Calculus AP/D
Rev 2017

§2.5: Implicit Differentiation
"I WILL…
… Apply the derivative rules using Implicit Differentiation"

I. Definitions

A. Implicit Differentiation takes the derivative with both sides with respects to x in a relationship between x, y, and $\frac{dy}{dx}$; essentially an application of the CHAIN RULE
B. An IMPLICIT function is written to which one has to complete additional steps to isolate the variable. Functions written are not solved for y as it is not clearly stated but it is implied.

II. Examples include:

A. $y = \frac{x}{2}$ whereas it can be written implicitly as $xy = 1$
B. $x^2 - 2y^3 + 4y = 2$ can be written with implicit differentiation

II. Steps

A. Differentiate both sides of the equation with respect to x. MUST Include a $\frac{dy}{dx}$ or y' every time to differentiate with y
B. Collect all $\frac{dy}{dx}$ on LEFT side and all other "stuff" to the right side
C. Factor out $\frac{dy}{dx}$
D. Divide both sides by the extra "stuff"

Ex 1: Solve $\frac{dy}{dx}$ of $x^2 + y^2 = 25$	Ex 2: Solve $\frac{dy}{dx}$ of $2x^3 = 2y^2 + 5$
Ex 3: Solve $\frac{dy}{dx}$ of $3y^3 + y^2 - 2y + 4x^2 = 7$	Your Turn: Solve $\frac{dy}{dx}$ for $y^3 + y^2 - 5y - x^2 = -4$

Ex 4: Solve $\frac{dy}{dx}$ of $x^3y^3 - y = x$	Ex 5: Solve $\frac{dy}{dx}$ of $\sin y \cos x = 5x$
Ex 6: Solve $\frac{dy}{dx}$ of $\sin x = e^y$ in $(0, \pi)$ in terms of x.	Your Turn: Solve $\frac{dy}{dx}$ of $2 \sin x \cos y = 1$

Ex 7: Solve the second derivative of $x^2 + y^2 = 16$.	Ex 8: Solve the equation of the tangent line to the graph of $x^2 + y^2 = 25$ at the point $(3, -4)$.

Ex 9: Find the equation of the tangent line to the graph of $7x^2 + xy + 7y^2 = 15$ at the point $(1,1)$.	Your Turn: Solve the equation of the tangent line to the graph of $x^2 + 4y^2 = 4$ at the point $\left(\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$.

AP1) If $5x^2 - 2xy + 7y^2 = 0$, then solve $\frac{dy}{dx}$

(A) $\frac{5x+7y}{x}$ (B) $\frac{y-5x}{7y}$ (C) $\frac{y-5x}{7y-x}$ (D) $10x - 2y + 14y$

Vocabulary	Connections and Process	Answer and Justifications

AP2) If $\tan(x + y) = x$, then $\frac{dy}{dx} =$

(A) $\tan^2(x + y)$ (B) $\sec^2(x + y)$ (C) $\sin^2(x + y) - 1$ (D) $\cos^2(x + y) - 1$

Vocabulary	Connections and Process	Answer and Justifications

Assignment: Worksheet