Extrema on an Interval

Section 3.1

Calculus AP/Dual, Revised ©2017

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DEFINITIONS

A. Relative Extrema vs Absolute Extrema

1. **Relative (Local) Extrema - occur on an open interval**
2. **Absolute (Global) Extrema - occur on a closed interval**
   A. **If there is an open interval containing c on which is a minimum, then $(c, f(c))$ is called a relative minimum of f, or you can say that f has a relative minimum at $(c, f(c))$. It is also called the local minimum. Relative Minimums are like *valleys* of the graphs.**
   B. **If there is an open interval containing c on which is a maximum, then $(c, f(c))$ is called a relative maximum of f, or you can say that f has a relative maximum at $(c, f(c))$. It is also called the local maximum. Relative Maximums are like *hills* of the graphs.**

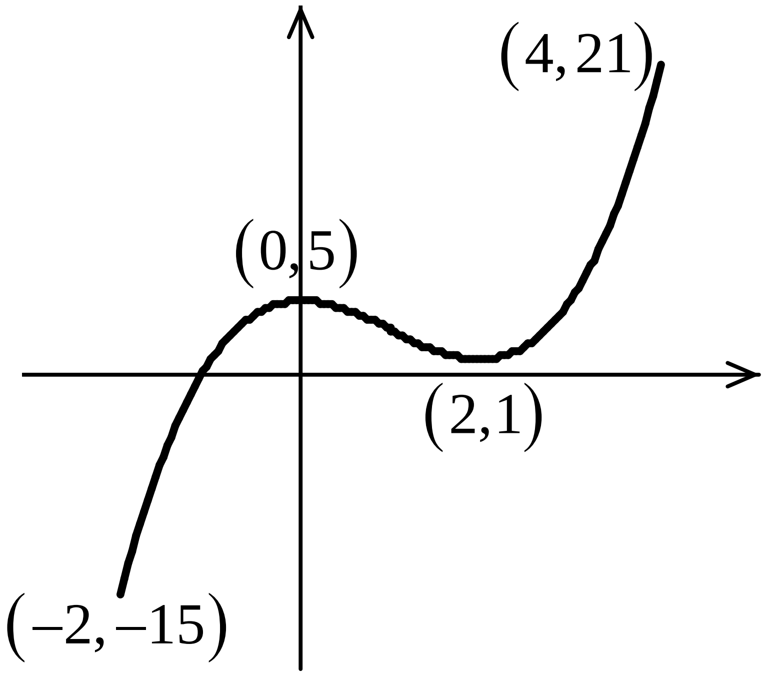
Extreme Value Theorem

<u>Extreme Value Theorem</u> is if f is continuous on a closed interval $[a, b]$, then f has both a minimum $f(c) \leq f(x)$ and a maximum $f(c) \geq f(x)$ on the interval.

1. f must be <u>continuous</u>
2. Holes do not qualify for maximums or minimums

Example 1

Where would the absolute and relative maxima be located on the graph, based on the interval $[-2, 4]$?



$Abs.Min.: (-2, -15)$

$Abs.Max.: (4, 21)$

Rel. $Min.: (2,1)$

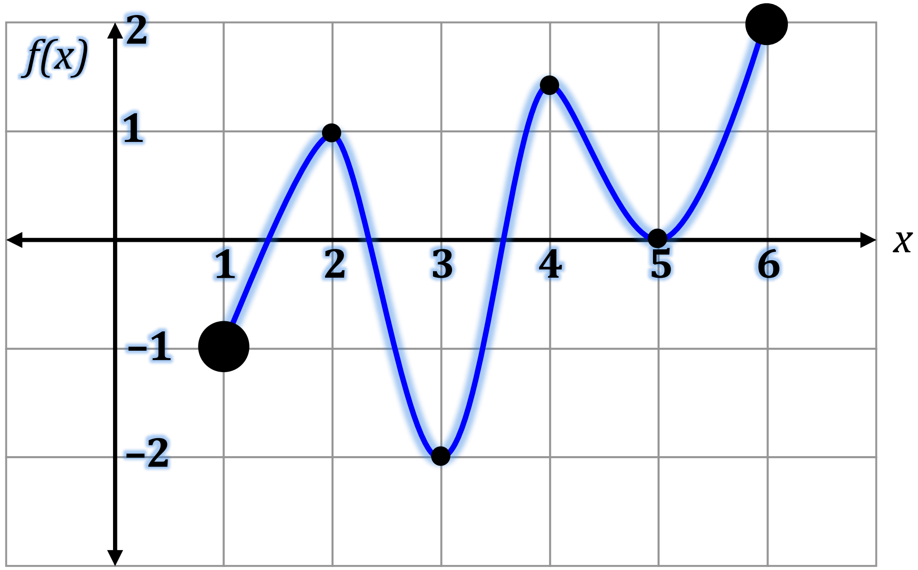
Rel. $Max.: (0,5)$

DETERMINE ABSOLUTE EXTREMA & CRITICAL POINTS

A. To Determine the Absolute Extrema of a Function given the equation:
    1. Identify the <u>Critical Points</u>
    2. Check the <u>Endpoints</u>
    3. Make a t-chart and plug it into the <u>ORIGINAL</u> equation
B. If $f'(c) = 0$ (where top equals zero) or if $f'(c) = DNE$ (where the bottom of the derivative equals zero) then c is a <u>critical Point</u> of f.
    1. Check where the derivative equals Zero (<u>Peak</u> or <u>Valley</u> on Graph)
    2. Results in horizontal tangent lines
    3. Therefore, a max or min and where the function is not differentiable

Example 2

Establish the Critical Point(s) given the $f(x)$ graph



$$CP: x = 2, x = 3, x = 4, x = 5$$

EXAMPLE 3

Identify any Critical Point(s) of $f(x) = x^3 + x^2 - 8x + 5$

$$f(x) = x^3 + x^2 - 8x + 5$$

$$f'(x) = 3x^2 + 2x - 8$$

$$3x^2 + 2x - 8 = 0$$

$$(3x - 4)(x + 2) = 0$$

$$x = -2, x = \frac{4}{3}$$

EXAMPLE 3

Identify any Critical Point(s) of $f(x) = x^3 + x^2 - 8x + 5$

$$x = -2, x = \frac{4}{3}$$

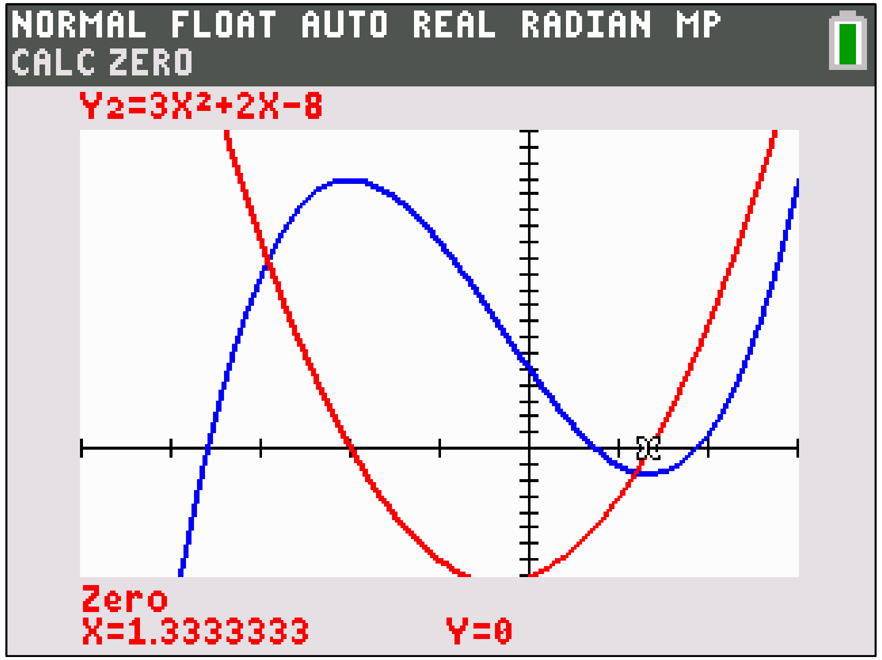
$$f(-2) = (-2)^3 + \left((-2)^2\right) - 8(-2) + 5 = 17$$

$$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 + \left(\left(\frac{4}{3}\right)^2\right) - 8\left(\frac{4}{3}\right) + 5 = -\frac{41}{27}$$

$$CP : (-2,17), \left(\frac{4}{3}, -\frac{41}{27}\right)$$

Example 3

Identify any Critical Point(s) of $f(x) = x^3 + x^2 - 8x + 5$



$$CP:(-2,17),\left(\frac{4}{3},-\frac{41}{27}\right)$$

EXAMPLE 4

Identify any Critical Point(s) of $\boldsymbol{f(x) = \frac{4x}{x^2-3}}$

$$f'(x) = \frac{(x^2-3)(4)-(4x)(2x)}{(x^2-3)^2}$$

$$f'(x) = \frac{4x^2-12-8x^2}{(x^2-3)^2}$$

$$f'(x) = \frac{-4x^2-12}{(x^2-3)^2}$$

EXAMPLE 4

Identify any Critical Point(s) of $f(x) = \frac{4x}{x^2-3}$

$$f'(x) = \frac{-4x^2 - 12}{\left(x^2-3\right)^2}$$

$$-4x^2 - 12 = 0$$

$$-4\left(x^2+3\right) = 0$$

$$x^2 + 3 = 0$$

~~$x = \pm\sqrt{3}$~~

$$\left(x^2-3\right)^2 = 0$$

$$x^2 = 3$$

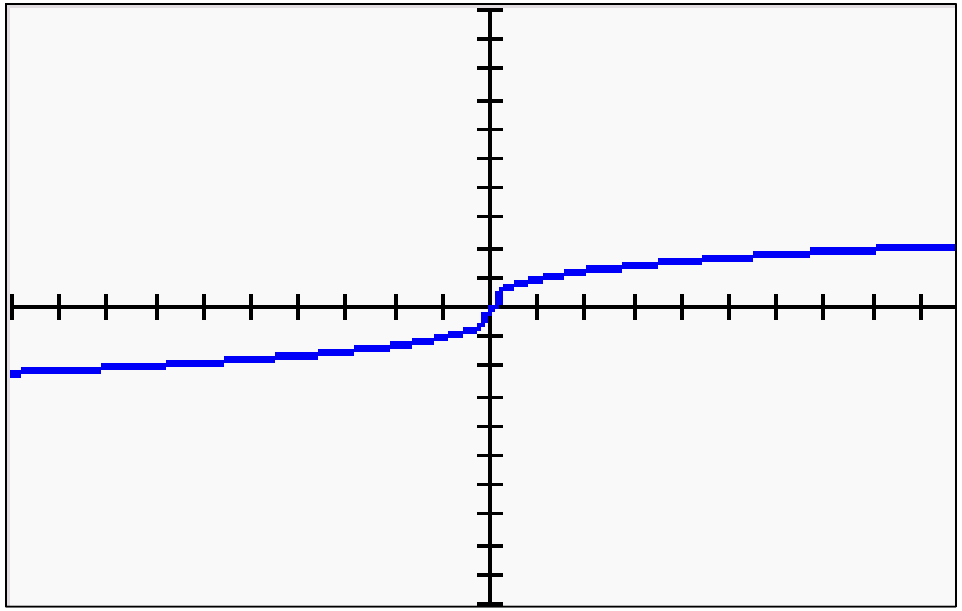
$$x = \pm\sqrt{3}$$

$$\left(\pm\sqrt{3}, und.\right)$$

Example 5

Identify any Critical Point(s) of $f(x) = x^{1/3}$

$$f'(x) = \frac{1}{3}x^{-2/3}$$



$$\frac{1}{3}x^{-2/3} = 0$$

$$\frac{1}{3x^{2/3}} = 0$$

$$x = 0$$

Your Turn

Identify any Critical Point(s) of $f(x) = \left(9 - x^2\right)^{3/5}$ (x −coordinate will be fine)

$$x = -3, 0, 3$$

CANDIDATES TEST

A. For closed intervals only, identify the Critical Points where $f\ '(x) = 0$ or $f'(x) = DNE$

B. Make a t-chart with x and $f(x)$

C. X-candidates are the endpoints and Critical Points

D. Find the associated y-Points and identify the max or min to the ORIGINAL equation

E. Reminder

   1. If the question is asking *WHAT* is the max or min value in the interval, LOOK AT THE $\underline{f(x) \text{ or } y\text{-Value}}$
   2. If the question is asking *WHERE* is the max or min value in the interval, LOOK AT THE $\underline{x\text{-Value}}$

Example 6

Identify the Absolute Extrema of $f(x) = x^3 - 12x$ on the interval $[0, 4]$

$$f(x) = x^3 - 12x$$

$$f'(x) = 3x^2 - 12$$

$$f'(x) = 3(x^2 - 4)$$

$$3(x^2 - 4) = 0$$

$$x = \pm 2$$

only C.P. is $x = 2$

because $x = -2$ is not in range

x	$f(x)$
0	$f(0) = (0)^3 - 12(0)$ $f(0) = 0 - 0$ $f(0) = 0$
2	$f(2) = (2)^3 - 12(2)$ $f(2) = 8 - 24$ $f(2) = -16$
4	$f(4) = (4)^3 - 12(4)$ $f(4) = 64 - 48$ $f(4) = 16$

$Abs.\ Min.: (2, -16)$

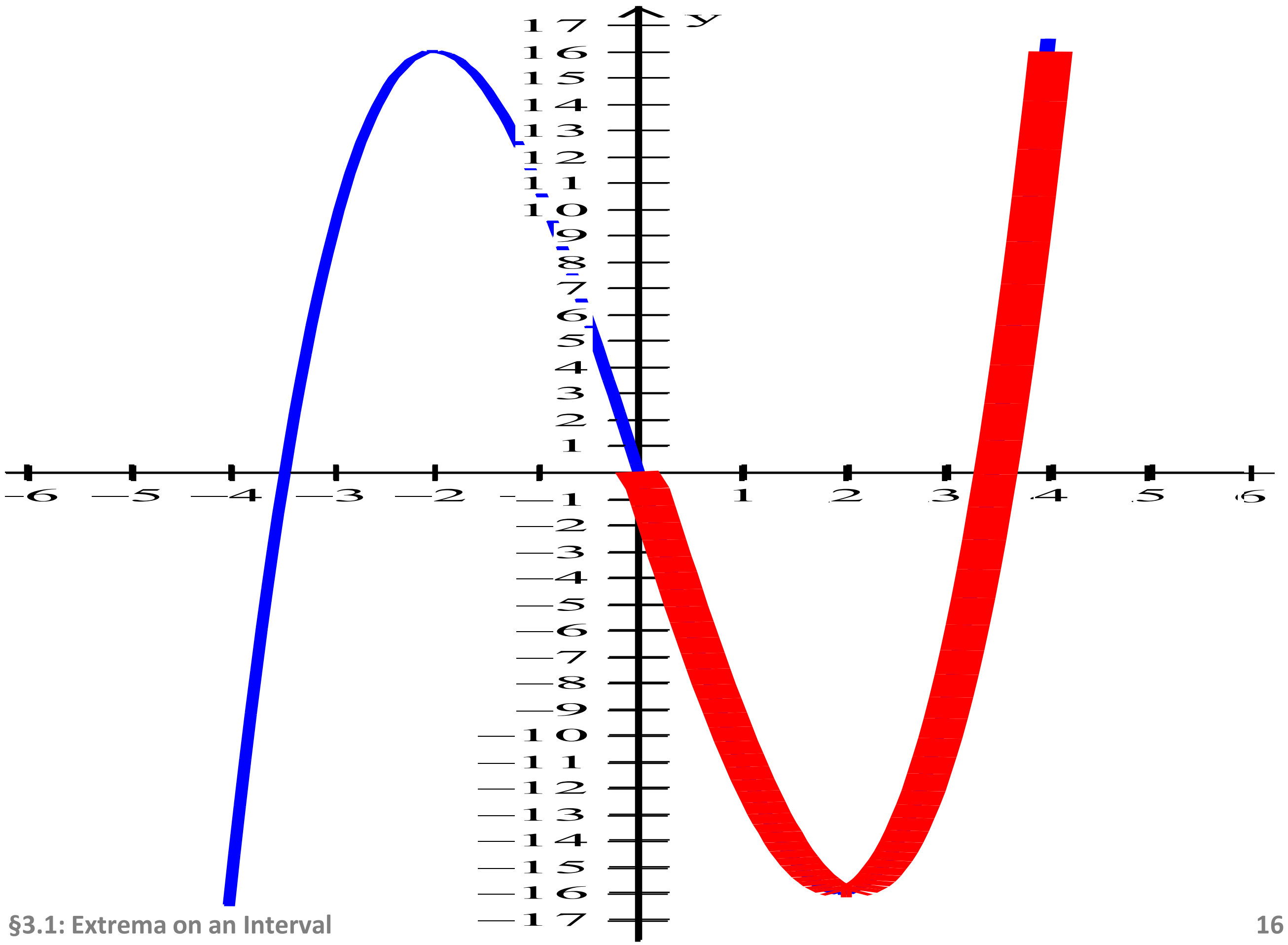
$Abs.\ Max.: (4, 16)$

Example 6

Identify the Absolute Extrema of $f(x) = x^3 - 12x$ on the interval $[0, 4]$

$Abs.\ Min.: (2, -16)$

$Abs.\ Max.: (4, 16)$



Example 7

Find the Absolute Extrema of $f(x) = 3x^{2/3} - 2x$ on the interval $[-1, 1]$

$$f'(x) = 2x^{-1/3} - 2$$

Critical Points

$$2x^{-1/3} - 2 = 0$$

$$\frac{2}{x^{1/3}} - 2 = 0$$

$$\frac{2}{x^{1/3}} - \frac{2x^{1/3}}{x^{1/3}} = 0$$

$$\frac{2 - 2x^{1/3}}{x^{1/3}} = 0$$

$$2 - 2x^{1/3} = 0$$

$$2x^{1/3} = 2$$

$$x^{1/3} = 1$$

$$x = 1$$

$$x^{1/3} = 0$$

$$x = 0$$

$$CP: x = 0,1$$

Example 7

Find the Absolute Extrema of $f(x) = 3x^{2/3} - 2x$ on the interval $[-1, 1]$

$Abs.Min.:(0,0)$

$Abs.Max.:(-1,5)$

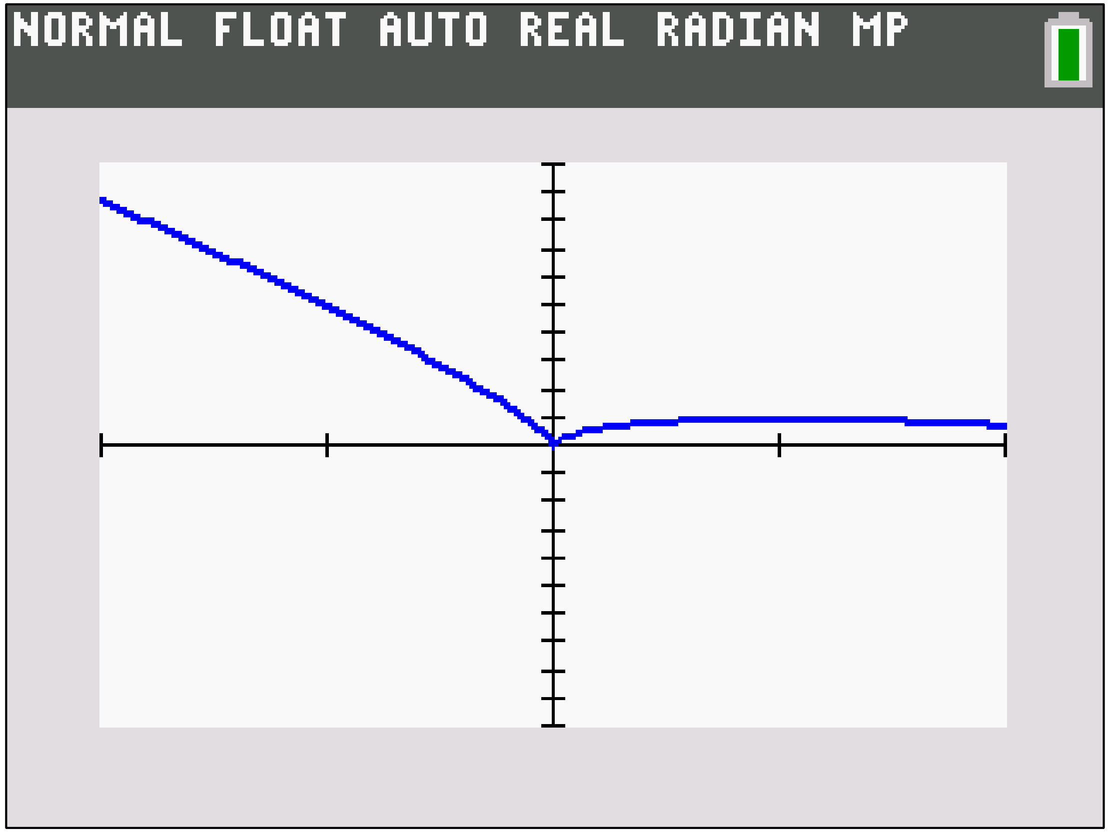
x	$f(x)$
-1	$f(-1) = 3(-1)^{2/3} - 2(-1)$ $f(-1) = 3(1) - 2(-1)$ $f(-1) = 5$
0	$f(0) = 3(0)^{2/3} - 2(0)$ $f(0) = 3(0) - 2(0)$ $f(0) = 0$
1	$f(1) = 3(1)^{2/3} - 2(1)$ $f(1) = 3 - 2$ $f(1) = 1$

Example 7

Find the Absolute Extrema of $f(x) = 3x^{2/3} - 2x$ on the interval $[-1, 1]$

$Abs.Min.: (0,0)$

$Abs.Max.: (-1,5)$



EXAMPLE 8

Identify the maximum value of $f(x) = -x - e^{1-x}$ on the interval $[0, 3]$ and justify

$$f(x) = -x - e^{1-x}$$

$$f'(x) = -1 + e^{1-x}$$

$$-1 + e^{1-x} = 0$$

$$e^{1-x} = 1$$

$$\ln e^{x-1} = \ln 1$$

$$x - 1 = 0$$

$$CP: x = 1$$

EXAMPLE 8

Identify the maximum value of $f(x) = -x - e^{1-x}$ on the interval $[0, 3]$ and justify

$CP : x = 1$

x	$f(x)$
0	$f(0) = -(0) - e^{1-(0)}$ $f(0) = 0 - e$ $f(0) = -e$
1	$f(1) = -(1) - e^{1-(1)}$ $f(1) = -1 - e^{0}$ $f(1) = -1 - 1 = -2$
3	$f(3) = -(3) - e^{1-(3)}$ $f(3) = -3 - e^{-2}$ $f(3) = -3 - \frac{1}{e^{2}}$

f has a maximum value of -2 when $x = 1$.

YOUR TURN

Find the Absolute Extrema of $f(t) = \frac{t}{t-2}$ **on the interval** $[1, 5]$

$$Abs.Min.: (1, -1)$$

$$Abs.Max.: \left(5, \frac{5}{3}\right)$$

EXAMPLE 9

Find the Absolute Extrema of $f(x) = \sin x + \cos x$ on the interval $[0, 2\pi]$

$$Abs.Min.: \left(\frac{5\pi}{4}, -\sqrt{2}\right)$$

$$Abs.Max.: \left(\frac{\pi}{4}, \sqrt{2}\right)$$

YOUR TURN

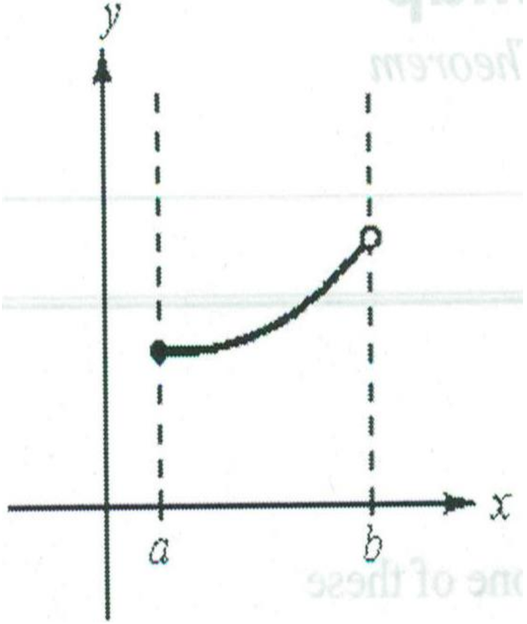
Find the Absolute Extrema of $f(x) = \cos^2 x + \sin x$ on the interval $[0, 2\pi]$

$$Abs.Min.:\left(\frac{3\pi}{2}, -1\right)$$

$$Abs.Max.:\left(\frac{5\pi}{6}\ or\ \frac{7\pi}{6}, \frac{5}{4}\right)$$

AP Multiple Choice Practice Question 1 (non-calculator)

Determine from the graph whether f possesses extrema on the interval $[a, b)$



(A) Maximum at $x = a$, minimum at $x = b$

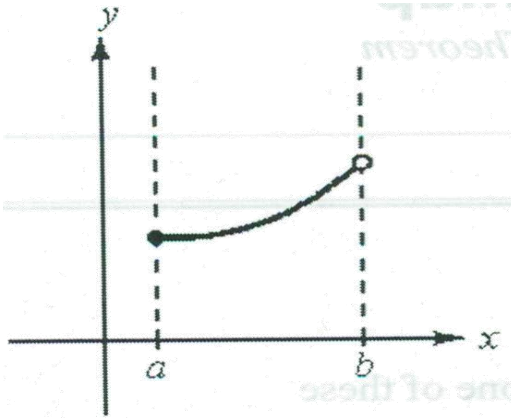
(B) Maximum at $x = b$, minimum at $x = a$

(C) No extrema

(D) No maximum, minimum at $x = a$

AP Multiple Choice Practice Question 1 (non-calculator)

Determine from the graph whether f possesses extrema on the interval $[a, b)$

Vocabulary	Connections and Process	Answer and Justifications
 EVT Relative Minimum Relative Maximum	$EVT:$ closed interval with relative min/max	D

Assignment

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1-27 odd, 33-43 odd, 45B