

§3.1: Extrema

“I WILL ...

...identify all critical values, absolute and relative extrema.”

I. Definitions

A. Relative Extrema vs Absolute Extrema

1. \_\_\_\_\_(Local) Extrema - occurs on an open interval
2. \_\_\_\_\_(Global) Extrema - occurs on a closed interval

A. If there is an open interval containing  $c$  on which is a minimum, then  $(c, f(c))$  is called a relative \_\_\_\_\_ of  $f$ , or you can say that  $f$  has a relative \_\_\_\_\_ at  $(c, f(c))$ . It is also called the local minimum. They are like \_\_\_\_\_ of the graphs.

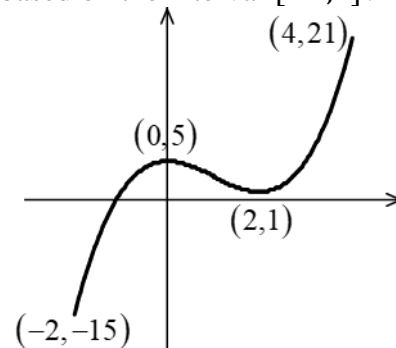
B. If there is an open interval containing  $c$  on which is a maximum, then  $(c, f(c))$  is called a relative \_\_\_\_\_ of  $f$ , or you can say that  $f$  has a relative \_\_\_\_\_ at  $(c, f(c))$ . It is also called the local maximum. They are like \_\_\_\_\_ of the graphs.

II. Extreme Value Theorem

A. Extreme Value Theorem is if  $f$  is \_\_\_\_\_ on a closed interval  $[a, b]$ , then  $f$  has both a \_\_\_\_\_  $f(c) \leq f(x)$  and a \_\_\_\_\_  $f(c) \geq f(x)$  on the interval.

1.  $f$  must be \_\_\_\_\_
2. Holes do not qualify for maximums or minimums

Ex 1: Where would the absolute and relative maxima be located on the graph, based on the interval  $[-2, 4]$ ?



II. Determine Absolute Extrema & Critical Numbers

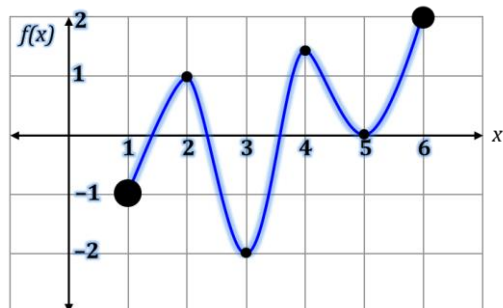
A. To Determine the Absolute Extrema of a Function given the equation:

1. Identify the \_\_\_\_\_
2. Check the \_\_\_\_\_

B. If  $f'(c) = 0$  (where top equals zero) or if  $f'(c) = DNE$  (where the bottom of the derivative equals zero) then  $c$  is a \_\_\_\_\_ of  $f$ .

1. Check where the derivative equals Zero (\_\_\_\_\_ or \_\_\_\_\_ on Graph)
2. Results in horizontal tangent lines
3. Therefore, a max or min and where the function is not differentiable

Ex 2: Establish the Critical Value and any absolute and relative extrema



Ex 3: Identify any Critical Point(s) of  $f(x) = x^3 + x^2 - 8x + 5$

Ex 4: Identify any Critical Numbers of  $f(x) = \frac{4x}{x^2-3}$

Ex 5: Identify any Critical Numbers of  $f(x) = x^{1/3}$

Your Turn: Identify any Critical Numbers of  $f(x) = (9 - x^2)^{3/5}$

### III. Candidates Test

- For closed intervals only, identify the critical numbers where \_\_\_\_\_ or \_\_\_\_\_
- Make a  $t$ -chart with  $x$  and  $f(x)$
- $X$ -candidates are the endpoints and Critical Numbers
- Find the associated  $y$ -Points and identify the max or min to the ORIGINAL equation
- Reminder
  - If the question is asking *WHAT* is the max or min value in the interval, LOOK AT THE \_\_\_\_\_ or \_\_\_\_\_-value
  - If the question is asking *WHERE* is the max or min value in the interval, LOOK AT THE \_\_\_\_\_-value

Ex 6: Identify the Absolute Extrema of  $f(x) = x^2 + 2x - 4$  on the interval  $[-1,1]$

Ex 7: Find the Absolute Extrema of  $f(x) = 3x^{2/3} - 2x$  on the interval  $[-1,1]$

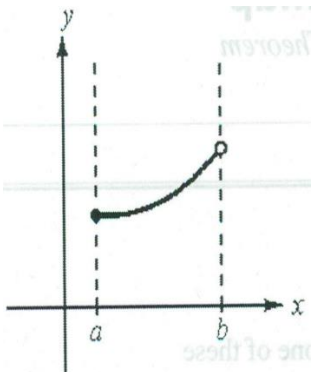
Ex 8: Identify the maximum value of  $f(x) = -x - e^{1-x}$  on the interval  $[0,3]$  and justify

Your Turn: Find the Absolute Extrema of  $f(t) = \frac{t}{t-2}$  on the interval  $[1, 5]$

Ex 9: Find the Absolute Extrema of  $f(x) = \sin x + \cos x$  on the interval  $[0, 2\pi]$

Your Turn: Find the Absolute Extrema of  $f(x) = \cos^2 x + \sin x$  on the interval  $[0, 2\pi]$

AP 1) Determine from the graph whether  $f$  possesses extrema on the interval  $[a, b]$



- (A) Maximum at  $x = a$ , minimum at  $x = b$
- (C) No extrema

- (B) Maximum at  $x = a$ , minimum at  $x = b$
- (D) No maximum, minimum at  $x = a$

Vocabulary	Connections and Process	Answer and Justifications