§2.2: Basic Derivatives and Derivative Data
"I WILL…
… Apply a derivative using the Constant and Power Rule"

I. Definitions

A. The derivative abbreviation is $\frac{d}{dx}$ or known as "the derivative of *d* with respect of *x*"

B. The Constant Rule: $\frac{d}{dx}[c] =$ ____________________

C. The Power Rule: $\frac{d}{dx}\left[x^n\right] =$ ____________________

D. The Constant Multiple Rule: $\frac{d}{dx}[cf(x)] =$ ____________________

E. The Sum and Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] =$ ____________________

II. Abbreviations:

A. y'

B. $\frac{dy}{dx}$

C. $\frac{d}{dx}[f(x)]$

D. $f'(x)$

E. INSTANTENOUS RATE OF CHANGE

Ex 1: Determine the derivative of $f(x) = 3$.	Ex 2: Determine the derivative of $f(x) = 2x - 3$
Your Turn: Determine the derivative of $f(x) = 3x^4 - 2x^3 + 7x - 9$	

III. Additional Hints

A. When dealing with the equations, ____________________

B. Simplify the expression

C. Revert to the form where exponents are positive

Ex 3: Determine the derivative of $f(x) = \frac{1}{x^2}$	Ex 4: Determine the derivative of $f(x) = \frac{2}{x} + \sqrt[3]{x}$
Your Turn: Determine the derivative of $f(t) = 3\sqrt[5]{t^3}$	Ex 5: Solve the derivative of $\lim_{h \to 0} = \frac{(x+h)^7 - 3(x+h)^5 - (x^7 - 3x^5)}{h}$

Your Turn: Solve for the original equation the derivative of a number in $\lim_{h\to 0} = \frac{\sqrt{9+h}-\sqrt{9}}{h}$	Ex 6: Let $f(x) = 5x^3 - 2x + 1$. Find the equation of the tangent line to $f(x)$ at $x = 1$.
Ex 7: Find the points on the graph of $f(x) = x^4 - 6x^2 + 4$ where the tangent line is horizontal.	Your Turn: Given $f(x) = 2x^3 - x^2 + 2$. Find the tangent line at the point $(1,3)$ and find where the tangent line is horizontal.

IV. Definitions

A. Main Trig Derivatives

1. $\frac{d}{dx}[\sin x] =$ ___________________________________ *Think of $\sin x$ to ______________ SIGN*
2. $\frac{d}{dx}[\cos x] =$ ___________________________________ *Think of $\cos x$ to ______________ SIGN*

B. Sum and Difference Rule

1. $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Ex 8: Solve the derivative of $f(x) = 2\sin x$	Ex 9: Solve the derivative of $\lim_{h\to 0} = \frac{\cos\left(\frac{\pi}{3}+h\right) - \cos\left(\frac{\pi}{3}\right)}{h}$
Ex 10: Identify slope at the particular point, $(\pi, -1)$ for the $g(x) = 2 + 3\cos t$	Your Turn: Solve the derivative of $f(x) = 3\sin x - 4\cos x + 6x^3$

III. Average Velocity

A. Rates of change play a role whenever we study the relationship between two changing quantities. A familiar example is velocity, which is the rate of change of position with respect to time.
B. If an object is traveling in a straight line, the average velocity over a given time interval from point A to point B
C. We cannot define instantaneous velocity as a ratio because we would have to divide by the length of the time interval, which is zero.

IV. Average Rate of Change

A. The average rate of change describes the constant rate at which a function would have to change over an interval if the function were to achieve the same vertical displacement over the length of the interval investigated.
B. Average Velocity Formula: $\bar{x} = \frac{\Delta s}{\Delta t} = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$

V. Steps

A. Plug a and b in to get the corresponding $f(a)$ and $f(b)$
B. Plug into the formula and reduce.
C. Average Rate of Change uses only endpoints of the interval. It does not reflect any fluctuations within the interval.
D. LABELS count!

Ex 11: A car travels along a straight road and the driver observes the mile markers along the trip as shown in the table. What are the units for the average speed over the 24 minutes? What is the average of the car in 24 minutes?

Time	0	8	20	24
Mile Marker	14	27	42	50

Ex 12: Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Using the table below, estimate $R'(2)$ and show all work that leads to the answer.

t (hours)	0	1	3	6	8
$R(t)$ (Liters/Hr)	1340	1190	950	740	700

Ex 13: A tennis ball is dropped from a height of 100 feet, its height s at time t is given by the position function, $d(t) = -16t^2 + 100$ where d is measured in feet and t is measured in seconds. Find the average velocity over the time interval of $[1,2]$.	Ex 14: If an object is dropped from a tall building, then the distance it has fallen after t seconds is given by the function $d(t) = -16t^2$. Find its average speed (average rate of change) between a and $a + h$ seconds.

Your Turn: Suppose you are driving to a friend's house. After 30 minutes, you are 15 miles from your house. After 3 hours, you are 60 miles from your house. What is your average rate of change between these times (in hours)? Label accordingly.

IV. Instantaneous Velocity

A. Velocity of an object at time t is found by taking the DERIVATIVE of the position function.

B. ____________ is a vector, which means it has direction. So, ____________ can be positive, negative, or zero.

C. ____________ is the absolute value of ____________, and cannot be negative.

D. Instantaneous Velocity Formula: $\lim_{\Delta t \to 0} \frac{s(t+\Delta t)-s(t)}{\Delta t} = s'(t) = v(t)$. It is also known as $\frac{ds}{dt}$.

Ex 15: At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$ where s is measured in feet and t is measured in seconds.

(a) Graph the function on the graphing calculator and sketch the result below.

(b) Find the average velocity (average rate of change) at which the diver is moving for the time interval $[1, 1.5]$.

(c) Use your answers to (b) to estimate the instantaneous rate of change (instantaneous velocity) at which the diver is moving at time $t = 1$ second.

(d) When does the diver hit the water?

(e) What is the diver's velocity at impact? What is their speed?

(f) When does the diver stop moving upward and start their descent?

Ex 16: The temperature T, in degrees Fahrenheit, of a cold potato placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the potato was put in the oven. What is the meaning of the statement $f'(20) = 2$?

AP 1)

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval, $[2,13]$. Estimate $f'(4)$. Show the work that leads to the answer.

AP 2)

Ex. Water is flowing into a tank over a 24-hour period. The amount of water $W(t)$ in the tank at various times is measured, and the results are given in the table below, where $W(t)$ is measured in gallons and t is measured in hours. There are 150 gallons of water in the tank at $t = 0$.

t (hours)	0	4	8	12	16	20	24
$W(t)$ (gallons)	150	184	221	257	294	327	357

(a) Use data from the table to find $W(8)$. Using appropriate units, explain the meaning of your answer.

(b) Use data from the table to find $W^{-1}(257)$. Using appropriate units, explain the meaning of your answer.

(c) Use data from the table to find an approximation for $W'(15)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

Assignment: Page 114: 3-51 EOO, 53A, 55A, 57-65 odd, 93-97 all, 105