

§3.2: Mean Value Theorem

Determine if the MVT applies to $f(x)$ on the given interval. If so, find the value(s) guaranteed by the theorem. If not, explain why. Make sure to test the end points and apply the equation.

1) Given: $f(x) = -x^2 + 3x$ at $[0,3]$

$$f(0) =$$

$$f(3) =$$

Is f continuous at $[0,3]$ and differentiable at $(0,3)$? **Yes** or **No**

Mean Value Equation:

$$C =$$

2) $f(x) = (x - 1)(x - 2)(x - 3)$ at $[1,3]$

3) $f(x) = x^{2/3} - 1$ at $[-8,8]$

4) $f(x) = \sin x$ at $[0,2\pi]$

5) $f(x) = \sin 3x$ at $\left[0, \frac{\pi}{3}\right]$

6) $f(x) = x^{2/3}$ at $[0,1]$

7) A student was completing the problem on the whiteboard. You are the teacher and are concerned. Identify the problem and correct his mistake on the right-hand side and solve. Then explain the error. BE SPECIFIC.

Student's work:

Consider the function $f(x) = x^k - 3x^2 + 1$ where k is a nonzero constant.

Let $k = 2$ so that $f(x) = -2x^2 + 1$. Explain why there must be a value c for $-1 < c < 2$, such that $f'(c) = -2$.

$$f(x) = -2x^2 + 1$$

$$f'(x) = -4x$$

$$f'(c) = -4c$$

$$\frac{-4c}{-4} = \frac{-2}{-4}$$

$$c = \frac{1}{2}$$

Your Corrections

Explanation of error: _____

7) (2008B-AB 5) Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown on the right for $-3 \leq x \leq 7$.

Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 \leq x \leq 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

