§2.6: Related Rates
"I WILL…
… apply Implicit Differentiation to solve real-world problems"

I. Definitions

A. Related rates are found when there are two or more variables that all depend on another variable, usually time
B. Two or more quantities change as time changes
C. Since the variables are related to each other, the rates at which they change (their derivatives) are also related
D. Real-life problems rarely involve just a single variable. Most are written in terms of multiple variables. Related rate problems are real-life situations based on equations defined by rates of change. We can differentiate these problems using IMPLICIT DIFFERENTATION.
E. Remember to replace one variable before differentiating.

Formulas

1) Distance Formula	
2) Area of Triangle	
3) Volume of a Sphere	
4) Pythagorean Theorem	
5) Area of a Circle	
6) Volume of a Cube	
7) Circumference of a Circle	
8) Surface Area of a Sphere	
9) Volume of a Cylinder	
10) Volume of a Cone	

II. Steps

A. Sketch and label the diagram and make sure the given rates are increasing (+) or decreasing (–)
B. Write the words "_______________" along with what you are finding out (for AP testing)
C. Write all given information
D. Write the equation which pertains to the problem (i.e. Pythagorean, Distance, Surface Area, etc…)
E. Differentiate with respect to time
F. Substitute all known values
G. Solve for the desired quantity
H. LABEL with appropriate units!

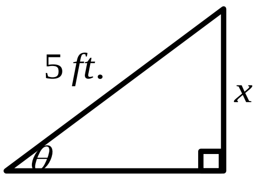
Ex 1: Solve $\frac{dy}{dt}$ at $x = 1$, given $y = x^2 + 3$ and $\frac{dx}{dt} = 2$ when $x = 1$	Your Turn: Solve $\frac{dy}{dt}$ at $x = 3$, given $y = 2(x^2 - 3x)$ and $\frac{dx}{dt} = 2$

Ex 2: Find $\frac{dV}{dt}$ of the volume of the cylinder.	Ex 3: A pebble is dropped into a calm pond, causing ripples in the shape of concentric circles. The radius of the outer ripple is increasing at a rate of 1 feet/sec. When the radius is 4 ft., find the rate at which the area disturbed water is changing.
Ex 4: The waves in a pond are circular. The radius increases at a rate of 2 ft./sec. When the radius is 5 ft., at what rate is the area changing?	Ex 5: Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

Your Turn: A balloon in the shape of a sphere is being blown up. The volume is increasing at the rate of 4 in^3/sec. At what rate is the radius increasing when the radius is exactly 1 in.?	Ex 6: Water is pouring into an inverted conical tank (right circular cone with height of 16 meters and base radius of 4 meters) at 2 cubic meters per minute. How fast is the water level rising when the water in the tank is 5 meters deep?
Ex 7: Water runs out of a conical tank at the constant rate of 2 cubic feet per minute. The radius at the top of the tank is 5 feet and the height of the tank is 10 feet. How fast is the water level sinking when the water is 4 feet deep?	Your Turn: A tank filled with water is in the shape of an inverted cone 20 feet high with a circular base (on top) whose radius is 5 feet. Water is running out of the bottom of the tank at a constant rate of 2 ft^3/min. How fast is the water level falling when the water is 8 ft. deep?

Ex 8: A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 in/sec. The volume is 128π in^3. At what rate is the length, h, changing when the radius is exactly 1.8 inches?	Ex 9: A 20 ft. ladder leans against a vertical building. If the bottom of the ladder slides away from the building at a rate of 2 $ft./sec$, how fast is the ladder sliding down the building when the ladder is exactly 12 feet above the ground?
Your Turn: A ladder 10 feet long rest against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 $ft./sec$, how fast is the top of the ladder sliding down at the wall when the bottom of the ladder is 6 ft. from the wall?	Ex 10: A balloon rises at a rate of 3 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

Ex 11: In the right-triangle shown, the angle θ is increasing at a constant rate of 2 radians per hour. At what rate is the side length of x increasing when $x = 4\ ft.$?



Your Turn: A balloon raises at the rate of 10 feet per second from a point on the ground 100 feet from an observer. Find the rate of change of the angle of elevation to the balloon from the observer when the balloon is 100 feet from the ground.

Ex 12: A boy 5 feet tall walks at the rate of 4 feet per second directly away from a street light which is 20 feet above the street. (a) At what rate is the tip of his shadow changing? (b) At what rate is the length of his shadow changing?

Ex 13: A flood light is on the ground 45 meters from a building. A thief 2 meters tall, runs from the flood light towards the building at 6 meters/sec. How rapidly is the length of the shadow on the building changing when he is 15 meters from the building?

AP 1) When the height of a cylinder is 12 cm and radius is 4 cm, the circumference of the cylinder is increasing at a rate of $\frac{\pi}{4}$ cm/min and the height of the cylinder is increasing four times faster than the radius. How fast is the volume of the cylinder changing? $V = \pi r^2 h$

(A) $\frac{\pi}{4}\ cm^3/min$ (B) $4\pi\ cm^3/min$ (C) $12\pi\ cm^3/min$ (D) $20\pi\ cm^3/min$

Vocabulary	Process and Connections	Answer and Justifications

Assignment: Worksheet