^2 - 3x$
$f'(-1) = 3(-1)^2 - 3(-1)$	$f(0) = (0)^3 - \frac{3}{2}(0)^2$	$f'(1/2) = 3(1/2)^2 - 3(1/2)$	$f(1) = (1)^3 - \frac{3}{2}(1)^2$	$f'(2) = 3(2)^2 - 3(2)$
$f'(-1) = (3) + (3)$	$f(0) = 0 - 0$		$f(1) = \frac{2}{2} - \frac{3}{2}$	$f'(2) = (12) - (6)$
$f'(-1) = 6$	$(0,0)$	$f'(1/2) = -\frac{3}{4}$	$\left(1, -\frac{1}{2}\right)$	$f'(2) = 6$
$+$ Increasing	$f(0): + \rightarrow -$ Relative MAX	$-$ Decreasing	$f(1): - \rightarrow +$ Relative MIN	$+$ Increasing

Example 4

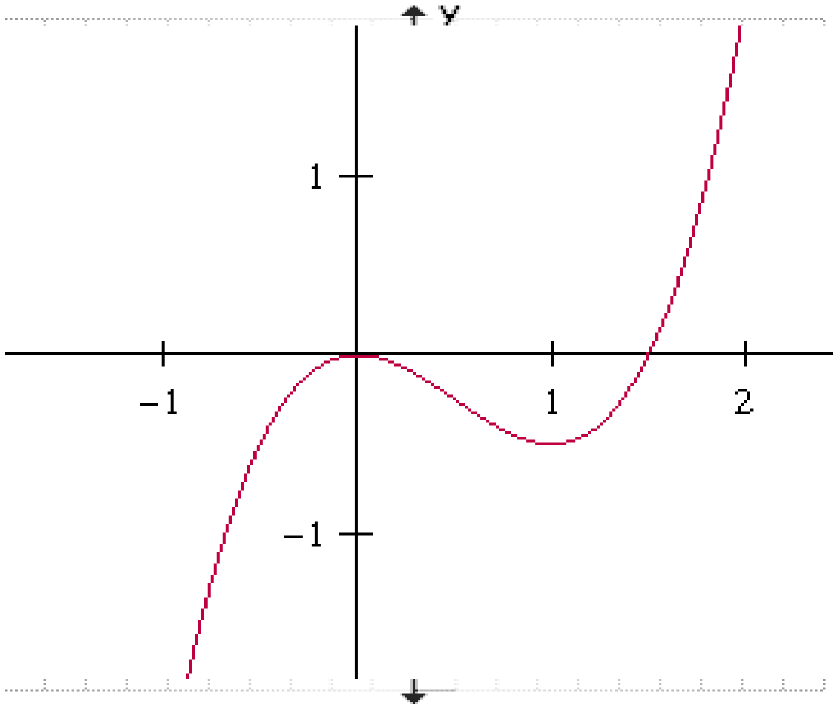
Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing and locate any Relative Extrema.

$f(x)$ is Increasing at $I(-\infty,0)\cup(1,\infty)$ when $f'(0) > 0$

$f(x)$ is Decreasing at $I(0,1)$ when $f'(0) < 0$

$f(x)$ has a Relative Maximum at $(0,0)$
because the sign of $f'(x)$ changes
from Positive to Negative.

$f(x)$ has a Relative Minimum at $\left(1,-\frac{1}{2}\right)$
because the sign of $f'(x)$ changes
from Negative to Positive.



EXAMPLE 5

Find the open intervals on which $h(x) = \frac{x+3}{x^2}$ is increasing or decreasing and locate any Relative Extrema.

$h(x)$ is Increasing at $I(-6, 0)$ when $h' > 0$

$h(x)$ is Decreasing at $I(-\infty . -6) \cup (0, \infty)$ when $h' < 0$

$h(x)$ has a Relative Minimum at $\left(-6, -\frac{1}{12}\right)$

because the sign of $h'(x)$ changes

from Negative to Positive.

$h(x)$ has no Relative Maximum at $x = 0$ because

there is a vertical asymptote.

YOUR TURN

Find the open intervals on which $f(x) = x^3 - 6x^2 + 15$ is increasing or decreasing and locate any Relative Extrema.

$f(x)$ is Increasing at $I(-\infty, 0) \cup (4, \infty)$ when $f' > 0$

$f(x)$ is Decreasing at $I(0, 4)$ when $f' < 0$

$f(x)$ has a Relative Maximum at $(0,15)$
because the sign of $f'(x)$ changes
from Positive to Negative.

$f(x)$ has a Relative Minimum at $(4, -17)$
because the sign of $f'(x)$ changes
from Negative to Positive.

EXAMPLE 6

Find the open intervals from $[0, 2\pi]$ on which $f(x) = \cos x - 1$ is increasing or decreasing and Extrema.

$$f(x) = \cos x - 1$$

$$f'(x) = -\sin x$$

$$-\sin x = 0$$

$$CP: x = 0, x = \pi, x = 2\pi$$

Example 6

Find the open intervals from $[0, 2\pi]$ on which $f(x) = \cos x - 1$ is increasing or decreasing and Extrema.

$x = 0$	$(0, \pi)$	$x = \pi$	$(\pi, 2\pi)$	$x = 2\pi$
$f(0)$ $f(0) = \cos(0) - 1$ $f(0) = (1) - (1)$ $f(0) = 0$ $(0,0)$ Absolute MAX	$f'\left(\frac{\pi}{4}\right)$ $f'\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$ $f'\left(\frac{\pi}{4}\right) = -\left(\frac{\sqrt{2}}{2}\right)$ $-$ Decreasing	$f(\pi)$ $f(0) = \cos(\pi) - 1$ $f(\pi) = (-1) - (1)$ $f(\pi) = -2$ $(\pi, -2)$ Absolute MIN	$f'\left(\frac{3\pi}{2}\right)$ $f'\left(\frac{3\pi}{2}\right) = -\sin\left(\frac{3\pi}{2}\right)$ $f'\left(\frac{\pi}{4}\right) = -(-1)$ $+$ Increasing	$f(2\pi)$ $f(2\pi) = \cos(2\pi) - 1$ $f(\pi) = (1) - (1)$ $f(2\pi) = 0$ $(2\pi, 0)$ Absolute MAX

Example 6

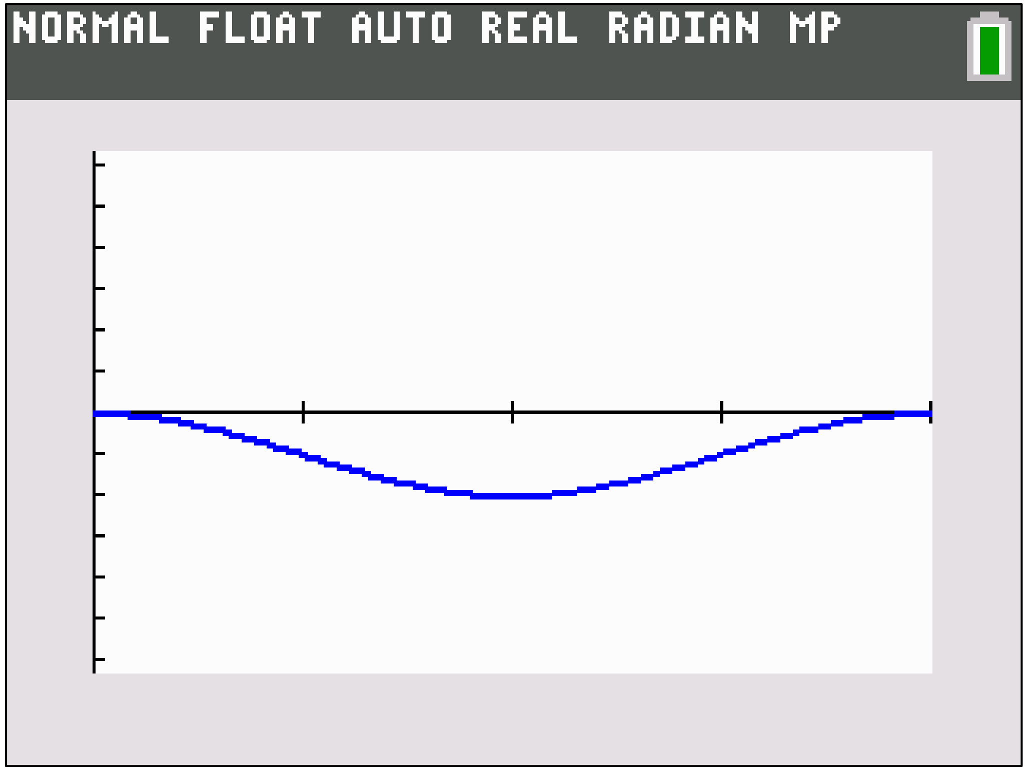
Find the open intervals from $[0, 2\pi]$ on which $f(x) = \cos x - 1$ is increasing or decreasing and Extrema.

$f(x)$ is increasing at $I(\pi, 2\pi)$ when $f' > 0$

$f(x)$ is decreasing at $I\ (0, \pi)$ when $f' < 0$

$f(x)$ *has* an Absolute Maximum at $(0,0)$ and $(2\pi, 0)$ because the sign of $f'(x)$ changes from POS to NEG.

$f(x)$ has an Absolute Minimum at $(\pi, -2)$ because the sign of $f'(x)$ changes from NEG to POS.



YOUR TURN

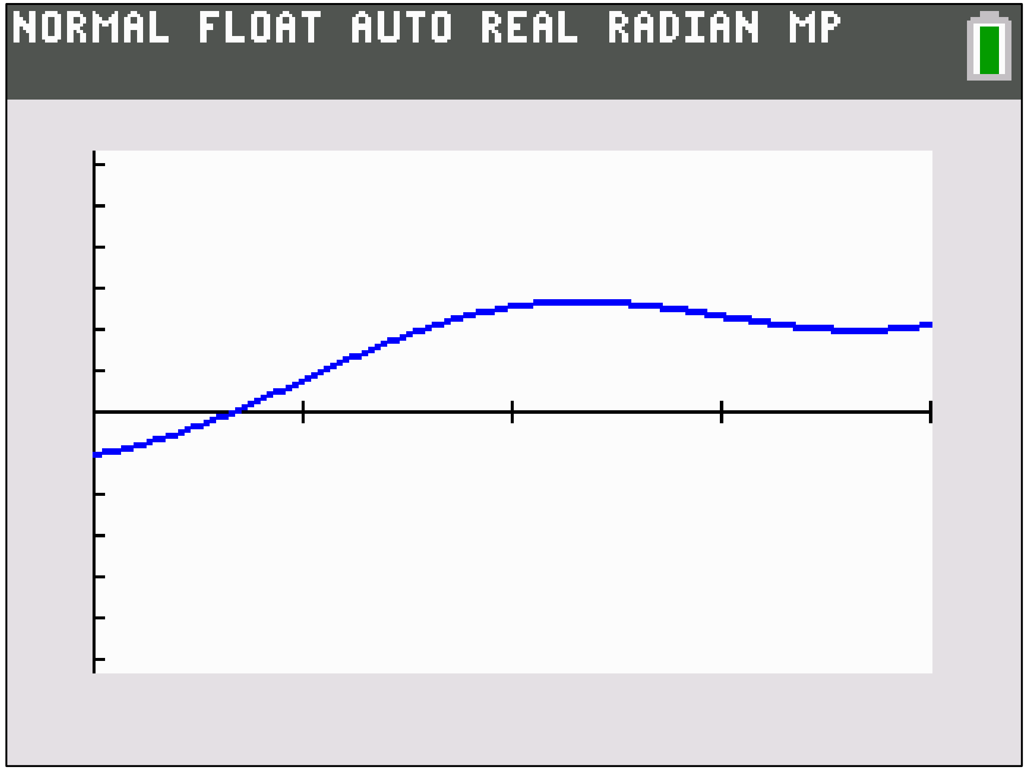
Find the open intervals from $[0, 2\pi]$ on which $f(x) = \frac{x}{2} - \cos x$ is increasing or decreasing and Extrema.

$f(x)$ is increasing at $I\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$ when $f' > 0$

$f(x)$ is decreasing at $I\left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$ when $f' < 0$

$f(x)$ *has* a Relative Maximum at $\left(\frac{\pi}{3}, \frac{\pi}{6} + \frac{1}{2}\right)$
because the sign of $f'(x)$ changes
from POS to NEG.

$f(x)$ *has* a Relative Minimum at $\left(\frac{5\pi}{3}, \frac{5\pi}{6} + \frac{1}{2}\right)$
because the sign of $f'(x)$ changes
from NEG to POS.



AP Multiple Choice Practice Question 1 (non-calculator)

The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

AP Multiple Choice Practice Question 1 (non-calculator)

The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

Vocabulary	Connections and Process	Answer and Justifications
First Der. Test Increasing	$f'(x) = 4x^3 + 2x$ $4x^3 + 2x = 0$ $2x(2x^2 + 1) = 0$ $x = 0$ **$(-\infty, 0)$**: $f'(-1) = 4(-1)^3 + 2(-1) = -4 - 2 = -6$ **Decreasing** **$(0, \infty)$**: $f'(1) = 4(1)^3 + 2(1) = 4 + 2 = 6$ **Increasing**	C f is increasing on any interval where $f'(x) > 0$.

ASSIGNMENT

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