

§2.6: Related Rates

- A. **Read** and **reread** the problem.
- B. Draw a diagram, if necessary
- C. Express the given information and required rate in terms of derivatives and state your “**find**” and “**when.**”
- D. **Write an equation** that relates the various quantities of the problem. Use geometry of the situation to eliminate one of the variables if necessary.
- E. Substitute all known values and rates of change into the equations and solve. Make sure to **WRITE** out the indicated units of measure.
- F. Write the answer in a complete sentence with Verb, Answer, Noun, Units, and Time.

Complete this assignment on a separate sheet of paper. Show all steps (including drawing a picture) and work or no credit will be awarded. Justify all responses.

- 1) Each side of x of a square is increasing at a constant rate of 3 in/sec. How fast is the area increasing when $x = 5$?
- 2) The radius r of a balloon is increasing at the constant rate of 4 in/min. Find the time rate of change of the volume of the balloon at the instant when $r = 2$ in. $V = \frac{4}{3}\pi r^3$
- 3) A 20-foot ladder leans against a vertical wall. If the base of the ladder is pulled away from the wall at the constant rate of 4 ft/sec, how fast is the top of the ladder sliding down the wall when the base of the ladder is 12 feet from the wall?
- 4) A right-circular cone for which $h = r$ is used as a reservoir. If water is drawn off at the constant rate of 8 cubic feet/min, how fast is the water level decreasing when the water is 15 feet deep? $V = \frac{\pi}{3}r^2h$
- 5) Sand poured upon the ground at the constant rate of 12 cubic feet/min forms a conical pile whose altitude is $\frac{2}{3}$ of the radius of the base. How fast is the height of the pile increasing when the base radius is 6 feet?
 $V = \frac{\pi}{3}r^2h$
- 6) Water leaking on a floor creates a circular puddle whose area increases at a rate of 7 square inches per minute. How fast is the radius of the puddle increasing when the radius is 10 inches?
- 7) A bag is tied to the top of a 5-meter ladder resting against a vertical wall. Suppose the ladder begins sliding down the wall in such a way that the foot of the ladder is moving away from the wall. How fast is the bag descending at the instant the foot of the ladder is 4 m from the wall and the foot is moving away at the rate of 2 meters per second?

- 8) A ladder is 25 feet long and leaning against a vertical wall. The bottom of the ladder is pulled horizontally away from the wall at 3 feet per second. Determine how fast the ladder is sliding down the wall when the bottom is 15 feet from the wall.
- 9) Water is flowing into a tank in the form of an inverted cone having an altitude of 16 meters and a radius of 4 meters at the rate of 2 meters³/min. How fast is the radius changing at the instant the radius of the tank is 2.5 meters? $V = \frac{\pi}{3} r^2 h$
- 10) A weather balloon is rising vertically at a rate of 15 feet per second. An observer is standing on the ground (not moving) at 600 feet horizontally from the point where the balloon was released. When the balloon is 800 feet in the air, find the rate at which:
- (a) The distance from the balloon to the observer is changing
- (b) The angle of elevation from the observer to the balloon is changing
- 11) All edges of a cube are expanding at a rate of 2 cubic centimeters per second. How fast is the volume changing when each edge is 7 cm?
- 12) A 4-foot child is walking away from streetlight 16 feet high at a rate of 2 feet per second. At what rate is the length of the child's shadow increasing?
- 13) Car A is traveling due west at 50 miles/hour and Car B in the intersection of the roads. Both are heading for the intersection of the two roads. At what rate are the cars approaching each other when Car A is 0.3 miles and Car B is 0.4 miles from the intersection?

Key:

- 1) 30 in²/sec 2) 64π in³/min 3) Decreasing at 3 ft/sec
- 4) Decreasing at $8/(225\pi)$ ft/min 5) $1/(3\pi)$ ft/min 6) $7/(20\pi)$ in/min
- 7) Descending at $8/3$ meters/sec. 8) sliding down the wall at $9/4$ feet/sec
- 9) Radius is changing at a rate of $2/25\pi$ meters/min 10A) Rate of 12 ft./sec. 10B) $9/1000$ rad./sec.
- 11) Volume is increasing at a rate of 294 cm³/sec 12) $2/3$ ft/sec. 13) -78 miles/hour